Name:______________________________

Class:______________________________

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

HSC ASSESSMENT TASK 3

JUNE 2010

MATHEMATICS Extension 1

Time Allowed: 70 minutes

Instructions:

- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new page.**
- Standard integrals can be found on the last page.

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/9	/8	/9	/8	/8	/8	**/50**

Question 1

a) Write the exact value of :

i) $\cos^{-1}(\cos\frac{3\pi}{2})$ 1

ii) $sin^{-1}0.4 + cos^{-1}0.4$ 1

b) Simplify $\frac{\sin(\pi-x)}{\sin(\frac{\pi}{2}-x)}$ 2

c) Evaluate $\log_9 30$ correct to 2 decimal places. 1

d) Solve, leaving answers in exact form:

i) $3^{x-1} = 7$ 2

ii) $\ln(x^2) + \ln x = 1$ 2

Question 2

a) Evaluate $\lim_{x\to 0} \frac{\frac{x}{3}}{sin\ 2x}$ 1

b) Differentiate:

i) $\tan 2x$ 1

ii) $\ln(\frac{x}{x^2+3})$ 2

iii) $cosec^2 x$ 2

iv) $sin^{-1}3x$ 2

Question 3

a) Find:

i) $\int e^{4x}\ dx$ 1

ii) $\int \frac{1+2x}{x^2}\ dx$ 2

iii) $\int \frac{x}{1+2x^2}\ dx$ 1

b) Using the substitution $u = \tan x$, or otherwise, find $\int sec^2x\, tan^2x\, dx$. 2

c) i) Find $\frac{d}{dx}(xe^{2x})$ 1

ii) Hence or otherwise, find $\int xe^{2x}dx$ 2

Question 4

a) On the same axes, sketch $y = \sin x$ and $y = \ln(\sin x)$ for $0 \leq x \leq \pi$. 2

Clearly label key features.

b) i) Express sin^2x in terms of $\cos 2x$ 1

ii) The curve $y = sin\, 2x,$ for $0 \leq x \leq \pi,$ is rotated about the x axis. 3

Find the total volume generated.

c) Evaluate $sin\left[tan^{-1}(\frac{5}{4})\right]$ in exact form. 2

Question 5

A function f is defined $f(x) = x^2 - 2x$.

a) State the largest positive domain for f to have an inverse function f^{-1}. 1

b) State the domain and range of f^{-1}. 2

c) Sketch f and f^{-1} on the same axes for the domains and ranges above. 2

Clearly show key points.

d) Find the inverse function $f^{-1}(x)$. 2

e) Find the value of x for which $f(x) = f^{-1}(x)$. 1

Question 6

a) Express $cos^{-1}\left(\frac{1}{3}\right) + cos^{-1}(\frac{1}{4})$ in the form $cos^{-1}M$. 2

b) i) Write the domain and range for $y = 3sin^{-1}(\frac{x}{2})$ 2

ii) Sketch the curve in i) 1

iii) Evaluate $\int_0^1 3sin^{-1}\left(\frac{x}{2}\right)dx$. 3

Leave your answer in exact form.

SOLUTIONS

1 a) i) $\cos^{-1} 0 = \pi/2$

ii) $\pi/2$

b) $\dfrac{\sin x}{\cos x} = \tan x$

c) $\dfrac{\log 30}{\log 9} \doteqdot 1.55$

d) $\log(3^{x-1}) = \log 7$

$(x-1)\log 3 = \log 7$

$x\log 3 - \log 3 = \log 7$

$x = \dfrac{\log 7 + \log 3}{\log 3}$

e) $\log_e(x^3) = 1$

$\therefore x^3 = e^1$

$\therefore x = \sqrt[3]{e}$

2 a) $\lim\limits_{x\to 0} \dfrac{2x}{\sin 2x} \times \dfrac{1}{6} = 1 \times \frac{1}{6}$

$= \dfrac{1}{6}$ 1

b) i) $2\sec^2 2x$ 1

ii) $\log x - \log(x^2+3)$

$\dfrac{dy}{dx} = \dfrac{1}{x} - \dfrac{2x}{x^2+3}$ 2

$\dfrac{3-x^2}{x(x^2+3)}$

iii) $y = \dfrac{1}{\sin^2 x}$

$-\dfrac{\sin 2x}{\sin^4 x}.$

$\dfrac{dy}{dx} = \dfrac{0 - 2\sin x\cos x}{\sin^4 x}$

2

$= \dfrac{-2\cos x}{\sin^3 x}$ or $-2\cot x\operatorname{cosec}^2 x$

iv) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-(3x)^2}} \times 3$

$= \dfrac{3}{\sqrt{1-9x^2}}$ 2

3 a) i) $\dfrac{e^{4x}}{4} + c$

ii) $\int\left(\frac{1}{x^2} + \frac{2x}{x^2}\right)dx$

$= \int\left(x^{-2} + \frac{2}{x}\right)dx$

$= -x^{-1} + 2\log x +$

$= -\frac{1}{x} + 2\log x + c$

iii) $\frac{1}{4}\int \dfrac{4x}{1+2x^2}\,dx$

$= \frac{1}{4}\log(1+2x^2) + c$

b) $\int \sec^2 x\tan^2 x\,dx = \int \cancel{\sec^2 x}\, u^2 \dfrac{du}{\cancel{\sec^2 x}}$

$u = \tan x$

$\dfrac{du}{dx} = \sec^2 x$

$dx = \dfrac{du}{\sec^2 x}$

$= \dfrac{u^3}{3} + c$

$= \dfrac{\tan^3 x}{3} + c$

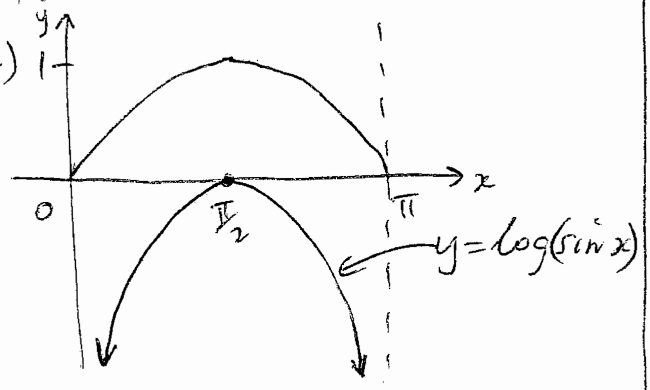
c) i) $\dfrac{d}{dx}(xe^{2x}) = e^{2x} + 2xe^{2x}$

ii) $2xe^{2x} = \dfrac{d}{dx}(xe^{2x}) - e^{2x}$

$\therefore \int xe^{2x} = \frac{1}{2}\int\frac{d}{dx}(xe^{2x}) - \frac{1}{2}\int e^{2x}\,dx$

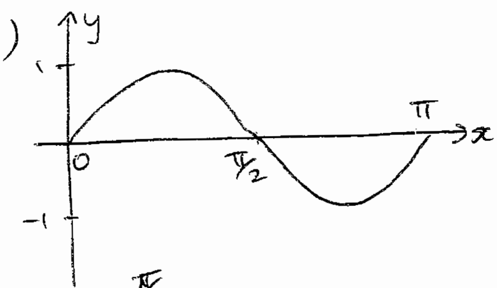
$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$

4) a)



b) i) $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

ii)



$$Vol = 2\pi\int_0^{\pi/2} \sin^2 2x \, dx$$

$$= 2\pi\int_0^{\pi/2} \tfrac{1}{2}(1-\cos 4x)\,dx$$

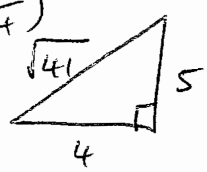
$$= \pi\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/2}$$

$$= \pi\left[(\pi/2 - 0) - (0-0)\right]$$

$$= \frac{\pi^2}{2} \text{ units}^3.$$

c) Let $\alpha = \tan^{-1}\left(\frac{5}{4}\right)$

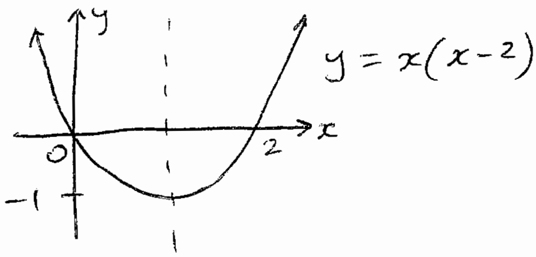
$\therefore \tan\alpha = \frac{5}{4}$



$\therefore \sin\left(\tan^{-1}\frac{5}{4}\right) = \sin\alpha$

$= \frac{5}{\sqrt{41}}$

5) a)

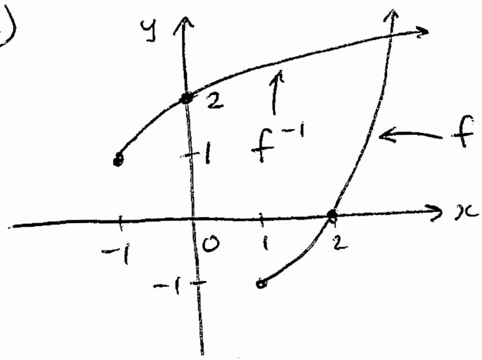


$\therefore$ domain: $x \geq 1$

b) For f^{-1}, $D: x \geq -1$

$R: y \geq 1$

c)



d) $f^{-1}(x) \Rightarrow x = y^2 - 2y$

$y^2 - 2y + 1 = x + 1$

$(y-1)^2 = x+1$

$y - 1 = +\sqrt{x+1}$ only

$y = \sqrt{x+1} + 1$

e) graphs intersect on $y = x$

$\therefore$ solve $x^2 - 2x = x$

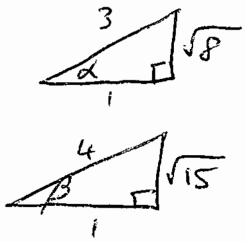
$x^2 - 3x = 0$

$x(x-3) = 0$

$\therefore \underline{x = 3 \text{ only}}$ ($x = 0$ not applicable).

(6) a) Let $\alpha = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow \cos\alpha = \frac{1}{3}$

$\beta = \cos^{-1}\left(\frac{1}{4}\right) \Rightarrow \cos\beta = \frac{1}{4}$



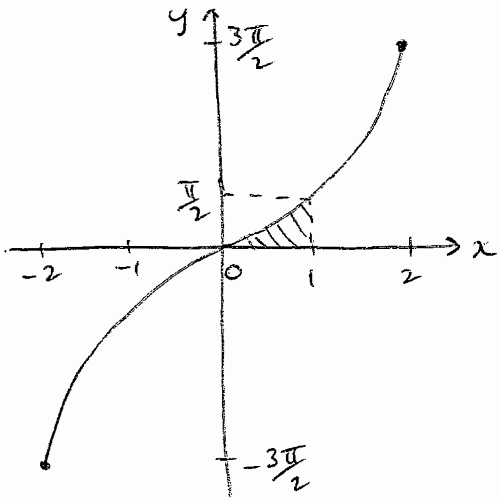
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \frac{1}{3}\times\frac{1}{4} - \frac{\sqrt{8}}{3}\times\frac{\sqrt{15}}{4}$$
$$= \frac{1}{12} - \frac{\sqrt{120}}{12}$$
$$= \frac{1-\sqrt{120}}{12}$$

$\therefore \alpha+\beta = \cos^{-1}\left(\frac{1-\sqrt{120}}{12}\right)$ as reqd.

b) i) $D: -2 \leq x \leq 2$

$R: -\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

ii)



iii) Shaded area = rectangle $- \int_0^{\pi/2} 2\sin\left(\frac{y}{3}\right) dy$

$$\therefore \int_0^1 3\sin^{-1}\left(\frac{x}{2}\right) dx = \frac{\pi}{2} - \left[-2\cos\left(\frac{y}{3}\right)\times 3\right]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi}{2} + 6\left[\cos\frac{y}{3}\right]_0^{\pi/2}$$
$$= \frac{\pi}{2} + 6\left(\frac{\sqrt{3}}{2} - 1\right)$$
$$= \frac{\pi}{2} + 3\sqrt{3} - 6$$