BAULKHAM HILLS HIGH SCHOOL

2015

YEAR 12 June - Task 3

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions
- Start a new page for each question

Total marks – 36
Exam consists of 5 pages.

Standard integrals provided on page 5

Question 1 **(9 marks) Start on the appropriate page of your answer booklet.** **Marks**

a) Find the indefinite integrals:

i) $\int \frac{dx}{\sqrt{x(x-4)}}$ **2**

ii) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$ **2**

iii) $\int \frac{1}{1+e^x} dx$ **2**

b) Evaluate: **3**

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2 - \sin x + 2\cos x}$$

Question 2 **(9 marks) Start on the appropriate page of your answer booklet.**

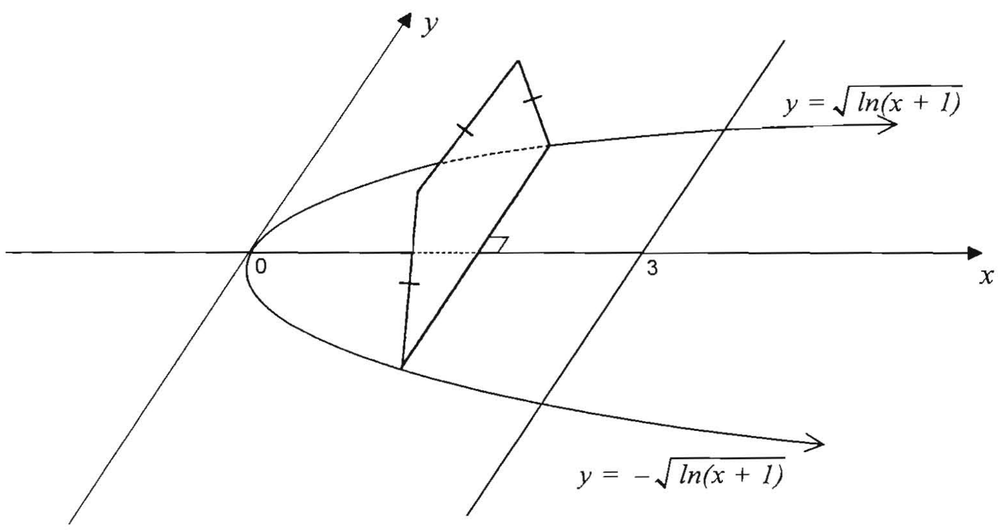
a) i) If $\dfrac{3x^2 - 4x + 3}{(x-1)(x^2 - x + 2)} \equiv \dfrac{A}{x-1} + \dfrac{Bx + C}{x^2 - x + 2}$ **2**

Find A, B and C

ii) Hence determine **2**

$$\int \frac{3x^2 - 4x + 3}{(x-1)(x^2 - x + 2)}\,dx$$

b) The base of a solid is the region bounded by $y = \sqrt{\ln(x+1)}$, $y = -\sqrt{\ln(x+1)}$ and $x = 3$



Each cross-section perpendicular to the x-axis is a trapezium, as shown in the diagram. The trapezium has three equal sides and its base is twice the length of any one of the equal sides.

i) Show that $V = \dfrac{3\sqrt{3}}{4}\displaystyle\int_0^3 \ln(x+1)\,dx$ **2**

ii) Find the volume of the solid. **3**

Question 3 **(8 marks) Start on the appropriate page of your answer booklet.** **Marks**

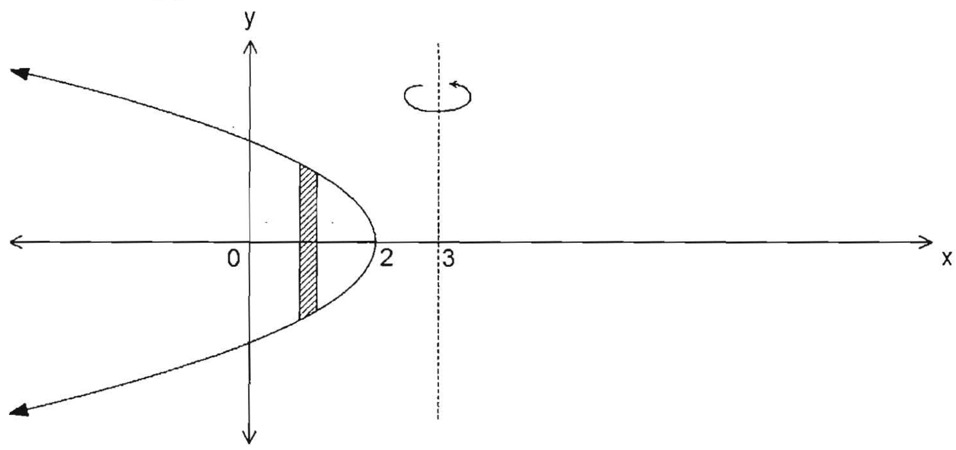
a) By using the relationship $\int_0^a f(a-x)dx = \int_0^a f(x)\,dx$ or otherwise, evaluate **4**

$$\int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}dx$$

b) Find $\int \frac{dx}{x^2\sqrt{x^2+4}}$ **4**

Question 4 **(10 marks) Start on the appropriate page of your answer booklet.** **Marks**

a) The area bounded by $y^2 = 2 - x$ and $x = 0$ is rotated about the line $x = 3$.



Using the method of cylindrical shells,

i) Show that the volume of a cylindrical shell at a distance x from the origin and thickness Δx is given by **1**

$$4\pi(3-x)\sqrt{2-x}\ \Delta x$$

ii) Find the volume of the solid. **4**

b) If $I_n = \int_0^1 x^n\sqrt{1-x^2}dx$

i) Show that **3**

$$I_n = \frac{n-1}{n+2}I_{n-2}$$

ii) Hence evaluate **2**

$$\int_0^1 x^4\sqrt{1-x^2}\,dx$$

-- End of Exam --