



SCEGGS Darlinghurst

Student Number: _____

Term 2, 2008
Wednesday 18th June

EXTENSION 1

MATHEMATICS

Task Weighting : 30 %

Outcomes Assessed: HE2, HE3, HE4, HE6, HE7

General Instructions

- Time allowed - 75 minutes
- Write your student number at the top of each page
- **Start each question on a new page**
- Attempt **all** questions and show all necessary working.
- Marks may be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used.
- A table of Standard Integrals is attached at the back of this paper.

Question	Calculus	Communication	Reasoning	TOTAL
1	/7	/3		/13
2	/6		/5	/14
3	/8	/3	/4	/16
4	/6		/4	/14
TOTAL	/27	/6	/13	/57

Parent's Signature _____

Question1 (13 Marks)**Marks**

- (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 7x}{5x}$ 1
- (b) Using the substitution $u^2 = x + 1$, or otherwise, find
 $\int x\sqrt{x+1} \, dx$ 3
- (c) (i) Differentiate $e^{2x}(\cos x - 2 \sin x)$ 2
- (ii) Hence, or otherwise, find
 $\int_0^{\pi} e^{2x} \sin x \, dx$ 2
- (d) (i) State the domain and range of $y = 3 \cos^{-1} 2x$ 2
- (ii) Sketch the function $y = 3 \cos^{-1} 2x$
 showing all important features. 1
- (e) Sketch the function $P = 10000 - 2000e^{-0.03t}$ for $t \geq 0$. 2

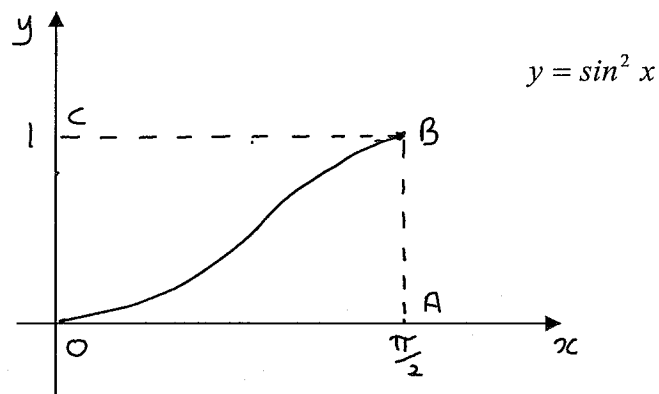
Question 2 (14 Marks)

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Marks

(a) Differentiate $x^2 \tan^{-1} 2x$ 2

(b) The diagram shows the curve $y = \sin^2 x$ between $x = 0$ and $x = \frac{\pi}{2}$



Show that this curve divides the rectangle $OABC$ into two regions of equal area. 4

- (c) For the function $y = \ln(1 + e^x)$
- (i) State the domain of $y = f(x)$ 1
- (ii) Find the inverse function $f^{-1}(x)$ 2
- (iii) State the domain of $f^{-1}(x)$ 1

Question 2 continues on the next page

Question 2 (continued)

Marks

- (d) During recent heavy rainfall, Angus monitored the volume V , in kilolitres, of water in a dam on his rural property.

The rate of change of the volume of water in the dam after t hours is given by $\frac{dV}{dt} = k(V - 15000)$, where k is a constant.

- (i) Show that $V = 15000 + Ae^{kt}$ satisfies the differential equation

$$\frac{dV}{dt} = k(V - 15000). \quad 1$$

- (ii) Initially, the dam contained 150000 kilolitres and after 10 hours of pouring rain the volume increased to 375000 kilolitres.

Find the values of A and k . 2

- (iii) What volume of water will be in the dam after 2 days?
(Answer to the nearest kilolitre.) 1

Question 3 (16 marks)

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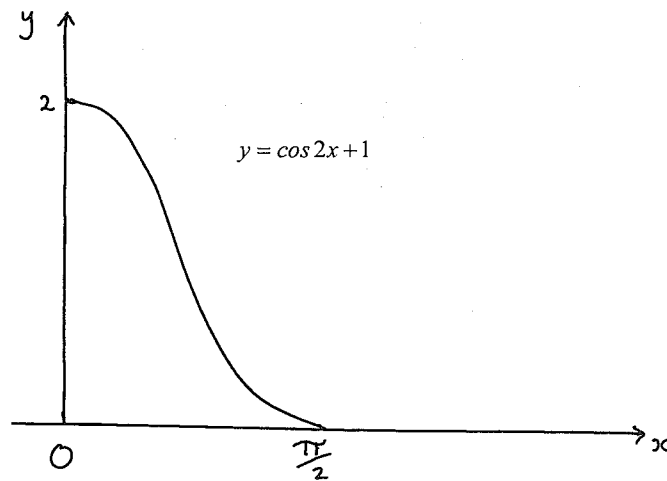
Marks

- (a) Use the substitution $u = \tan x$, or otherwise, to evaluate

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$$

3

- (b) The graph shows the curve $y = \cos 2x + 1$ for $0 \leq x \leq \frac{\pi}{2}$



Find the exact volume of the solid of revolution formed when the region bounded by the curve $y = \cos 2x + 1$ and the x -axis between

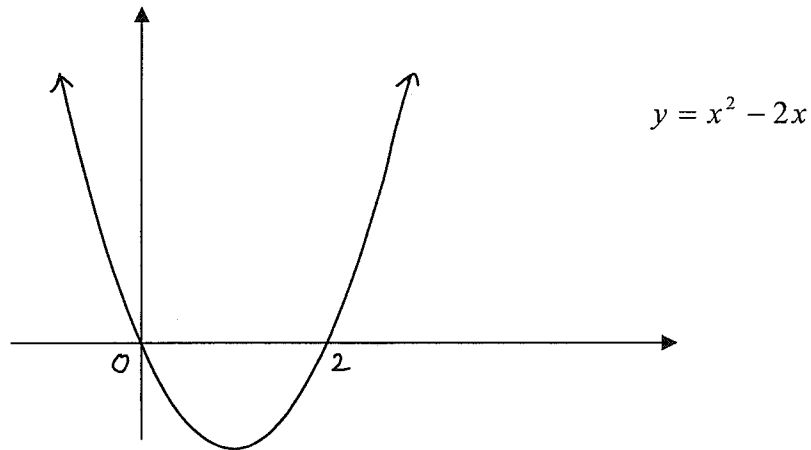
$x=0$ to $x = \frac{\pi}{2}$ is rotated about the x -axis.

3

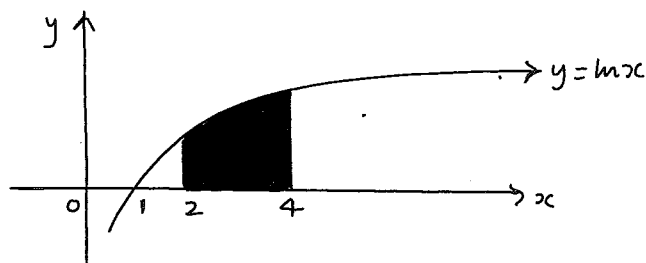
Question 3 continues on the next page.

Question 3 (continued)**Marks**

- (c) The function $y = x^2 - 2x$ is shown in the diagram.



- (i) State why the function $y = x^2 - 2x$ does not have an inverse function. 1
- (ii) State the largest possible domain over which the function $y = x^2 - 2x$ is monotonic increasing. 1
- (iii) Show that the inverse function $f^{-1}(x)$ over this restricted domain is given by $y = 1 + \sqrt{x+1}$ 2
- (iv) On the same set of axes sketch $y = f(x)$ and $y = f^{-1}(x)$ showing all important features. 2
- (b) The diagram shows the shaded area bounded by the curve $y = \ln x$, the x -axis and the lines $x=2$ and $x=4$.



Show that the exact area of the shaded region is given by $(\ln 64 - 2)$ units²

Question 4 (14 Marks)

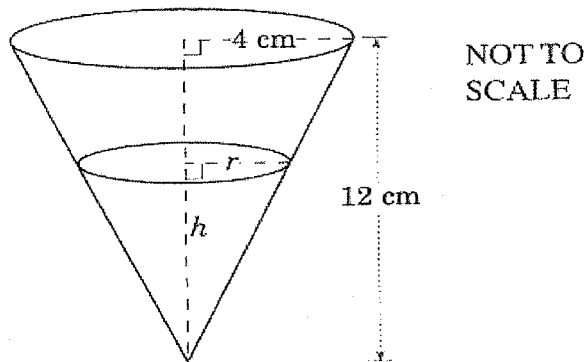
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Marks

- (a) Use an appropriate double angle formula to find the exact value of 2

$$\sin\left(2\cos^{-1}\frac{6}{7}\right)$$

(b)



The diagram shows a conical drinking cup of height 12 cm and radius 4 cm. The cup is being filled with water at the rate of 3 cm^3 per second.

The height of the water at time t seconds is h cm and the radius of the water's surface is r cm.

- (i) Show that $r = \frac{h}{3}$ 1
- (ii) Show that the volume is given by $V = \frac{\pi}{27}h^3$ 1
- (iii) Find the rate at which the height is increasing when the height of the water is 9 cm. 2

Question 4 (continued)**Marks**

- (c) (i) Show that $\frac{d}{dx} \left(x \sin^{-1} x + \sqrt{1-x^2} \right) = \sin^{-1} x$ 2
- (ii) Hence find the exact area bounded by the curve $y = \sin^{-1} x$, the x -axis and the lines $x = 0$ and $x = 1$. 2
- (d) (i) Show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ 2
- (ii) Given $\sin^{-1} \left(-\frac{2}{3} \right) - \cos^{-1} \left(-\frac{2}{3} \right) = k$ and by using the expressions
 $\sin^{-1}(-x) = -\sin^{-1}(x)$ and $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$
find an expression for $\cos^{-1} \left(\frac{2}{3} \right)$ in terms of k . 2

End of paper.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x$, $x > 0$

Question 1

13

Calc/1 Comm/3

Comments.

a) $\lim_{x \rightarrow 0} \frac{\sin 7x}{5x}$
 $= \frac{7}{5} \lim_{x \rightarrow 0} \frac{\sin 7x}{7x}$
 $= \frac{7}{5} \times 1$
 $= \frac{7}{5}$

Do you understand what you're doing?
 For very small angles $\sin x \doteq x \doteq \tan x$
 ✓ Well done but some confused working steps.

b) $u^2 = x + 1$
 $x = u^2 - 1$
 $\frac{dx}{du} = 2u$
 $dx = 2u du$

$$\int x \sqrt{x+1} dx$$

$$= \int (u^2 - 1) \cdot \sqrt{u^2} \cdot 2u du$$

$$= \int (u^2 - 1) \cdot u \cdot 2u du$$

$$= \int 2u^2 (u^2 - 1) du$$

$$= \int (2u^4 - 2u^2) du$$

$$= \frac{2u^5}{5} - \frac{2u^3}{3} + C$$

$$= \frac{2(\sqrt{x+1})^5}{5} - \frac{2(\sqrt{x+1})^3}{3} + C$$

$$= \frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$$

Well done but some of you need to practise your index rules!
 ✓
 ✓
 must be in terms of x
 ✓ Don't forget this last step. It's an easy mark.
 (Calc 3)

c) i) $\frac{d}{dx} (e^{2x} (\cos x - 2 \sin x))$

$$\begin{cases} u = e^{2x} & v = \cos x - 2 \sin x \\ u' = 2e^{2x} & v' = -\sin x - 2 \cos x \end{cases}$$

Product rule $vu' + uv'$

$$= 2e^{2x}(\cos x - 2 \sin x) + e^{2x}(-\sin x - 2 \cos x)$$

$$= 2e^{2x} \cos x - 4e^{2x} \sin x - e^{2x} \sin x - 2e^{2x} \cos x$$

$$= -5e^{2x} \sin x$$

✓ Well done!
 Practise collecting like terms if you got this wrong.
 (Calc 2)

ii) $\int_0^{\pi} e^{2x} \sin x dx$

$$= -\frac{1}{5} \int_0^{\pi} -5e^{2x} \sin x dx$$

$$= -\frac{1}{5} [e^{2x} (\cos x - 2 \sin x)]_0^{\pi}$$

$$= -\frac{1}{5} \{ e^{2\pi} (\cos \pi - 2 \sin \pi) - e^0 (\cos 0 - 2 \sin 0) \}$$

$$= -\frac{1}{5} \{ e^{2\pi} (-1 - 0) - 1(1 - 0) \}$$

$$= -\frac{1}{5} (-e^{2\pi} - 1)$$

$$= \frac{1}{5} (e^{2\pi} + 1)$$

✓ well done.
 ✓ (Calc 2)

d) $y = 3 \cos^{-1} 2x$

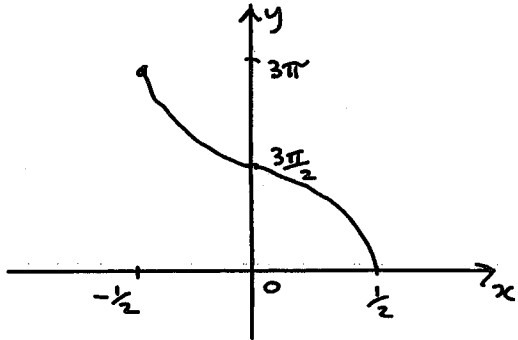
Domain $-1 \leq 2x \leq 1$
 $-\frac{1}{2} \leq x \leq \frac{1}{2}$

✓

Range $0 \leq \cos^{-1} 2x \leq \pi$

$0 \leq 3 \cos^{-1} 2x \leq 3\pi$

$0 \leq y \leq 3\pi$



✓

This part was overall very well done.

✓

(Comm1)

Question 2 Calc /6, Reas /5

a)

$\frac{d}{dx} (x^2 + \tan^{-1} 2x)$

$u = x^2$

$u' = 2x$

$v = \tan^{-1} 2x$

$v' = \frac{1}{1+(2x)^2} \times 2$

$= \frac{2}{1+4x^2}$

using product rule $vu' + uv'$

$= 2x \tan^{-1} 2x + \frac{2x^2}{1+4x^2}$

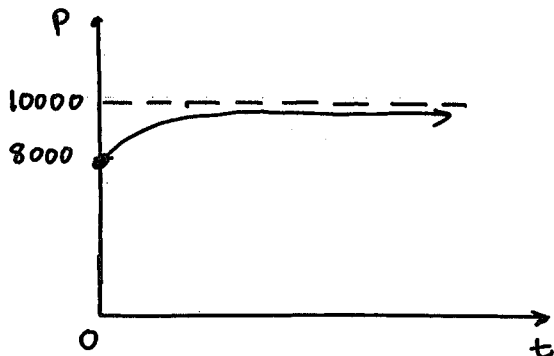
Some forgot to use the chain rule when differentiating $\tan^{-1}(2x)$

✓

✓ correct use of product rule (Calc 2)

e)

$P = 10000 - 2000e^{-0.03t}$



$t = 0, P = 10000 - 2000 = 8000$
 $t \rightarrow \infty, e^{-0.03t} \rightarrow 0 \therefore P \rightarrow 10000$

Not particularly well done. This is a 2U question!

Think about how the graph evolves.

$e^{-0.03t} \rightarrow -e^{-0.03t}$



Then shift it up!

or substitute in a few values and find some points.

✓ Shape/start at 8000

✓ Horizontal Asymptote

(Comm2)

b)

Area below curve

$= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 2x) \, dx$

$= \frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2} \left(\left(\frac{\pi}{2} - \frac{1}{2} \sin \pi \right) - \left(0 - \frac{1}{2} \sin 0 \right) \right)$

$= \frac{1}{2} \left(\frac{\pi}{2} - 0 \right)$

$= \frac{\pi}{4} \text{ units}^2$

Generally well done

✓

✓

(Calc 3)

Area of rectangle OABC

$= \frac{\pi}{2} \times 1$

$= \frac{\pi}{2} \text{ units}^2$

\therefore The curve divides the rectangle area in half.

✓ for calculating (Reas 1) the area AND putting it all together.

c) i) $y = \ln(1+e^x)$

Domain $1+e^x > 0$
 $e^x > -1$

This is true for all real x
 \therefore Domain is all real x

Parts (i) & (iii) were very poorly done!

✓ (Reas 1)

ii) $f^{-1}(x): y = \log_e(1+e^x)$

Interchange x and y

$x = \log_e(1+e^y)$

make y the subject

use the definition $\begin{cases} x = \log_a N \\ a^x = N \end{cases}$

$e^x = 1+e^y$

$e^y = e^x - 1$

Take logs both sides or use the defn.

$\log_e e^y = \log_e(e^x - 1)$

$y = \log_e(e^x - 1)$

✓ (Reas 2)

iii) Domain of $f^{-1}(x)$

$e^x - 1 > 0$

$e^x > 1$

Take logs both sides

$\log_e e^x > \log_e 1$

$x > 0$

✓ (Reas 1)

i) $V = 15000 + Ae^{kt}$

$\frac{dV}{dt} = k \cdot Ae^{kt}$

$= k(V - 15000)$

$\therefore V$ satisfies the equation.

Standard exponential growth question that was done well.

✓ (Calc)

ii) When $t=0$ $V=15000$

Find A

$15000 = 15000 + Ae^0$

$\therefore A = 135000$

✓

When $t=10$, $V=375000$

$V = 15000 + Ae^{kt}$

$375000 = 15000 + 135000e^{10k}$

$360000 = 135000e^{10k}$

$e^{10k} = \frac{360000}{135000}$

$e^{10k} = \frac{8}{3}$

Take logs both sides.

$\ln e^{10k} = \ln\left(\frac{8}{3}\right)$

$10k = \ln\left(\frac{8}{3}\right)$

$k = \frac{1}{10} \ln\left(\frac{8}{3}\right)$

$\doteq 0.09808\dots$ (store in calculator)

✓

iii) After $t=2$ days = 48 hours

$V = 15000 + 135000e^{k \times 48}$

$= 14\,976\,794$ kilolitres

✓

Question 3 16 Calc 8 Comm 3 Reas 4

a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$

$u = \tan x$
 $\frac{du}{dx} = \sec^2 x$
 $du = \sec^2 x dx$

Change limits
 $x = \frac{\pi}{3} \quad u = \tan \frac{\pi}{3} = \sqrt{3}$
 $x = \frac{\pi}{4} \quad u = \tan \frac{\pi}{4} = 1$

First mark $\int \frac{1}{u} du$ was well done but after that...
 Look at the standard integrals or your rule page! $\int \frac{1}{u} du = \ln u$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x} dx$
 $= \int_1^{\sqrt{3}} \frac{du}{u}$
 $= [\ln u]_1^{\sqrt{3}}$
 $= \ln \sqrt{3} - \ln 1$
 $= \ln \sqrt{3}$

✓
 ✓ No need to write $\ln(\tan x)$ because you changed the limits from x to u .
 ✓
 (Calc 3)

Volume about the x-axis

$V = \pi \int_a^b y^2 dx$
 $= \pi \int_0^{\frac{\pi}{2}} (\cos 2x + 1)^2 dx$
 $= \pi \int_0^{\frac{\pi}{2}} (\cos^2 2x + 2\cos 2x + 1) dx$
 ↓
 replace by
 $= \pi \int_0^{\frac{\pi}{2}} \left(\frac{1}{2}(1 + \cos 4x) + 2\cos 2x + 1 \right) dx$
 $= \pi \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2} \cos 4x + 2\cos 2x + 1 \right) dx$

Note that the $\frac{1}{2}$ in this substitution only affects the $(1 + \cos 4x)$ not the other terms in that integration.

Terrible! If you can't expand a quadratic you need to perfect this skill.
 ✓ $(a+b)^2 = a^2 + 2ab + b^2$
 Note $(\cos 2x)^2 \neq \cos^2 4x$
 It is just $= \cos^2 2x$

$= \pi \left[\frac{1}{2}x + \frac{1}{2}x \cdot \frac{1}{4} \sin^4 2x + 2x \cdot \frac{1}{2} \sin 2x + x \right]_0^{\frac{\pi}{2}}$
 $= \pi \left(\frac{\pi}{4} + \frac{1}{8} \sin^4 \pi + \sin \pi + \frac{\pi}{2} \right) - 0$
 $= \pi \left(\frac{\pi}{4} + 0 + 0 + \frac{\pi}{2} \right)$
 $= \pi \left(\frac{3\pi}{4} \right)$
 $= \frac{3\pi^2}{4} \text{ units}^3$

✓
 Well done if you got here.
 ✓ (Calc 3)

c) i) The horizontal line test fails. There is not one-to-one correspondence

✓ Very well done. (Comm 1)

ii) $x > 1$

✓ ($x > 1$ is OK)

iii) $y = x^2 - 2x$
 Interchange x and y
 $x = y^2 - 2y$
 Complete the square
 $x + 1 = y^2 - 2y + 1$
 $x + 1 = (y - 1)^2$
 $y - 1 = \pm \sqrt{x + 1}$
 $y = 1 \pm \sqrt{x + 1}$

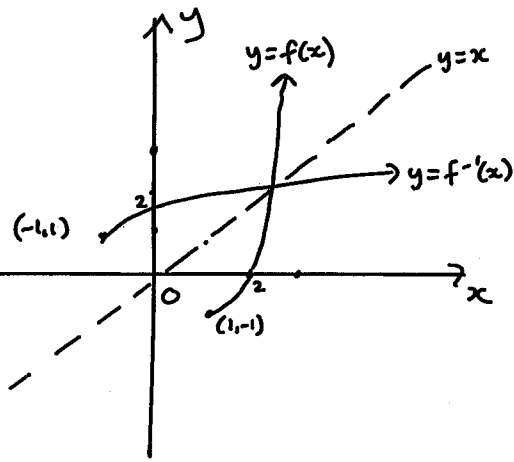
This question comes straight from your textbook examples! Completing the Square method is used in questions like this.

Since the range of the inverse function is $y > 1$

← This mark was harder to get. Do you understand the logic.

$\therefore y = 1 + \sqrt{x + 1}$

✓ (Reas 2)

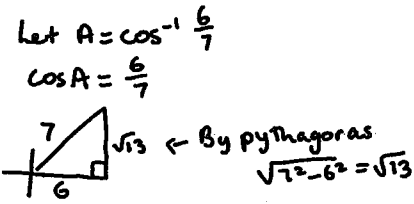


Although not all
Originals and Inverse
Functions intersect
on the line $y=x$,
this one does.
✓ intersect
on $y=x$

Well done.
(Comm 2)

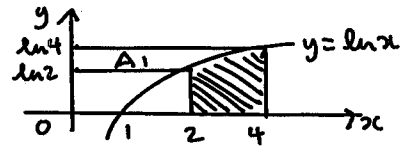
Question 4 Calc /6, Reas /4

a) $\sin(2 \cos^{-1} \frac{6}{7})$



$$\begin{aligned} \sin(2 \cos^{-1} \frac{6}{7}) &= \sin 2A \\ &= 2 \sin A \cos A \\ &= 2 \times \frac{\sqrt{13}}{7} \times \frac{6}{7} \\ &= \frac{12\sqrt{13}}{49} \end{aligned}$$

It was surprising
how many students
could not correctly
applying Pythagoras
or calculate 7×7
✓ Overall, however
this was done well



Find area to the y-axis. A_1
 $y = \log_e x$
 $x = e^y$

$$\begin{aligned} A_1 &= \int_{\ln 2}^{\ln 4} e^y dy \\ &= [e^y]_{\ln 2}^{\ln 4} \\ &= e^{\ln 4} - e^{\ln 2} \\ &= 4 - 2 \\ &= 2 \quad u^2 \end{aligned}$$

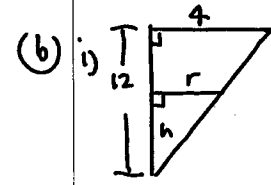
Shaded area = Large Rectangle - A_1 - Small rectangle
 $= 4 \ln 4 - 2 - 2 \ln 2$
 $= \ln \frac{4^4}{2^2} - 2 = \ln 64 - 2$

This is definitely
a harder 2M
question.
Everyone should
be able to find
area to the y-axis.

Note $\int \ln x dx \neq \frac{1}{5c} x$
 ✓ but $\int \frac{1}{x} dx = \ln x$.
 Clear up your
rules please!

✓
(Calc 2)

✓ Draw a diagram.
Use some labelled
(coloured) sections.
✓ (Reas 2)



Using similar Δ s

$$\frac{12}{4} = \frac{h}{r}$$

$$\begin{aligned} \therefore 3r &= h \\ r &= \frac{h}{3} \end{aligned}$$

It is not good enough
to simply state
 $h = 3 \times r$ because
 $12 = 3 \times 4$.
You must state you
are using similar
 Δ s!

ii) Volume of
a cone

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \left(\frac{h}{3}\right)^2 h \\ &= \frac{1}{3} \pi \times \frac{h^2}{9} \times h \\ &= \frac{\pi h^3}{27} \end{aligned}$$

✓

iii) $\frac{dv}{dt} = 3 \text{ cm}^3/\text{sec}$

$\frac{dv}{dh} = \frac{3\pi h^2}{27} = \frac{\pi h^2}{9}$

Find $\frac{dh}{dt}$

$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt}$
 $3 = \frac{\pi h^2}{9} \times \frac{dh}{dt}$

when $h=9$

$\frac{dh}{dt} = \frac{3}{\frac{\pi \times 9^2}{9}}$

$= \frac{3}{9\pi}$

$= \frac{1}{3\pi}$

$\hat{=} 0.106 \text{ cm/sec}$



Don't forget the units!
(Calc 2)

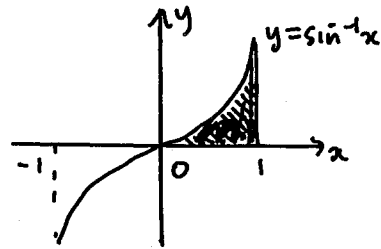
c)

$\frac{d}{dx} (x \sin^{-1} x + \sqrt{1-x^2})$
 $\frac{d}{dx} ((x \sin^{-1} x + (1-x^2)^{1/2})$
 product rule
 $\begin{cases} u=x & v=\sin^{-1}x \\ u'=1 & v'=\frac{1}{\sqrt{1-x^2}} \\ v u' + u v' \end{cases}$

$= \sin^{-1} x \cdot 1 + x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2} (1-x^2)^{-1/2} \cdot -2x$
 $= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$
 $= \sin^{-1} x$

Please be sure to set out your work clearly. You don't get marks for fudging & if your work is unclear correct use of product rule it will look like you're fudging it!
(Calc 2)

c) ii)

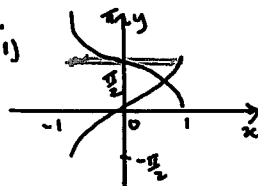


Area = $\int_0^1 \sin^{-1} x \, dx$
 $= [x \sin^{-1} x + \sqrt{1-x^2}]_0^1$
 $= (1 \sin^{-1} 1 + \sqrt{1-1}) - (0 + \sqrt{1})$
 $= (\frac{\pi}{2} - 1) \text{ units}^2$

(Calc 2)

This was an easy two marks for most.

d) i)



It can be shown graphically by adding the ordinates that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

OR show $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = 0$

$\therefore \sin^{-1} x + \cos^{-1} x = \text{constant}$

then evaluate that constant by substituting e.g. $x=0$

ii) $\sin^{-1}(\frac{2}{3}) - \cos^{-1}(-\frac{2}{3}) = k$

Replace

$\sin^{-1}(\frac{2}{3}) = -\sin^{-1}(\frac{2}{3})$

$\cos^{-1}(-\frac{2}{3}) = \pi - \cos^{-1}(\frac{2}{3})$

from (i) $\sin^{-1}(\frac{2}{3}) + \cos^{-1}(\frac{2}{3}) = \frac{\pi}{2}$

$\therefore \sin^{-1}(-\frac{2}{3}) - \cos^{-1}(-\frac{2}{3}) = k$

$-\sin^{-1}(\frac{2}{3}) - (\pi - \cos^{-1}(\frac{2}{3})) = k$

$-\frac{\pi}{2} - \cos^{-1}(\frac{2}{3}) - \pi + \cos^{-1}(\frac{2}{3}) = k$

$2 \cos^{-1}(\frac{2}{3}) - \frac{\pi}{2} - \pi = k$

While this is a nice way to convince yourself $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ it does not constitute a proof. You really need to show that the function you get by adding the ordinates is a constant. (Reas 2)

Note: there are lots of ways to prove (i)

Those who ended up with the result $\cos^{-1}(\frac{2}{3}) = k + \sin^{-1}(\frac{2}{3}) + 1/2$. To get the extra mark you needed to link this part with part (i).

$2 \cos^{-1}(\frac{2}{3}) = k + \frac{3\pi}{2}$
 $\therefore \cos^{-1}(\frac{2}{3}) = \frac{k}{2} + \frac{3\pi}{4}$