Number:	
Class:	

YEAR 12

ASSESSMENT TEST 3
TERM 2, 2014

MATHEMATICS
EXTENSION 1

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper
- This is an open book test, any printed or hand written materials are allowed must be placed on desk

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc. Each question must show your Candidate Number.

ATTEMPT ALL 7 QUESTIONS (70 marks)

QUESTION 1 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

a) Five cards numbered 1, 2, 3, 4, 5 are arranged at random.

(i) How many arrangements are there? **1**

(ii) What is the probability that the arrangement finishes with a 24? **2**

(iii) What is the probability that the last number in the arrangement is odd? **2**

(b) The equation of motion of a particle moving along the x-axis is $x = 2\sin(t + \frac{\pi}{6})$, where x is in metres and t in seconds.

(i) Draw the displacement-time graph for $0 \le t \le \frac{11\pi}{6}$. **2**

(ii) When does the particle first change direction and where is it at this time? **2**

(iii) Find the distance travelled by the particle in the first $\frac{4\pi}{3}$ seconds. **1**

QUESTION 2 (10 Marks) **COMMENCE A NEW PAGE**

(a) A particle moves in a straight line so that its acceleration as a function of displacement is given by $\frac{d^2x}{dt^2} = 1 - 4x$. Initially $x = 1.25$ cm and $v = 0$.

(i) Show that the motion is simple harmonic. **1**

(ii) Find the amplitude, period and centre of motion. **3**

(iii) Find the velocity when $t = 0.2$ seconds. **2**

(iv) Describe briefly what would have happened if the motion had commenced at $x = 0.25$ with $v = 0$. **1**

(b) A container with capacity A litres is being filled with water. After t minutes the volume V litres of water in the container is given by $V = A(1 - e^{-kt})$ for some constant $k > 0$. **3**

If one quarter of the container is filled in the first two minutes, find what fraction of the container is filled in the next two minutes.

QUESTION 3 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

(a) During a chemical reaction the amount, A grams, of a substance unconverted after t hours is given by the formula $A = 4 + e^{-3kt}$.

(i) Show that $\frac{dA}{dt} = -3k(A-4)$. **1**

(ii) If initially A decreases at the rate of 0.04 grams per hour, find the value of k. **2**

(iii) Using k found in part (ii), sketch the graph of $A = 4 + e^{-3kt}$ and indicate the values that A can take. **3**

(b) Eight people attend a meeting. They are provided with two circular tables, one seating 3 people, the other 5 people.

(i) How many seating arrangements are possible? **2**

(ii) If the seating is done randomly, what is the probability that a particular couple are on different tables? **2**

QUESTION 4 (10 Marks) **COMMENCE A NEW PAGE**

(a) A container of water, heated to 100^0C, is placed in a coolroom where the temperature is maintained at -5^0C. After t minutes, the rate of change of the temperature, $T^0\ C$, of the water is given by $\frac{dT}{dt} = -k(T+5)$, where k is a constant.

(i) Assuming the function $T = Ae^{-kt} - 5$, where A is a constant, is a solution to the above differential equation, find the value of A. **1**

(ii) After 20 minutes, the water temperature falls to 30^0C. Find, to the nearest degree, the water temperature after a further 10 minutes. **2**

(iii) Find, to the nearest minute, the time the water will need to be in the coolroom before its temperature reaches 0^0C. **2**

(b) A small cube has one red face, two blue faces and three green faces. It is rolled three times.

(i) Find the number of distinct probability values of the possible outcomes. **1**

(ii) Find the probability that only one colour appeared in three rolls of the cube. **2**

(iii) Find the probability that exactly one blue face appeared in three rolls of the cube. **2**

QUESTION 5 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

(a) Two bags each contain three marbles, one red and two blue. A marble is is drawn at random from the first bag and placed in the second. A marble is is the drawn from the second and placed in the first. **2**

Find the probability that each bag still contains one red and two blue marbles.

(b) The letters of the word PERSEVERE are randomly arranged in a row. Find the probability that the E's are together and the R's are together? **2**

(c) A particle vibrates in Simple Harmonic Motion, making 100 oscillations per second.

(i) Show that the acceleration is given by $\ddot{x} = -40\,000\pi^2 x$. **2**

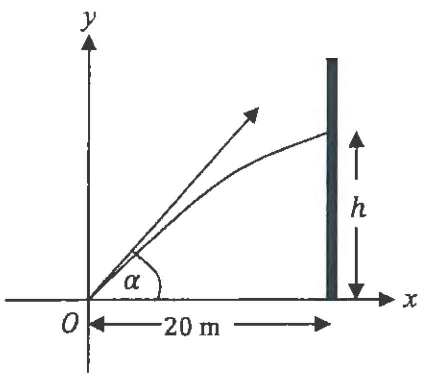
(ii) If the amplitude of the motion is 20 cm, calculate the speed of the particle at

α) the centre of its motion. **3**

β) the extremities of its motion. **1**

QUESTION 6 (10 Marks) **COMMENCE A NEW PAGE**

(a) A garden hose releases a stream of water with a velocity of 20 m s^{-1} at an angle of α to the horizontal. The water streams toward a high wall 20 m away on level ground.



Given $x = 20t\cos\alpha$ and $y = 20t\sin\alpha - 5t^2$,
where x and y are the horizontal and vertical displacements of the stream of water from O at any time t, $g = 10$ m s^{-2} and the coordinate axes are taken as shown.

(i) Find the equation of the path of the stream of water in terms of x, y and α. **1**

(ii) Show that the height h at which the water hits the wall is given by **2**

$$h = 20\tan\alpha - 5(1 + \tan^2\alpha).$$

(iii) Using part (ii), show that the maximum value of h occurs when $\tan\alpha = 2$ and find this maximum height. **2**

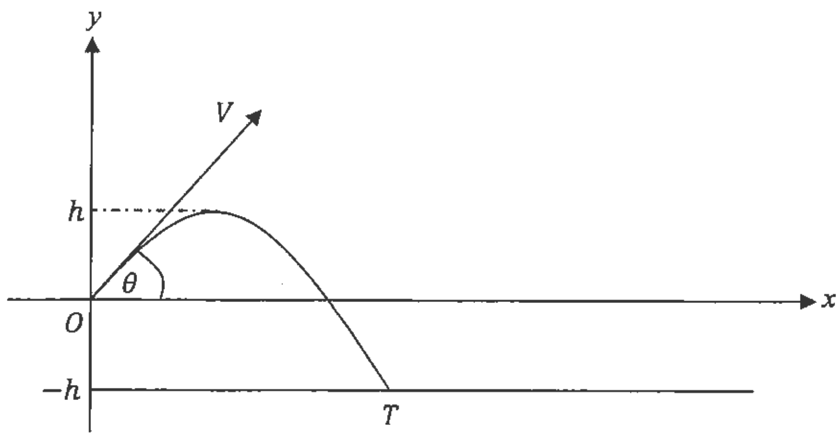
(b) A particle moves in a straight line so that after t seconds its velocity is v m s^{-1} and its displacement is x.

(i) Given that $\frac{d^2x}{dt^2} = 10x - 2x^3$ and that $v = 0$ when $x = -1$, find v in terms of x. **3**

(ii) Explain why the motion cannot exist between $x = -1$ and $x = 1$. **2**

QUESTION 7 (10 Marks) **COMMENCE A NEW PAGE**

The diagram below shows the path of a projectile fired from the top of a cliff, O. Its initial velocity is V m s^{-1}, its initial angle of elevation is θ and it rises to a maximum height h metres above O. It strikes a target T situated on a horizontal plane h metres below O.



a) Given that $\ddot{y} = -g$ and $\ddot{x} = 0$, derive equations for y and x as functions of time. **2**

b) Prove that $h = \dfrac{V^2 \sin^2 \theta}{2g}$, where g is acceleration due to gravity. **3**

c) Prove that the time taken for the projectile to reach its target is **3**

$$\frac{V(1+\sqrt{2})\sin\theta}{g} \text{ seconds.}$$

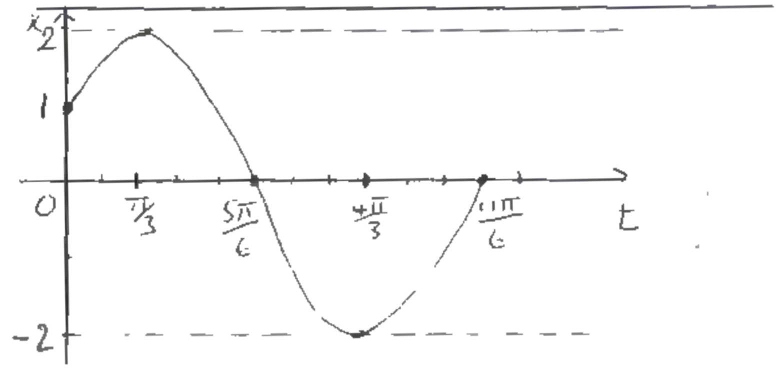
d) Show that the distance from the target to the base of the cliff is **2**

$$\frac{V^2(1+\sqrt{2})\sin 2\theta}{2g} \text{ metres.}$$

☺ **END OF EXAMINATION** ☺

TERM 2 2014

X I **MATHEMATICS: Question** ...1...

Suggested Solutions	Marks	Marker's Comments
a) i) First card chosen in 5 ways Second " " " 4 ways ⋮ Last " " " 1 way $5!$ ways $= \underline{\underline{120}}$ ways	1	$5!$ was not enough.
ii) Last 2 cards already placed Others can be placed in $3! = 6$ ways $\therefore p(E) = \frac{3!}{5!} = \frac{6}{120} = \underline{\underline{\frac{1}{20}}}$	1 1	
iii) Last number is odd in 3 ways out of 5. $\therefore p(E) = \underline{\underline{3/5}}$	2.	Some people found 72 ways of doing this
b) i) 	2	This diagram shows the least minimum for 2 marks. Any omissions [illegible] lost 1 mark.
ii) From the graph, first direction change is when $\underline{t = \pi/3}$ sec. and $\underline{x = 2 \text{ m}}$.	1 1	
iii) Between 0 and $\pi/3$ travels 1 (+ve) $\pi/3$ and $4\pi/3$ travels 4 (−ve) total distance $= \underline{\underline{5 \text{ metres}}}$	1	

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
a) (i) $\frac{d^2x}{dt^2} = 1-4x$ $= -4(x-\frac{1}{4})$ $= -2^2(x-\frac{1}{4})$ Since of form $\ddot{x} = n^2 X$ where $X = (x - x_0)$ it is in SHM	1	need to make a clear statement as to why it's in SHM.
(ii) From (i) centre is $\frac{1}{4}$	1	
(ii) period $= \frac{2\pi}{2}\left(\frac{2\pi}{n}\right)$ $= \pi$	1	
amplitude since extremity is 1.25 and centre 0.25 amplitude $= 1.25 - 0.25$ $= 1$	1	Too many students used $v^2 = n^2(a^2-x^2)$ which only works when centre is origin
Alternative not as elegant $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 1-4x$ $\frac{1}{2}v^2 = x - 2x^2 + c_1$ $v^2 = 2x - 4x^2 + c_2$ at $x = 1.25$ $v = 0$ $0 = 2\times(1.25) - 4\times(1.25)^2 + c_2$ $\therefore c_2 = 3.75$		

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
$v^2 = 2x - 4x^2 + 3.75$ $v = 0$ $0 = 4x^2 - 2x - 3.75$ $0 = 16x^2 - 8x - 15$ $(4x+3)(4x-5) = 0$ $x = -\frac{3}{4}$ or $\frac{5}{4}$ $\therefore$ amplitude $\frac{1}{2} \times \left(\frac{5}{4} - -\frac{3}{4}\right)$ $= 1$		Students using this method made algebraic errors.
(iii) Assume general equation $x = b + a\cos(nt + \alpha)$ $b = \frac{1}{4}$ when $t = 0$ $x = 1.25$ $a = 1$ $n = 2$ $1.25 = \frac{1}{4} + \cos(2t + \alpha)$ $1 = \cos\alpha$ $\therefore \alpha = 0$ $x = \frac{1}{4} + \cos(2t)$ or $\left(x = \frac{1}{4} + \sin\left(2t + \frac{\pi}{2}\right)\right)$	1	Some students found this but substituted in $v^2 = n^2(a^2 - x^2)$ rather than just differentiating

MATHEMATICS Extension 1: Question 2

Suggested Solutions	Marks	Marker's Comments
$\frac{dx}{dt} = -2\sin 2t$		
when $t = 0.2$		• need to evaluate $\sin(0.4)$ in radians
$v = -2\sin(0.4)$		• students substituted $t=0.2$ into $v^2 = n^2(a^2 - x^2)$
$= -0.78\,m/s.$		
(b) $V = A - Ae^{-kt}$		
$\frac{A}{4} = A - Ae^{-2k}$		
$\frac{1}{4} = 1 - e^{-2k}$.		Some students used $V = \frac{1}{4}$ OR $V = \frac{V}{4}$ which made question difficult.
$-2k = \ln\left(\frac{3}{4}\right)$		
$k = \frac{1}{2}\ln\left(\frac{4}{3}\right)$	1	
$V = A\left(1 - e^{-\frac{1}{2}\ln\left(\frac{4}{3}\right)4}\right)$		
$V = A\left(1 - e^{-2\ln\left(\frac{4}{3}\right)}\right)$		
$V = A\left(1 - e^{\ln\frac{9}{16}}\right)$		
$= A\left(1 - \frac{9}{16}\right)$		
$= \frac{7}{16}A$	1	
In next 2 minutes		Many students failed to answer question asked
$\frac{7}{16}A - \frac{1}{4}A = \frac{3}{16}A$		
so $\frac{3}{16}$ will be filled in next 2 minutes	1	

3-unit Mathematics

Question 3

(i) show that $\frac{dA}{dt} = -3k(A-4)$

$A = 4 + e^{-3kt}$

$\frac{dA}{dt} = -3k\,e^{-3kt}$

since $A = 4 + e^{-3kt}$

$e^{-3kt} = A-4$

$\therefore \frac{dA}{dt} = -3k(A-4)$

1

(1)

Alternatively

$\frac{dA}{dt} = -3k(e^{-3kt})$

$= -3k(4+e^{-3kt}-4)$

$= -3k(A-4)$

(ii) Find the value of k if initially A decreases at a rate of 0.04 g/hr.

when $t=0$, $\frac{dA}{dt} = -0.04$.

$\frac{dA}{dt} = -3k\,e^{-3k(0)}$

$-0.04 = -3k\,e^{0}$

$-0.04 = -3k$

$\therefore k = \frac{-0.04}{-3}$

$k = \frac{1}{75}$

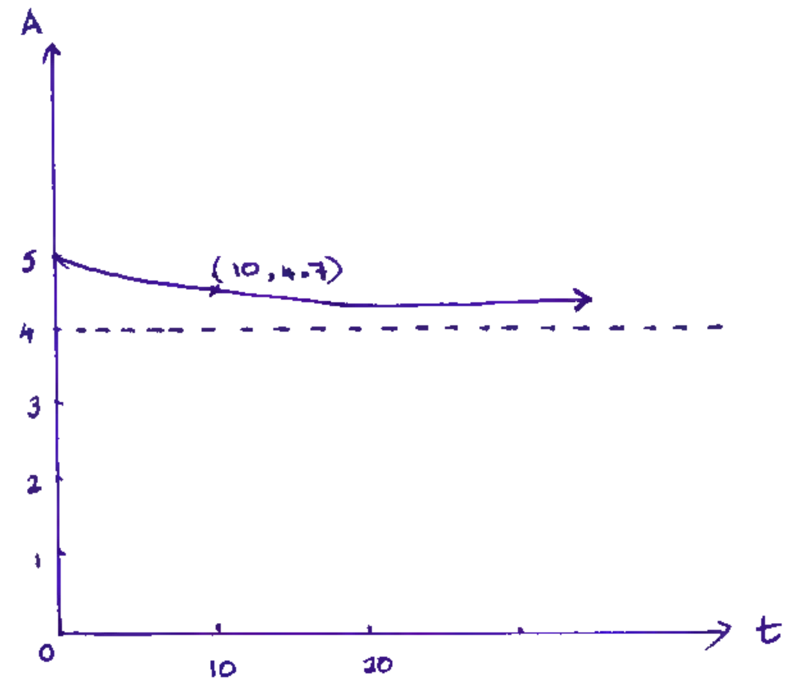
1

1

(1) If used $\frac{dA}{dt}$ as 0.04 then got $\frac{-1}{75}$ as answer (got only 1 mark).

(2) If used $\frac{dA}{dt} = -3k(A-4)$ then had to calculate A as 5.

(iii)



Other points mentioned were (3, 4.89), (200, 4.0) (5, 4.82) (17.3, 4.5) - accepted (25, $4+\frac{1}{e}$)

3

1 mark for y-intercept

1 mark for asymptote $y=4$

1 mark for shape and must show one other point apart from y-intercept

Note

Took away marks if curve was touching the asymptote.

b) (i) Number of seating arrangements possible

$${}^{8}C_{3} \times (3-1)! \times {}^{5}C_{5} \times (5-1)!$$

choose 3 from 1st table | arranging 3 on first table | choosing remaining 5 on table 2 | arranging 5 on second table

$$= \frac{8!}{5!3!} \times 2! \times 1 \times 4!$$

$= 2688$ ways

OR

$${}^{8}C_{5} \times 4! \times {}^{3}C_{3} \times 2! = 2688 \text{ ways}$$

2

②

Some did

- split 3 people and 5 people as

$${}^{8}C_{3} \times {}^{5}C_{5} = 56$$

and arranging around table as

$$2! \times 4! = 48$$

$\therefore$ Total ways $= 56 \times 48$
$= 2688$

✓1 mark for choosing 3 or 5 on a table

ie ${}^{8}C_{3} \times {}^{5}C_{5}$

or

${}^{8}C_{5} \times {}^{3}C_{3}$

✓1 mark arranging 3 or 5 on a table

ie $2! \times 4!$

Some got:

* ${}^{8}C_{3} \times 2! \times {}^{5}C_{4} \times 4! = 13440$

No need as choosing 5 people on 2nd table is fixed

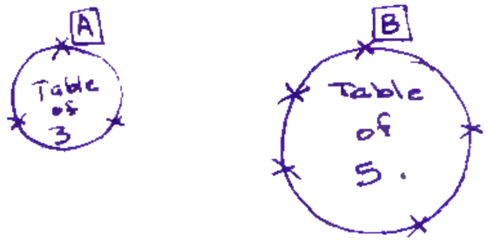
* 40320 means you did ${}^{8}C_{5} \times {}^{3}C_{3} \times 3! \times 5!$

* $5040 = 7!$

* $8! = 40320$

b(ii) P(particular couple are on different tables)

say A are B are couples



2 are already seated ie A and B

Now we have 6 to choose from.

$^6C_2 \times (3-1)! \times (5-1)! \times 2$.

select 2 more to sit on 1st table

arranging seating small table

arranging seating on large table

swap couple

= 1440 ways.

$\therefore$ P(couple on different tables) = $\frac{1440}{2688}$

$= \frac{15}{28}$

Method II

P(couples on different tables) = 1 − P(couple on same table)

$= 1 - \left({}^6C_3 \times {}^3C_3 \times 4!\,2! + {}^4C_5 \times {}^1C_1 \times 4! \times 2!\right) \div 2688$

Couple together on table of 5

Couple together on table of 3

$= 1 - \frac{1248}{2688}$

$= 1 - \frac{13}{28}$

$= \frac{15}{28}$

2

(3)

Different ways.

A on table 3

$^6C_2 \times 2! \times 4! = 720$

A on table of 5

$^6C_4 \times 2! \times 4! = 720$

Case 1 A on table 3 and B on table 5

$^6C_2 \times 2! \times {}^4C_4 \times 4!$

Case 2 B on table of 3 and A on table of 5

$^6C_2 \times 2! \times {}^4C_4 \times 4!$

$^2C_1 \times {}^6C_2 \times 2! \times 4!$

choose which table each person will sit in

chose 2 on table of 3

ways of arranging on table 3

ways of arranging on table 5

Note: if got $\frac{15}{56}$ means you did not swap the seating ie ×2

Method III

Case 1
Couple on same table of 3

$${}^{6}C_{1} \times 2! \times {}^{5}C_{5} \times 4! = 288$$

Case 2 couple on same table of 5.

$${}^{6}C_{3} \times 4! \times {}^{3}C_{3} \times 2! = 960$$

$$\therefore p\left(\begin{matrix}\text{couple on}\\ \text{different}\\ \text{tables}\end{matrix}\right) = 1 - p\left(\begin{matrix}\text{couples on}\\ \text{same table}\end{matrix}\right)$$

$$= \quad - \frac{1248}{2688}$$

$$= \frac{1440}{2688}$$

$$= \frac{15}{28}$$

3U **MATHEMATICS: Question** ..4..

Suggested Solutions	Marks	Marker's Comments
(i) (1) $T = Ae^{-kt} - 5$ When $T = 100$ $t = 0$ $100 = Ae^{0} - 5$ $\therefore A = 105$	1 mk	Two common careless errors ① Forgot the -5 ② Subtracted 5 from 100 & got $A = 95$.
(ii) When $t = 20$ $T = 30$ $30 = 105e^{-20k} - 5$ $35 = 105e^{-20k}$ $-20k = \ln\frac{35}{105}$ $k = -\frac{1}{20}\ln\frac{35}{105}$ $= -\frac{1}{20}\ln\frac{1}{3}$ OR $\frac{\ln 3}{20}$	1 mk	Many students confused t and T in the substitution
When $t = 30$ $T = 105e^{-30k} - 5$ $= 105e^{-30(\frac{\ln 3}{20})} - 5$ $= 15.20725942$ $= \underline{15^{\circ}}$ (nearest degree)	1 mk	Common error - forgot to subtract the 5 Mark not deducted for not rounding.
(iii) When $T = 0$ $0 = 105e^{-kt} - 5$ $e^{-kt} = \frac{5}{105}$ $e^{-kt} = \frac{1}{21}$ $-kt = \ln\frac{1}{21}$ $t = -\frac{1}{k}\ln\frac{1}{21}$ OR $\frac{\ln 21}{k}$	1 mk	
$t = \frac{\ln\frac{1}{21}}{\frac{1}{20}\ln\frac{1}{3}}$ OR $\frac{20\ln 21}{\ln 3}$ $= 55.42487498$ $= 55$ min (nearest min)	1 mk	Note: When $t = 55$ $T = 0.12$ $t = 56$ $T = 0.155$

However because the temperature is dropping it would not ~~pass~~ reach 0° until 56 min.

3U **MATHEMATICS: Question** 4

Suggested Solutions	Marks	Marker's Comments

(b)(i) $RRR = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$, $RRB = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{3}$, $RRG = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{2}$

$BBR = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{6}$, $BBB = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}$, $BBG = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}$

$GGR = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6}$, $GGB = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}$, $GGG = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$RBG = \frac{1}{6}\cdot\frac{1}{3}\cdot\frac{1}{2}$

There are <u>10</u> distinct probability values — 1mk

A common incorrect answer was 27.
Note: There are 27 different combinations but not distinct.
eg. $BBR = RBB = \frac{1}{54}$
$GGB = GBG = \frac{1}{12}$ etc

(ii) $R(RRR) + P(BBB) + P(GGG)$

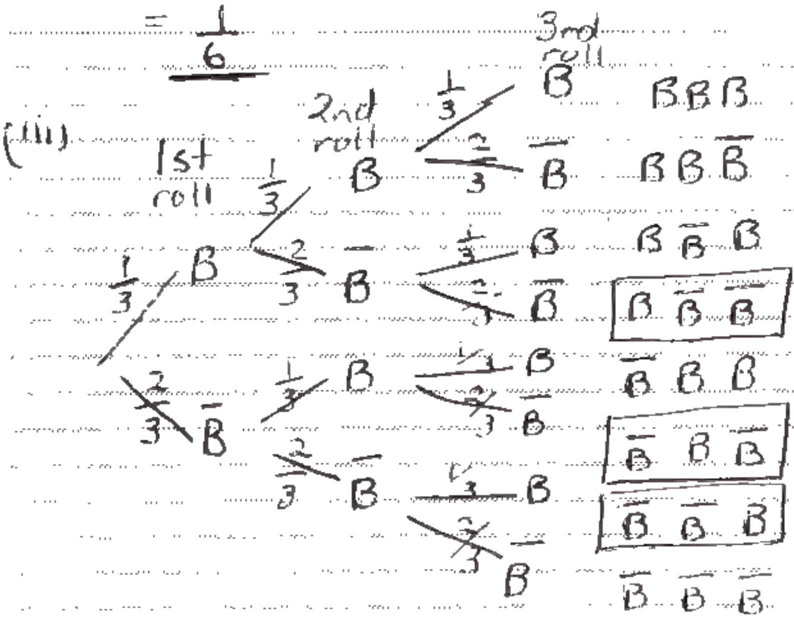
$= \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$ — 1mk

$= \left(\frac{1}{6}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{1}{216} + \frac{1}{27} + \frac{1}{8}$ — 1mk

$= \frac{1}{6}$

(iii)



1mk

Students who drew a tree diagram had a better understanding of the question

Note: If you were looking at all colours
(3) RRB, RBR, BRR
(3) GGB, GBG, BGG
(6) RBG, RGB, BRG, BGR, GRB, GBR

$P(\text{exactly one blue}) = P(B\bar{B}\bar{B}) + P(\bar{B}B\bar{B}) + P(\bar{B}\bar{B}B)$

$= \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}$

$= 3\left(\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\right)$

$= \frac{4}{9}$ — 1mk

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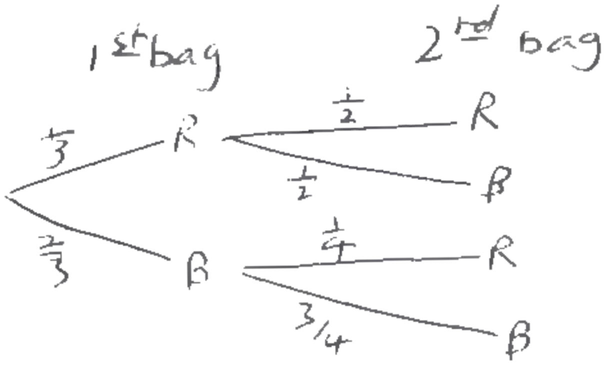
a)



$P(E) = P(RR) + P(BB)$

$= \frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{6} + \frac{1}{2}$

$= \frac{2}{3}$ #

1 m for $\frac{1}{6}$ or $\frac{1}{2}$

1 m for $\frac{2}{3}$

Well

Done

b) PERSEVERE (4 E) (2 R) (1 P) (1 V) (1 S)

9 letters

No of arrangements with E's together & R's together

is $5! = 120$

No of arrangements w/o restriction $= \frac{9!}{4!\,2!} = 7560$

(4 E) (2 R)

$P(E) = \frac{120}{7560} = \frac{1}{63}$ #

1 m for $5!$ or $\frac{9!}{4!\,2!}$

1 m for $\frac{1}{63}$

c i) $T = \frac{2\pi}{n} = \frac{1}{100}$ $\therefore n = 200\pi$

$\ddot{x} = -n^2 x = -40{,}000\pi^2 x$

1 m for n

1 m for sub.

ii) α) centre of motion $x = 0$

$v^2 = n^2(a^2 - x^2)$

$= 40000\pi^2(20^2 - 0)$

$v^2 = 40{,}000\pi^2 \times 400$

$v = \pm 4000\pi$ cm/s

speed $= |v| = 4000\pi$ cm/s # or 4π m/s #

C.O.M 1 m

$x=0$

1 m

1 m must justify speed is positive & must have correct unit

β) when $x = \pm 20$,

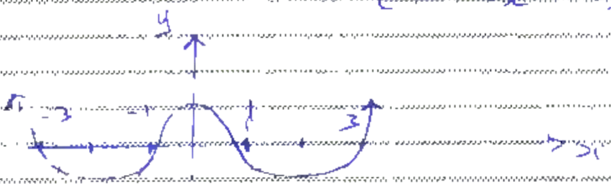
$v = 0$ cm/s #

EXTENSION I

MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments
a)(i) $x = 20t\cos\alpha \quad y = 20t\sin\alpha - 5t^2$	1	Correct Equation in any form involving x, y, α. NOT t
$\therefore \quad = \dfrac{x}{20\cos\alpha}$		
$y = 20\left[\dfrac{x}{20\cos\alpha}\right]\sin\alpha - 5\left[\dfrac{x}{20\cos\alpha}\right]^2$		
$\therefore = x\tan\alpha - \dfrac{5x^2}{400\cos^2\alpha}$		
$= x\tan\alpha - \dfrac{x^2}{80\cos^2\alpha}$		
(ii) $y = x\tan\alpha - \dfrac{x^2\sec^2\alpha}{80}$	2	1 for changing $\dfrac{1}{\cos^2\alpha}$ to $(1+\tan^2\alpha)$
$= x\tan\alpha - \dfrac{x^2(1+\tan^2\alpha)}{80}$		1 sub values and simplifying
when $y = h \quad x = 20$		Answer was given so substitution must be shown
$h = 20\tan\alpha - \dfrac{(20)^2(1+\tan^2\alpha)}{80}$		
$= 20\tan\alpha - \dfrac{400}{80}(1+\tan^2\alpha)$		
$\therefore h = 20\tan\alpha - 5(1+\tan^2\alpha)$		
(iii) Method (1) Equation for h is a quadratic in $\tan\alpha$.	2	1 for showing max occurs when $\tan\alpha = 2$
Let $\tan\alpha = t$		must show why it is maximum
$\therefore h = 20t - 5 - 5t^2$		
$= -5t^2 + 20t - 5$		
as coefficient of $t^2 < 0$ function is concave down		
maximum h occurs when		
$t = \dfrac{-b}{2a} = \dfrac{-20}{2\times -5} = 2$		
$\therefore$ max when $\tan\alpha = 2$		
Method (2) $\dfrac{dh}{dt} = -10t + 20$		Alternatively And $dh/d\alpha$ and test for max
for stat point $\dfrac{dh}{dt} = 0 \quad t = 2$		
$\dfrac{d^2h}{dt^2} = -10 < 0$ concave down		
local max = absolute max		
$\therefore \tan\alpha = 2$		

EXTENSION I
MATHEMATICS: Question 6

Suggested Solutions	Marks	Marker's Comments
maximum height $\tan x = 2$ $h = 20\times 2 - 5(1+2^2)$ $= 40 - 25$ $= 15$ Max height is 15 m	1	1 correct answer.
b) (i) $\frac{d^2x}{dt^2} = 10x - 2x^3$ $\frac{d}{dx}(\frac{1}{2}v^2) = 10x - 2x^3$ $\frac{1}{2}v^2 = \frac{10x^2}{2} - \frac{2x^4}{4} + C$ $v = 0 \quad x = -1$ $0 = 5 - \frac{1}{2} + C$ $C = -\frac{9}{2}$ $\therefore \frac{1}{2}v^2 = 5x^2 - \frac{1}{2}x^4 - \frac{9}{2}$ $v^2 = 10x^2 - x^4 - 9$ $\therefore v = \pm\sqrt{10x^2 - x^4 - 9}$	3	using $\frac{d}{dx}(\frac{1}{2}v^2)$ 1 + integrating 1 C value 1 complete "v" expression including $\pm\sqrt{\ }$
(ii) For motion to exist $v^2 > 0$ Method 1 $\therefore 10x^2 - x^4 - 9 > 0$ $x^4 - 10x^2 + 9 < 0$ $\therefore$ Consider $y = x^4 - 10x^2 + 9$ $= (x^2-9)(x^2-1)$ $= (x-3)(x+3)(x-1)(x+1)$  $\therefore x^4 - 10x^2 + 9 > 0$ for $-1 < x < 1$ $v^2 < 0$ for $-1 < x < 1$ $\therefore$ No motion Method 2 $v = 0$ at $x = \pm 3$, $x = \pm 1$ at $x = -1$ $v = 0$ $\ddot{x} = -8$ object moves to left and stops at x at $x = -3$ $\ddot{x} = 24$ object moves to right and stops at $x = -1$ then moves to left oscillating between $x = -1$ and $x = -3$ $\therefore$ No motion for $-1 < x < 1$	2	1 stating $v^2 > 0$ 1 SHOWING that $v^2 < 0$ for $-1 < x < 1$ Alternatively 1 showing particle moves to left then returning 1 showing particle only moves between $-3 \le x \le -1$

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
(a) $\ddot{x} = 0$ $\quad$ $\ddot{y} = -g$		* If the students didn't show how they evaluated the constants of integration, they got 0.
$\dot{x} = c_1$ $\quad$ $\dot{y} = -gt + c_2$		
at $t = 0$, $x = 0$, $\dot{x} = v\cos\theta$, $\dot{y} = v\sin\theta$		
$\therefore c_1 = v\cos\theta$ $\quad$ $c_2 = v\sin\theta$		
$\therefore \dot{x} = v\cos\theta$ $\quad$ $\therefore \dot{y} = -gt + v\sin\theta$		
$x = vt\cos\theta + c_3$ $\quad$ $y = -\frac{g}{2}t^2 + vt\sin\theta + c_4$		1 for 'x'
at $t = 0, x = 0, y = 0$ $\therefore c_3 = 0$ and $c_4 = 0$		1 for 'y'
$\therefore x = vt\cos\theta$ $\quad$ $y = -\frac{g}{2}t^2 + vt\sin\theta$		
(b) Max height when $\dot{y} = 0$		
$\therefore 0 = -gt + v\sin\theta$		
$t = \frac{v\sin\theta}{g}$	1 mk	
sub into y		
$\therefore h = -\frac{g}{2}\left(\frac{v\sin\theta}{g}\right)^2 + v\left(\frac{v\sin\theta}{g}\right)\sin\theta$	1	
$= -\frac{v^2\sin^2\theta}{2g} + \frac{v^2\sin^2\theta}{g}$		
$= \frac{-v^2\sin^2\theta + 2v^2\sin^2\theta}{2g}$	1	
$= \frac{v^2\sin^2\theta}{2g}$		
(c) when $y = -h$		
$-h = -\frac{1}{2}gt^2 + vt\sin\theta$		
$\frac{-v^2\sin^2\theta}{2g} = -\frac{1}{2}gt^2 + vt\sin\theta$	1	

2/2

MATHEMATICS Extension 1 : Question 7 cont.

Suggested Solutions	Marks	Marker's Comments
$\therefore 0 = gt^2 - 2vt\sin\theta - \frac{v^2\sin^2\theta}{g}$		
$\therefore t = \frac{2v\sin\theta \pm \sqrt{(-2v\sin\theta)^2 - 4g\left(\frac{-v^2\sin^2\theta}{g}\right)}}{2g}$	1	
$= \frac{2v\sin\theta \pm \sqrt{4v^2\sin^2\theta + 4v^2\sin^2\theta}}{2g}$		
$= \frac{2v\sin\theta \pm 2\sqrt{2}v\sin\theta}{2g}$		
$= \frac{v\sin\theta \pm \sqrt{2}v\sin\theta}{g}$		
$= \frac{v\sin\theta(1+\sqrt{2})}{g}$ as $t > 0$ time is positive	1	had to explain why it was positive
(d) when $t = \frac{v\sin\theta(1+\sqrt{2})}{g}$, $x = ??$		
$\therefore x = vt\cos\theta$		
$= v \cdot \frac{v\sin\theta(1+\sqrt{2})}{g}\cos\theta$	1	
$= \frac{v^2\sin\theta\cos\theta(1+\sqrt{2})}{g}$		
$= \frac{v^2\, 2\sin\theta\cos\theta(1+\sqrt{2})}{2g}$	1	
$= \frac{v^2\sin 2\theta(1+\sqrt{2})}{2g}$		