Name:	
Class:	

YEAR 12

ASSESSMENT TEST 3
TERM 2, 2012

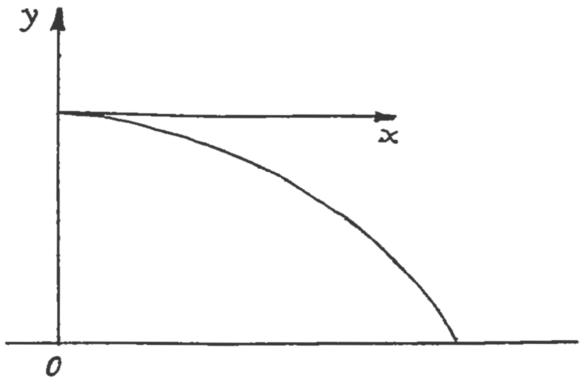
MATHEMATICS
EXTENSION 1

Time Allowed *– 90 Minutes*
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators are permitted
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

Question 1 (9 Marks) **START A NEW PAGE** **Marks**



a) A cat sitting on the top of a wall 2.45 metres high, sees a bird on the ground 4.2 metres from the base of the wall. The cat launches itself horizontally from the top of the wall with an initial velocity of 5m/s. Take the acceleration due to gravity as 10m/s^2 and air resistance is ignored.

(i) Find how long the cat takes to reach the ground. **2**

(ii) Show that the cat misses the bird. **1**

(iii) If the cat is to land on the bird; re-calculate the initial velocity. Hence, find the speed (to the nearest m/s) and angle (to the nearest minute) to the horizontal that the cat has on landing on the bird. **4**

b) After the introduction of myxomatosis in Australia, earlier last century the rabbit population declined rapidly at first. Eventually the rabbits developed resistance and the rabbit population started to increase again. **2**
If R is the rabbit population, describe what happens to the population in terms of $\frac{dR}{dt}$ and $\frac{d^2R}{dt^2}$ over the time since myxomatosis was introduced.

Question 2 (9 Marks) **START A NEW PAGE** **Marks**

a) For a body falling under gravity, in air, the rate of change of velocity is given by:
$\frac{dv}{dt} = -k(v - c)$ where c and k are constants and air resistance is ignored.

(i) Show that $v = C + Ae^{-kt}$ is a solution of the above equation, where A is a constant. **1**

(ii) If the body starts from rest, given $C = 1000$ and after 5 seconds its velocity is 30m/s find A and k. **2**

Question 2 continues on the next page.

Marks

(iii) Find the velocity after 20 seconds (to 1 decimal place). **1**

(iv) Find the maximum velocity as t approaches infinity. Justify your answer. **2**

(v) Sketch the rate, $\frac{dv}{dt}$ against the velocity, v. **1**

b) The letters of the word GLENELG are arranged at random in a straight line. What is the probability that the sequence reads the same from right to left as from left to right? **2**

Question 3 (9 Marks) **START A NEW PAGE**

a) The acceleration of a particle, a ms^{-2} moving in a straight line is given as $a = 2x - 3$. The initial displacement is 4 m to the right of the origin and velocity is zero.

(i) Find an expression for the velocity of the particle. **3**

(ii) Does the particle pass through the origin? Justify your answer. **1**

(iii) Find the displacement of the particle when the velocity is 10 ms^{-1}. Justify your answer. **2**

b) A particle is moving with S.H.M. When it is at a distance d from the centre, its speed is V. If its speed is $\frac{V}{2}$ when the distance from the centre is $2d$, show that the period of the motion is $\frac{4\pi d}{V}$ and the amplitude is $d\sqrt{5}$. **3**

Question 4 (9 Marks) **START A NEW PAGE** **Marks**

a) The farewell committee of eight is to be formed from a selection of 10 boys and 12 girls.

(i) Find the number of ways of selecting 3 boys and 5 girls for the committee. **1**

(ii) Find the number of ways of selecting the committee with the majority of members being girls, but there must be at least one boy. **2**

(iii) Sharon and Sidney are two of the twenty-two present. To be fair, it is decided that there should be an equal number of boys and girls on the committee. Everyone has voted that Sharon should be on the committee of 8 students. Sharon accepts, but does not want Sidney on the committee. Find the number of ways in which the committee can be formed with Sharon on the committee. **1**

Question 4 continues on the next page.

Marks

b) For a safe passage a ship needs 9.5 metres of water. At low tide, which occurs at noon, it is 6 metres and at high tide, which occurs at 6:40pm, it is 10.5 metres. Assume that the surface of the water moves with Simple Harmonic Motion.

(i) Show that the mean depth is 8.25 metres. **1**

(ii) Given $x = b + a\cos(nt + \alpha)$, where x is the height (in metres) of the water level from the mean position, where $0 \leq \alpha \leq 2\pi$. Find an expression for x. **2**

(iii) Hence, find the earliest time t, in hours, to two decimal places, at which the ship can pass the point safely. **2**

Question 5 (9 Marks) **START A NEW PAGE**

a) There are 4 suits in a deck of playing cards; consisting of 13 red diamonds, 13 black clubs, 13 black spades and 13 red hearts cards. A hand of 5 cards is dealt out at random.

(i) Find the probability that at least 2 cards are diamonds. **2**

(ii) Find the probability that the hand of 5 cards includes every suit. **2**

(iii) Find the probability of obtaining exactly 4 black cards, if it is know that at least 2 are black. **3**

b) Four boys and five girls are to be seated around a circular table. A particular boy X does not want to sit next to any of the girls and a particular girl Y does not want to sit next to any of the boys. How many such permutations are possible? **2**

Question 6 (9 Marks) **START A NEW PAGE**

a) Five letters are chosen from the letters of the word WRITING. These five letters are then placed alongside one another to form a five-letter arrangement. Find the number of distinct five-letter arrangements which are possible. **3**

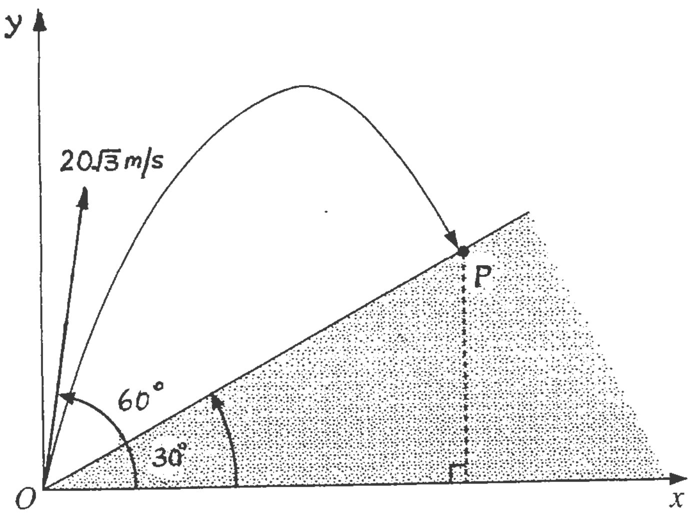
b) A particle is released from rest $\frac{5\pi}{3}$ metres to the left of the origin and travels in a straight line. Its velocity is given by $v^2 = 3 - 6\cos x$.

(i) Find the acceleration and the direction in which the particle first moves. Explain your answer. **2**

(ii) Where is the particle stationary again? **3**

(iii) Does the particle exhibit Simple Harmonic Motion? **1**

Question 7 (9 Marks) **START A NEW PAGE** **Marks**



The diagram shows an inclined plane that makes an angle of 30° with the horizontal. A projectile is fired from O, at the bottom of the incline, with a speed of $20\sqrt{3}$ m/s at an angle of elevation of 60° to the horizontal as shown and given $\ddot{x} = 0$ and $x = 10\sqrt{3}t$. Assume negligible air resistance and let $g = 10$.

(i) Derive the vertical equations of motion for this projectile. **2**

(ii) Hence, show that the path of the projectile is $y = x\sqrt{3} - \frac{x^2}{60}$ **1**

(iii) Find the range of the projectile, OP metres, up the incline. **3**

(iv) Determine the time of flight. **1**

(v) Given that the range, in part (iii), is the maximum range up the incline for the trajectory of the particle; show that the initial direction is perpendicular to the direction in which the projectile hits the inclined plane. **2**

The End

2012 HSC Trn 2

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments

a) Equations of Motion

$\ddot{x} = 0$ $\qquad$ $\ddot{y} = -10$

$\dot{x} = 5$ $\qquad$ $\dot{y} = -10t$

$x = 5t$ $\qquad$ $y = -5t^2 + 2.45$

$-\frac{1}{2}$ mark if you did not mention $t > 0$

i) The cat reaches the ground when $y = 0$

ie $-5t^2 + 2.45 = 0$

$5t^2 = 2.45$

$t^2 = 0.49$

$t = 0.7$ as $t > 0$

Thus it takes 0.7 sec to reach the ground. **2.**

ii) when $t = 0.7$

$x = 5 \times 0.7$

$x = 3.5$

The bird is 4.2 m from the wall, thus the cat misses the bird if it lands 3.5 m from the wall. **1**

Alternatively – find time.

$x = 5t$

$4.2 = 5t$

$t = 0.84$ sec

– different from $t = 0.7$ s $\therefore$ miss.

iii) For the cat to land on the bird $x = 4.2$.

Since $x = Vt$ where V is the initial velocity

$4.2 = V \times 0.7$

$V = 6$

$\therefore$ to catch the bird, the cat's initial velocity would be 6 m/s ✓ 1 mark

no unit $-\frac{1}{2}$ mark.

Now $\dot{x} = 6$ and $\dot{y} = -10 \times 0.7 = -7$

$V^2 = 6^2 + (-7)^2$

$= 36 + 49$

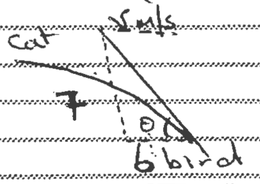
$= 85$

$V = \sqrt{85} = 9.2195 = 9$ (to nearest whole no) ✓ 1 mark.

no rounding $-\frac{1}{2}$ mark

no unit $-\frac{1}{2}$ mark

Let θ be the acute angle.



MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments

$\tan\theta = \frac{7}{6}$

$\theta = 49.3987$

$\theta = 49^\circ 24'$ (to the nearest minute)

1 mark for $49^\circ 24'$

Angle $= 180^\circ - 49^\circ 24' = 130^\circ 36'$

$\frac{1}{2}$ mark for rounding to nearest minute

Hence the cat will land with a speed of 9 m/s at an angle of $130^\circ 36'$ to the horizontal

$\frac{1}{2}$ mark for getting $130^\circ 36'$

b) After the introduction of myxomatosis.

$\frac{dR}{dt} < 0$ ✓

dropping at a decreasing rate $\left(\frac{d^2R}{dt^2} > 0\right)$ ✓

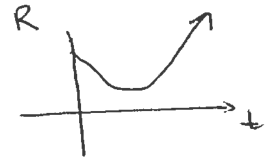
then once they developed resistance

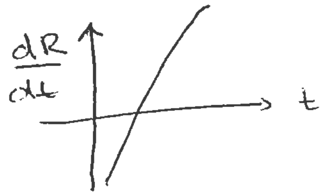
$\frac{dR}{dt} > 0$ ✓

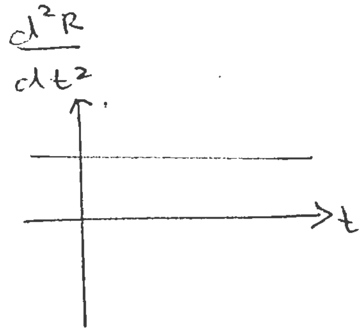
increasing at an increasing rate $\left(\frac{d^2R}{dt^2} > 0\right)$

Note

$\frac{d^2R}{dt^2}$ will alway be a postive constant as the rate at which the change in population is always positive



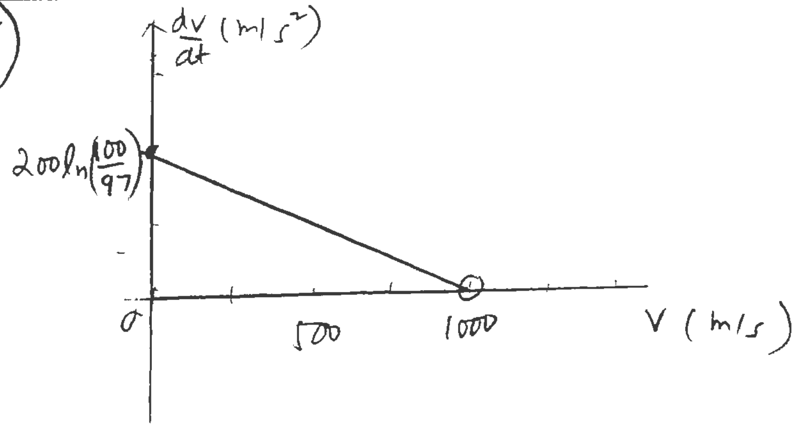




Yr 12 T2 2012 Ext 1 **MATHEMATICS: Question** 2

Suggested Solutions	Marks	Marker's Comments
a i) $V = C + Ae^{-kt}$		
$\frac{dv}{dt} = -k(Ae^{-kt})$		forget
$\frac{dv}{dt} = -k(V-c)$ since $V - c = Ae^{-kt}$	1	$V - c = Ae^{-kt}$ $-\frac{1}{2}m$
ii) $t=0$, $V=0$, $C=1000$		
$0 = 1000 + A\cancel{e^{-0}}1$		
$A = -1000$ #	1	No half mark
$30 = 1000 - 1000e^{-5k}$		for A and k.
$k = -\frac{1}{5}\ln(0.97)$ or $\frac{1}{5}\ln\left(\frac{100}{97}\right)$	1	
$k \doteqdot 0.006092$		
iii) $t = 20$ $V = 1000 - 1000e^{-20\left[\frac{\ln(0.97)}{-5}\right]}$		
$V = 1000(1 - e^{4\times\ln 0.97})$		some forget to
$= 114.7071\ldots$		show $114.7071\ldots$
$\therefore$ velocity is 114.7 m/s (1 d.p.)	1	$-\frac{1}{2}m$
iv) As $k \doteqdot 0.006092 > 0$		
$e^{-kt} \to 0$ as $t \to \infty$	1	
$1000e^{-kt} \to 0$ as $t \to \infty$	1	
$\therefore$ max velocity is 1000 m/s		
Alternatively		
$-\frac{1}{5}\ln\left(\frac{100}{97}\right) < 0$	or	
$\therefore e^{-\frac{t}{5}\ln\left(\frac{100}{97}\right)} \to 0$ as $t \to \infty$	1	
$\therefore$ max velocity is 1000 m/s	1	

Yr 12 T2 2012 Ext 1 **MATHEMATICS: Question** 2

Suggested Solutions	Marks	Marker's Comments
v) 	1	y-intercept is $200\ln\frac{100}{97} \doteqdot 6.09$ Some think $200\ln\frac{100}{97} \doteqdot -6.09$ x-intercept is 1000 1 m many draw with correct intercepts non-straight line graph in 1st quadrant max $\frac{1}{2}$m.
b) GLENELG : 2G, 2E, 2L, 1N } 7 letters No restrictions, # ways to arrange GLENELG $= \frac{7!}{2!\,2!\,2!} = 630$ GLE can be arranged in 3! ways (ie 6 ways) $\therefore$ Prob $= \frac{6}{630} = \frac{1}{105}$ #	1 1	The probability Question is well done. 630 1 m (denominator) $3! = 6$ 1 m (numerator) Some write 3! only $\frac{1}{2}$m

MATHEMATICS Extension 1 : Question 3 $(x=4)$ p1/3

Suggested Solutions	Marks	Marker's Comment
a) i) $v\frac{dv}{dx} = 2x-3$		
$\int v\,dv = \int 2x-3\,dx$		
$\frac{v^2}{2} = x^2-3x+k$	1	
But, when $x=4$, $v=0$		
$\therefore 0 = 16-12+k$		
$k=-4$		
$\therefore v^2 = 2x^2-6x-8$	1	Surprising num[ber] of people stop[ped] here.
$\therefore v = \pm\sqrt{2x^2-6x-8}$		
$= \pm\sqrt{2(x-4)(x+1)}$		
Motion is only possible for $x \geq 4$ or $x \leq -1$. In this case, motion starts at $x=4$ so only $x \geq 4$ is possible	1	
Particle moves to right from $x=4$ ($acc^n = 5$), thus v is always positive.		
$\therefore v = \sqrt{2(x-4)(x+1)}$		
ii) As shown above motion only occurs for $x \geq 4$. Velocity is always positive. It never passes the origin.	1	
iii) When $v=10$, $100 = 2x^2-6x-8$		
$2x^2-6x-108=0$		
$x^2-3x-54=0$		
$(x-9)(x+6)=0$ $x=9$ or -6	1	
But $x \geq 4$ only so displacement is 9 m to right of origin.	1	

Suggested Solutions	Marks	Marker's Comment
a) i) $v\frac{dv}{dx} = 2x-3$		
$\int v\,dv = \int 2x-3\,dx$		
$\frac{v^2}{2} = x^2-3x+k$	1	
When $x=0.04$, $v=0$		
$\therefore 0 = 0.0016-0.12+k$		generally more lenient as numbers messy.
$k = 0.1184$		
$\therefore v^2 = 2x^2-6x+0.2368$	1	
$v = \pm\sqrt{2x^2-6x+0.2368}$		
$= \pm\sqrt{2(x-0.04)(x-2.96)}$		
Motion only possible for $x \le 0.04$ or $x \ge 2.96$. In this case, motion starts at $x=0.04$ so only $x \le 0.04$ is possible.		
Particle moves to left from $x=0.04$ ($acc^n = -2.92$), thus v is always negative.	1	
$v = -\sqrt{2(x-0.04)(x-2.96)}$		
ii) As shown above, motion only occurs for $x < 0.04$. Velocity is negative and increasing indefinitely in size. Thus it does pass through the origin.	1	
iii) As shown above, velocity is always negative, so it never has velocity $+10$ m/s.	2	For people who came this way I allowed the answer $x=-5$ which is strictly where $v=-10$
[By squaring it can be shown that $\lvert v\rvert = 10$ when $x = -5.72$]		

Suggested Solutions	Marks	Marker's Comments

b) Exhibiting SHM $\Rightarrow$ $\ddot{x} = -n^2 x$

$$\frac{d}{dx}\left(\frac{v^2}{2}\right) = -n^2 x$$

$$\therefore \frac{v^2}{2} = -\frac{n^2 x^2}{2} + k$$

When $v = 0$, $x = a$, the amplitude.

$$\therefore k = \frac{n^2 a^2}{2}$$

$$\therefore v^2 = n^2(a^2 - x^2)$$

When $x = d$, $v = V$ $\quad V^2 = n^2(a^2 - d^2)$ — ①

$x = 2d$, $v = V/2$ $\quad \frac{V^2}{4} = n^2(a^2 - 4d^2)$ — ②

① − ② $\Rightarrow$ $\frac{3V^2}{4} = n^2(-d^2 + 4d^2)$

$$\frac{3V^2}{4} = n^2 3d^2$$

$$n^2 = \frac{V^2}{4d^2}$$

$$n = \frac{V}{2d} \quad (n > 0)$$

Period $= \frac{2\pi}{n} = 2\pi \div \frac{V}{2d}$

$$= \frac{4\pi d}{V}$$

① ÷ ② $\Rightarrow$ $4 = \frac{a^2 - d^2}{a^2 - 4d^2}$

$$4a^2 - 16d^2 = a^2 - d^2$$

$$3a^2 = 15d^2$$

$$a^2 = 5d^2$$

$$a = d\sqrt{5} \quad (a > 0)$$

$\therefore$ Amplitude is $d\sqrt{5}$.

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
(a) 10 boys choose 3 and 12 girls choose 5		${}^{10}C_3 + {}^{12}C_5 =$ ½ mk.
$\therefore {}^{10}C_3 \times {}^{12}C_5 = 95\,040$	1 mk	
(ii) Possibilities: (3B, 5G) or (2B, 6G) or (1B, 7G)	½	* If you did it as a probability – max of one providing full working was shown.
$= 95040 + ({}^{10}C_2 \times {}^{12}C_6) + ({}^{10}C_1 \times {}^{12}C_7)$	½	
$= 95040 + 41580 + 7920$ $= 144\,540$	1	
(iii) equal Boys & Girls; Sharon on Sidney off.		(If you assumed Sidney was a girl the ${}^{10}C_3 \times {}^{10}C_4 = 25\,200$)
${}^{9}C_4 \times {}^{11}C_3$	½	
$\therefore = 20\,790$	½	
(b) (i) $\dfrac{10.5 + 6}{2} = 8.25$	1	you HAD to show the numbers/working to get the mark
(ii) GIVEN $x = b + a\cos(nt + \alpha)$		alternatively
$a = 2.25$ but starts to low tide $\therefore a = -2.25$ $b = 8.25$ (centre of motion)	½	$a = 2.25$ $b = 8.25$ $n = \frac{3\pi}{20}$
when $t = 0$, $x = 6$ $\therefore 6 = 8.25 - 2.25\cos(\alpha)$ $\cos\alpha = 1$ $\alpha = 0$ for $0 \leq \alpha \leq 2\pi$	½	when $t = 0$, $x = 6$ $6 = 8.25 + 2.25\cos\alpha$ $\cos\alpha = -1$ $\therefore \alpha = \pi$
period $= 2 \times 6\frac{2}{3}$ $= 13\frac{1}{3}$ hours (800 minutes)	½	
$T = \frac{2\pi}{n}$ $\frac{40}{3} = \frac{2\pi}{n}$ $\rightarrow n = \frac{3\pi}{20}$. $\left(\frac{\pi}{400}\right)$	½	$\therefore x = 8.25 + 2.25\cos\left(\frac{3\pi}{20}t + \right.$
$\therefore x = 8.25 - 2.25\cos\left(\frac{3\pi}{20}t\right)$		
(iii) when $x = 9.5$		
$9.5 = 8.25 - 2.25\cos\left(\frac{3\pi}{20}t\right)$	½	
$-\frac{5}{9} = \cos\left(\frac{3\pi}{20}t\right)$		
$\cos^{-1}\left(-\frac{5}{9}\right) = \frac{3\pi}{20}t$	½	

MATHEMATICS Extension 1 : Question……..

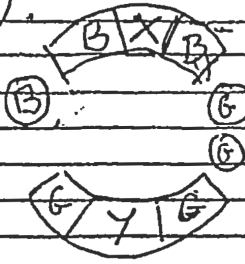
Suggested Solutions	Marks	Marker's Comm
$= \pm 2.159827296$		
$\frac{3\pi}{20}t = 2.159827296$ as $t > 0$	$\frac{1}{2}$	* deducted $\frac{1}{2}$ they started x=9 instead
$t = 4.583295872$		
$t = 4.58$ hours	$\frac{1}{2}$	
O.C.		
$9.5 = 8.25 + 2.25\cos\left(\frac{3\pi}{20}t + \pi\right)$		
$\frac{5}{9} = \cos\left(\frac{3\pi}{20}t + \pi\right)$		$\alpha = 2m\pi \pm \cos^{-1}$
$\cos^{-1}\left(\frac{5}{9}\right) = \frac{3\pi}{20}t + \pi$		where m is a
$\therefore \frac{3\pi}{20}t + \pi = \pm 0.9817653566 + 2m\pi$		
when $m=0$,		
$\frac{3\pi}{20}t + \pi = -2.159827297$		* most stud who did it way got $t = 8.75$
but $t > 0$ $\therefore$ disregard		
when $m=1$,		
$\frac{3\pi}{20}t + \pi = 5.301419951$		
$\frac{3\pi}{20}t = 2.159827297$		
$t = 4.583295874$		
$\therefore t = 4.58$ hours		

Suggested Solutions	Marks	Marker's Comments
(i) P(at least 2 diamonds) $= 1 - (P(\text{no diamonds}) + P(1\text{ diamond}))$ $= 1 - \left[\frac{^{39}C_5}{^{52}C_5} + \frac{^{39}C_4 \times {}^{13}C_1}{^{52}C_5}\right]$ $= 1 - \left[\frac{575757 + 1069263}{2598960}\right]$ $= 1 - \frac{1645020}{2598960}$ $= \frac{953940}{2598960} = \frac{1223}{3332}$	2	($\frac{1}{2}$) method; ($\frac{1}{2}$) $^{52}C_5$; ($\frac{1}{2}$) numerator; ($\frac{1}{2}$) numerical values.
Alternatively $P = \frac{^{13}C_2 \times {}^{39}C_3 + {}^{13}C_3 \times {}^{39}C_2 + {}^{13}C_4 \times {}^{39}C_1 + {}^{13}C_5}{^{52}C_5}$ $= \frac{953940}{2598960} = \frac{1223}{3332}$		
Alternatively $P = 1 - \left[\frac{39 \times 38 \times 37 \times 36 \times 35}{52 \times 51 \times 50 \times 49 \times 48} + \frac{13 \times 39 \times 38 \times 37 \times 36 \times 5}{52\ 51\ 50\ 49\ 48}\right]$ $= 1 - \frac{2109}{3332} = \frac{1223}{3332}$		Note $\boxed{\times 5}$
(ii) P(1 card of every suit) $= \left[{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_2 \times 4\right] / {}^{52}C_5$ $= \frac{685464}{2598960} = \frac{2197}{8330}$	2	($\frac{1}{2}$) $({}^{13}C_1)^3$; ($\frac{1}{2}$) $^{52}C_5$; ($\frac{1}{2}$) correct an; ($\frac{1}{2}$) numerical calculatio
Alternatively $P = \frac{13 \times 13 \times 13 \times 13 \times 12 \times 4 \times 5!}{52 \times 51 \times 50 \times 49 \times 48 \times 2}$ $= \frac{2197}{8330}$.		
Alternatively D – D – D – S; D – D – S – D; D – D – S – D – D; D – S – D – D – D. D = different; S = same as one chosen previously		*note $^{13}C_1 \times {}^{48}C_1$ gi repeated ga
$\frac{52}{52} \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49} \times \frac{48}{48} + \frac{52}{52} \times \frac{39}{51} \times \frac{26}{50} \times \frac{36}{49} \times \frac{13}{48} +$	$\frac{52}{52} \times \frac{39}{51} \times \frac{24}{50} \times \frac{26}{49} \times \frac{13}{48}$	

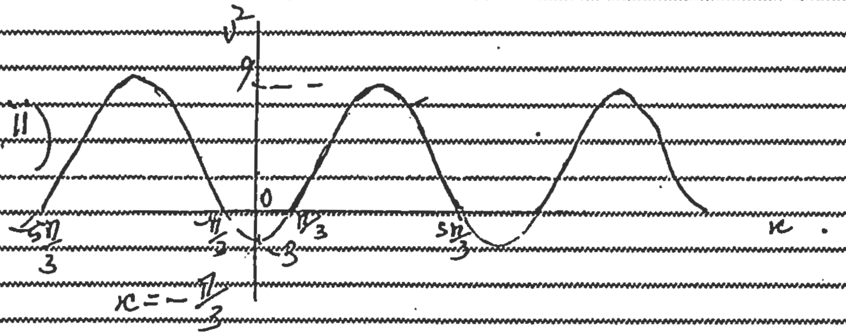
$\frac{52}{52} \times \frac{12}{51} \times \frac{39}{50} \times \frac{26}{49} \times \frac{13}{48} = \frac{219}{83}$

$= \boxed{\frac{2197}{833}}$

MATHEMATICS Extension 1 : Question 5

Suggested Solutions	Marks	Marker's Comments
(iii) P(Exactly 4 black given 2 cards are black) Number of card hands with at least 2 black cards $= {}^{52}C_5 - ({}^{26}C_5 + {}^{26}C_4 \times {}^{26}C_1)$ no black / 4 red. 1 black / 4 red. or ${}^{26}C_2 \times {}^{26}C_3 + {}^{26}C_3 \times {}^{26}C_2 + {}^{26}C_4 \times {}^{26}C_1 + {}^{26}C_5$ $= 2144480$	2	There are 26 black cards not 13. ${}^{50}C_3 \neq 2144480$. ${}^{26}C_2 \times {}^{50}C_3 \neq 2144480$
Number of hands with 4 black cards ${}^{26}C_4 \times {}^{26}C_1$ (4 black / 1 red). $= 388700$	1	${}^{26}C_4 \neq {}^{26}C_2 \times {}^{24}C_2$. as ${}^{26}C_2 \times {}^{24}C_2$ contains repeated groups.
P(4 black / at least 2 black). $= \frac{388700}{2144480} = \frac{1495}{8248}$		$(-\frac{1}{2})$ if not written as a numerical fraction
b)  Place Boy X at top of table. $= 1$ way Place 2 boys either side of X $= {}^{3}C_2 \times 2!$ ways Place 2 girls either side of girl Y $= {}^{4}C_2 \times 2!$ Arrange remaining 2 girls, 1 boy and girl group in 4! ways. Total $= {}^{3}C_2 \times 2 \times {}^{4}C_2 \times 2 \times 4!$ $= 1728$ Other explanations possible	2	1 mark for $\boxed{BXB}$ and $\boxed{GYG}$ ${}^{3}C_2 \times 2 \times {}^{4}C_2 \times 2$. $\frac{1}{2}$ for 4! arrangements $\frac{1}{2}$ answer. Explanation required for cfe marks.

MATHEMATICS Extension 1 : Question 6

Suggested Solutions	Marks	Marker's Comments
c) WRITING		
All different {W R I T N G} $\ {}^6C_5 \times 5! = 720$	2	Some people broke this into 2 steps
2 I {W R T N G} $\ {}^5C_3 \times \frac{5!}{2!} = 600$.	1	80% success.
TOTAL 1320		
d) (i) $v^2 = 3 - 6\cos x$	1	to clearly state $\ddot{x} = \frac{d}{dx}\left(\frac{v^2}{2}\right)$
$\ddot{x} = \frac{d}{dx}\left(\frac{v^2}{2}\right)$		
$= \frac{d}{dx}\frac{1}{2}(3 - 6\cos x)$		
$\ddot{x} = 3\sin x$		
At $x = -\frac{5\pi}{3}$ $\ddot{x} = \frac{3\sqrt{3}}{2} > 0$ Particle moves right	1	50% students chose $x = \frac{5\pi}{3}$ No marks. — students could not evaluate $\ddot{x}$ poor effort.
ii) 	2	Graph v^2 against x
	1	Deduce from particular branch student had extremely poor understanding of this Question
NB $\cos x = \frac{1}{2}$ General soln $x = 2n\pi \pm \frac{\pi}{3}$ (1) $n \in \mathbb{Z}$		
iii) $\ddot{x} \neq -n^2(x - a)$ $\therefore$ NOT SHM	1	— NO one graphed v^2 against x to identify branches of motion — Few understood particle had to stay in that branch. — Similar to $\dot{x}^2 = (x-1)(x-2)(x$

Y12 M. Ext 1 T2 ASSESSMENT TASK 3. 2012.

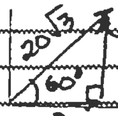
MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments

(i) Given $t=0$, $\angle\alpha = 60^\circ$, $x=0$, $V = 20\sqrt{3}$, $y=0$

$x = 10t\sqrt{3}$

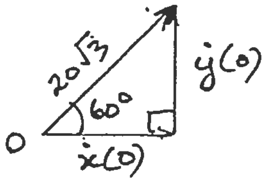
$\therefore \dot{x} = 10\sqrt{3}$



$\underline{t=0}$

$\sin 60^\circ = \dfrac{\dot{y}}{20\sqrt{3}}$

$\dot{y} = 20\sqrt{3} \times \dfrac{\sqrt{3}}{2} = 30$



$\frac{1}{2}$ For showing why $\dot{y}(0) = 3$

VERTICAL: $m\ddot{y} = -mg = -10m$

$\ddot{y} = -10$

$\dot{y} = \int -10\,dt = -10t + c_1$ — $\frac{1}{2}$

$t=0$, $\dot{y} = 30$ $\therefore 30 = c_1$

$\therefore \dot{y} = -10t + 30$

$y = \int (30 - 10t)\,dt$

$y = 30t - 5t^2 + c_2$ — $\frac{1}{2}$

$t=0$, $y=0$ so c_2

$\therefore y = 30t - 5t^2$ — $\frac{1}{2}$

$\boxed{2}$

(ii) As $x = 10t\sqrt{3}$

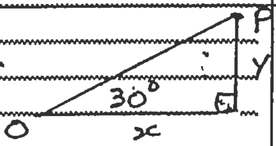
$\therefore t = \dfrac{x}{10\sqrt{3}}$

so $y = 30 \times \dfrac{x}{10\sqrt{3}} - 5\times\left(\dfrac{x}{10\sqrt{3}}\right)^2$ — $\frac{1}{2}$

$y = \dfrac{3}{\sqrt{3}}x - \dfrac{5x^2}{300} = x\sqrt{3} - \dfrac{x^2}{60}$ QED — $\frac{1}{2}$

$\boxed{1}$

(iii) RTF the max length OP.



METHOD 1 : $P(x, y)$

$\cos 30^\circ = \dfrac{x}{OP} \Rightarrow x = \dfrac{\sqrt{3}}{2}OP$ — $\frac{1}{2}$

$\sin 30^\circ = \dfrac{y}{OP} \Rightarrow y = \frac{1}{2}OP$ — $\frac{1}{2}$

So $\frac{1}{2}OP = \dfrac{\sqrt{3}}{2}OP \times \sqrt{3} - \dfrac{1}{60}\times\dfrac{3}{4}OP^2$ — $\frac{1}{2}$

$\frac{1}{2} = \frac{3}{2} - \frac{1}{80}OP$, $OP \neq 0$ — $\frac{1}{2}$

$\dfrac{OP}{80} = 1$

$\therefore OP = 80\text{ m}$

Range(max) $= 80\text{ m}$

METHOD 2:

Eqn of OP: $m_{OP} = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\therefore y = \dfrac{1}{\sqrt{3}}x$ — $\frac{1}{2}$

so $x = y\sqrt{3}$

so $y = y\sqrt{3}\times\sqrt{3} - \dfrac{3y^2}{60}$

$y = 3y - \dfrac{y^2}{20}$

$\dfrac{y^2}{20} = 2y$

$y = 40$. $(y \neq 0)$

$\therefore 40 = \dfrac{OP}{2}$

$\therefore OP = 80\text{ m}$

$\boxed{3}$

Suggested Solutions	Marks	Marker's Comments

CONT (iii)

METHOD 3:

$y = \frac{x}{\sqrt{3}}$ $\frac{1}{2}$ $\therefore 40\sqrt{3} = \frac{\sqrt{3}}{2} OP$

$\therefore \underline{\underline{OP = 80m}}$ 1

$\therefore \frac{x}{\sqrt{3}} = x\sqrt{3} - \frac{x^2}{60}$

$\underline{OR}$ $y = \frac{40\sqrt{3}}{\sqrt{3}} = 40$ $(\frac{1}{2})$

$x = 3x - \frac{x^2\sqrt{3}}{60}$ $\frac{1}{2}$

$\frac{x^2\sqrt{3}}{60} = 2x$ $\therefore OP^2 = (40\sqrt{3})^2 + 40^2$ (Pyth. Thm)

$x\sqrt{3} = 120$ $(x \neq 0)$ $OP^2 = 40^2 \times 3 + 40^2 = 40^2 \times 4$

$x = \frac{120}{\sqrt{3}} = \underline{40\sqrt{3}}$ 1 $OP = 40 \times 2 = \underline{80m}$. 1

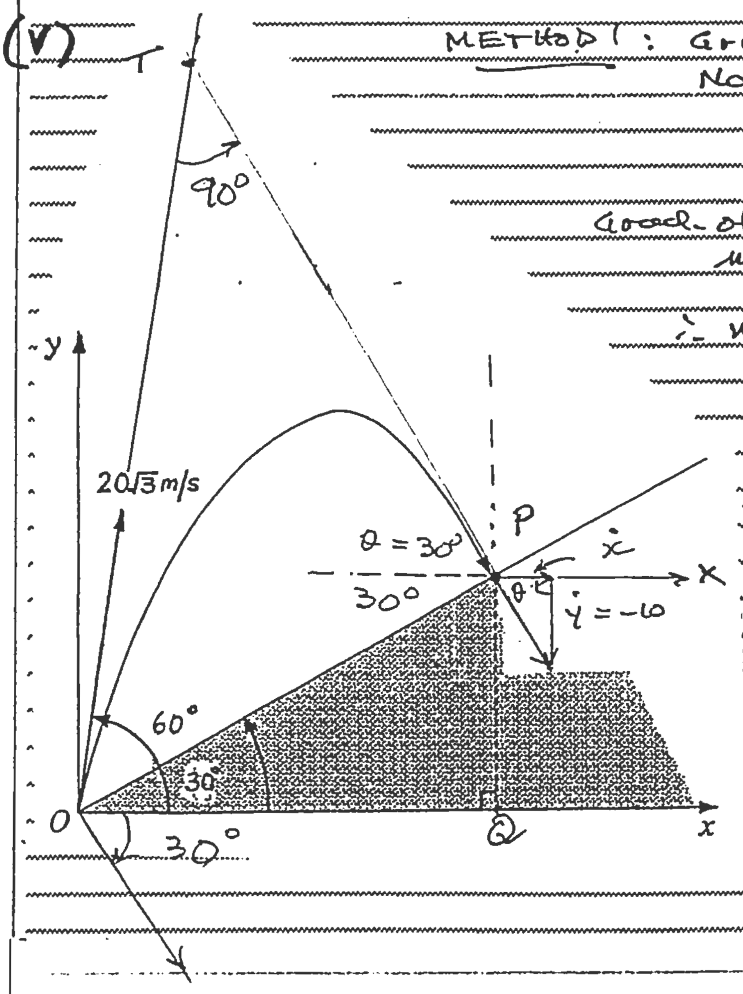
(iv) Time of flight to P

$x = 10t\sqrt{3} = 40\sqrt{3}$ horizontal $\frac{1}{2}$

$\therefore t = 4$

$\therefore$ time of flight is $\underline{4 \text{ seconds}}$ $\frac{1}{2}$ $\boxed{1}$

(v)



$\underline{\text{METHOD 1}}$: Gradient at O $m_O = \tan 60° = \sqrt{3}$

Now $y = x\sqrt{3} - \frac{x^2}{60}$

$\frac{dy}{dx} = \sqrt{3} - \frac{x}{30}$ $(\frac{1}{2})$

Grad. of tangent at $x = 40\sqrt{3}$ $(t = 4)$

$m_P = \sqrt{3} - \frac{40\sqrt{3}}{30} = -\frac{1}{\sqrt{3}}$ $(\frac{1}{2})$

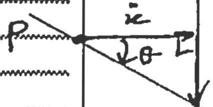
$\therefore m_O \times m_P = \sqrt{3} \times (-\frac{1}{\sqrt{3}}) = -1$ $(\frac{1}{2})$

$\therefore$ initial direction is at $90°$ to direction at impact point

$\underline{\text{METHOD 2}}$: at P $t = 4$ $\boxed{2}$

$\dot{x} = 10\sqrt{3}$

$\dot{y} = 30 - 10 \times 4 = -10$ $(\frac{1}{2})$



magnitude of θ: $\tan\theta = \left|\frac{\dot{y}}{\dot{x}}\right| = \left|\frac{-10}{10\sqrt{3}}\right|$

$\therefore \angle\theta = 30°$

$30°$ to the horizontal directed below "-3

$\angle XPT = 150°$

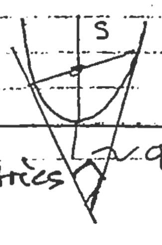
A clear description using a diagram to show that they are at $90°$ (

JUSTIFICATION

NOTE

Y11

Parametrics



OP is a FOCAL CHORD !!