NORTH SYDNEY GIRLS HIGH SCHOOL

YEAR 12 - TERM 2 ASSESSMENT

2003

MATHEMATICS

EXTENSION 1 COURSE

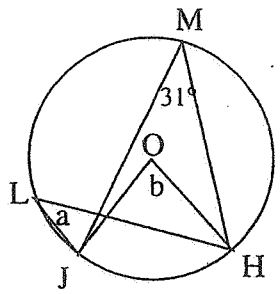
TIME ALLOWED: 60 MINUTES

INSTRUCTIONS:

- This test is worth 20% of the HSC assessment
- Start each question on a new page
- Show all necessary working
- There are six questions

QUESTION 1 (9 Marks) Marks

a) O is the centre of the circle 2
Find the value of a and b, giving reasons



b) Consider the polynomial $P(x) = x^3 - 5x + c$
(i) Find the value of c if $x + 2$ is a factor of $P(x)$ 1
(ii) For this value of c, find $Q(x)$ such that $P(x) = (x + 2)Q(x)$ 2

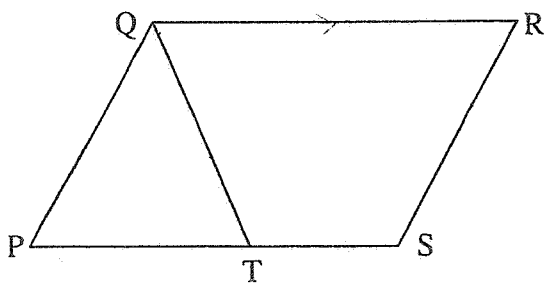
c) Solve for $0° \leq \theta \leq 360°$ 4
$5\cos\theta - 2\sin\theta = 2$

QUESTION 2 (8 Marks)

a) A function is given by the rule $f(x) = \dfrac{x+1}{x+2}$ 2
Find the rule for the inverse function $f^{-1}(x)$

b) State the general solution for θ if $\cos\theta = -\dfrac{1}{\sqrt{2}}$ 2

c) 4



$PQRS$ is a parallelogram. T is a point on PS such that $PQ = QT$ and $\hat{QRS} = \theta$
(i) Copy the diagram showing the above information
(ii) Show that $QRST$ is a cyclic quadrilateral.

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QUESTION 3 (9 Marks) — Marks

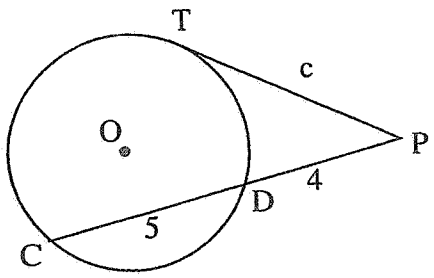
a) Find the exact value of $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx$ — 2

b) Consider the function $y = 2\cos^{-1}(1-x)$
(i) Find the domain and range of the function. — 2
(ii) Sketch the graph of the function. — 2

c) Using the substitution, $u = 4x^2 - 1$, find an indefinite integral of — 3
$\frac{8x}{\sqrt{4x^2-1}}$

QUESTION 4 (8 Marks)

a) O is the centre of the circle and PT is a tangent to the circle — 1
Find the value c



b) Find the gradient of the tangent to the curve $y = \tan^{-1}\left(\frac{1}{x}\right)$ at the point — 2
on the curve where $x = 1$

c) P $(2t, t^2)$ is a point on the parabola $x^2 = 4y$ with focus F
The point M divides the interval FP externally in the ratio 3 : 1
(i) Show that the co-ordinates of M are $(3t, \frac{3t^2-1}{2})$ — 1
(ii) Show that as P moves on the parabola $x^2 = 4y$, then M moves on the parabola $x^2 = 6y + 3$ — 2
(iii) Find the co-ordinates of the focus and the equation of the directrix of the locus of M — 2

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Marks

QUESTION 5 (8 Marks)

a) Find the value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ in terms of π 1

b) If α, β and γ are the roots of the equation $3x^3 + 5x^2 - 7x + 4 = 0$, find the values of

(i) $\alpha\beta + \alpha\gamma + \beta\gamma$ 1

(ii) $\alpha\beta\gamma$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ 1

c) Use the substitution $x = u^2$, $u > 0$, to express the value of $\int_1^{100} \frac{1}{x + 2\sqrt{x}}dx$ 4

in the form $\ln a$ for some constant $a > 0$

QUESTION 6 (8 Marks)

a) The length of the tangent from an external point T to a circle, radius 2·5 cm is 6 cm. Calculate the radius of the circle, centre T, which touches the given circle externally. Include a diagram with your answer. 3

b) The equation of the normal at the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ is given by $x + py = ap^3 + 2ap$ 3
Show that this normal meets the parabola again at the point with parameter $\left[-\frac{2 + p^2}{p}\right]$

c) The function $\frac{\ln x}{x} = -2$ has a root near $x = 0{\cdot}5$. Starting with this value use one application of Newton's method to find an improved approximation for the value of the root, giving the answer correct to 2 decimal places. 2

END OF TEST