Ext 2

Number of Pages Used________

NORTH SYDNEY BOYS HIGH SCHOOL

2009

ASSESSMENT TASK 3

Mathematics

Extension 2

Examiner: GB

General Instructions

- Working time – 60 minutes
- Write in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Barrett
- O Mr Fletcher
- O Mr Weiss

Student Number:

Question No	1	2	3	4	5	6	Total	Total
Mark	$\overline{17}$	$\overline{3}$	$\overline{10}$	$\overline{5}$	$\overline{8}$	$\overline{5}$	$\overline{48}$	$\overline{100}$

(To be used by the exam markers only.)

Question 1 (17 marks) **Marks**

Integrate the following

a) $\displaystyle\int \frac{dx}{x(\log_e x)^2}$ **2**

b) $\displaystyle\int \frac{x + 2}{x^2 + 6}\,dx$ **2**

c) $\displaystyle\int \frac{e^x}{\sqrt{e^{2x} + 1}}\,dx$ **3**

d) $\displaystyle\int_0^{\frac{\pi}{4}} x\sin 2x \, dx$ **3**

e) $\displaystyle\int \frac{3x^2 + 10x - 5}{(x+1)^2(x-2)}\,dx$ Use partial fractions **4**

f) $\displaystyle\int \frac{dx}{\sin x + \tan x}$ **3**

Question 2 (3 Marks)

Solve: $\sin x - \cos x = \dfrac{\sqrt{6}}{2}$ $0 \le x \le 2\pi$

Leave your answer in exact form.

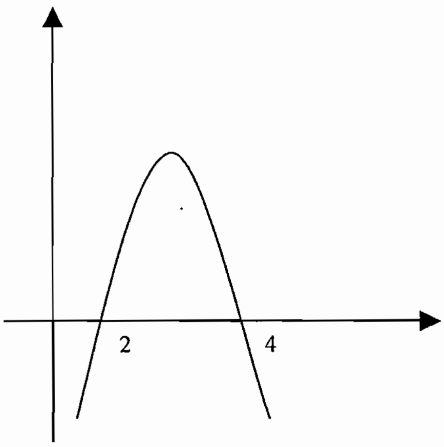
Continued on page 2

Question 3 (10 Marks)

The diagram below shows the graph of the function $y = -x^2 + 6x - 8$.

On separate diagrams and without using calculus, sketch the following graphs.

Indicate clearly any asymptotes and intercepts with the axes.



a) $y = |f(x)|$ 2

b) $y^2 = f(x)$ 3

c) $y = \dfrac{1}{f(x)}$ 3

d) $y = e^{f(x)}$ 2

Continued on page 3

Question 4 (5 Marks)

If $I_n = \int x(\ln x)^n \, dx$

a) Prove that $I_n = \dfrac{x^2(\ln x)^n}{2} - \dfrac{n}{2} I_{n-1}$ **2**

b) Hence find $\int_1^3 x(\ln x)^2 \, dx$. **3**

Question 5 (8 Marks)

Sketch the following curves on separate number planes, showing all their important features.

a) $y^2 = (x - 1)(x - 3)^2$ **3**

b) $|y| = 16 - x^2$ **2**

c) $y = \sin(\cos^{-1} x)$ **3**

Question 6 (5 Marks)

The area bounded by the curve $y = 4 - x^2$ and the x axis is rotated about the line $x = 2$. Use the method of slices to find the volume of the solid formed. Show all your working.

END OF EXAMINATION

2003 EX 2 TASK 3

Q1 a $\int \frac{1}{x} \times (\log_e x)^{-2}\,dx$

$= -1 \times (\log_e x)^{-1} + c$

$= \frac{-1}{\log_e x} + c$ | $\frac{1}{2}$

b) $\int \frac{x+2}{x^2+6}\,dx = \int \frac{x}{x^2+6}\,dx + \int \frac{2}{x^2+6}\,dx$ | 1

$= \frac{1}{2}\log_e(x^2+6) + \frac{2}{\sqrt{6}}\tan^{-1}\left(\frac{x}{\sqrt{6}}\right) + c$ | 1/2

c) $\int \frac{e^x\,dx}{\sqrt{(e^x)^2+1}}$ let $u = e^x$, $du = e^x dx$ | 1

$\Rightarrow \int \frac{du}{\sqrt{u^2+1}} = \ln(u + \sqrt{u^2+1}) + c$ | 1

$= \ln(e^x + \sqrt{e^{2x}+1}) + c$ | 1/3

d) $\int_0^{\frac{\pi}{4}} x\sin 2x\,dx$

$= \left[x \times -\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 1 \times -\frac{1}{2}\cos 2x\,dx$ | 1

$= 0 + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{4}}$ | 1

$= \frac{1}{4}$

1/3

e) $\int \frac{3x^2+10x-5}{(x+1)^2(x-2)}\,dx$

$$\int \frac{ax+b}{(x+1)^2} + \frac{c}{x-2}\,dx$$

$\Rightarrow (ax+b)(x-2)+c(x+1)^2 = 3x^2+10x-5$ 1

$(a+c)=3$ ①

let $x=2 \Rightarrow 9c=27$

$c=3$ ②

② into ① $\Rightarrow a=0$ 1.

$x=0 \Rightarrow -2b+c=-5$

$-2b=-8$

$b=4$

$$\int\left(\frac{4}{(x+1)^2}+\frac{3}{x-2}\right)dx$$

$= \frac{-4}{x+1} + 3\ln(x-2)+c$ 1,1/4

f) $\int \frac{dx}{\sin x + \tan x}$

Let $t = \tan\frac{x}{2}$ $\therefore dx = \frac{2dt}{1+t^2}$

$$\Rightarrow \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2} + \frac{2t}{1-t^2}}$$ 1

$$= \int \frac{\frac{dt}{1+t^2}}{\frac{t(1-t^2) + t(1+t^2)}{(1+t^2)(1-t^2)}}$$

$$= \int \frac{1-t^2 dt}{2t}$$ 1

$$= \int \left(\frac{1}{2t} - \frac{t}{2}\right) dt$$

$$= \frac{1}{2}\ln\left(\tan\frac{x}{2}\right) - \frac{1}{4}\left(\tan\frac{x}{2}\right)^2 + C$$ 1/3 $= \boxed{17}$

Q2

$\sin x - \cos x = \dfrac{\sqrt{6}}{2}$

Let $\quad A\sin(x-\alpha) = \sin x - \cos x = \dfrac{\sqrt{6}}{2}$

$\therefore \quad A\cos\alpha = 1$

$A\sin\alpha = 1$

$\therefore \tan\alpha = 1 \Rightarrow \alpha = \dfrac{\pi}{4}$ | 1 method.

$A^2\cos^2\alpha + A^2\sin^2\alpha = A^2$

$A^2 = 1+1$

$\therefore A = \sqrt{2}$

$\sqrt{2}\sin\left(x - \dfrac{\pi}{4}\right) = \dfrac{\sqrt{6}}{2}$ | 1

$\sin\left(x - \dfrac{\pi}{4}\right) = \dfrac{\sqrt{3}}{2}$

$x - \dfrac{\pi}{4} = \dfrac{\pi}{3}$

or

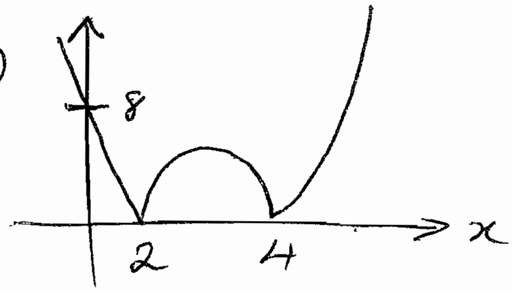
$= \dfrac{2\pi}{3}$

$\therefore \; x = \dfrac{7\pi}{12}$ or $\dfrac{11\pi}{12}$ | 1

1/3 $\boxed{3}$

Q3

a)



1 for y intercept = 8

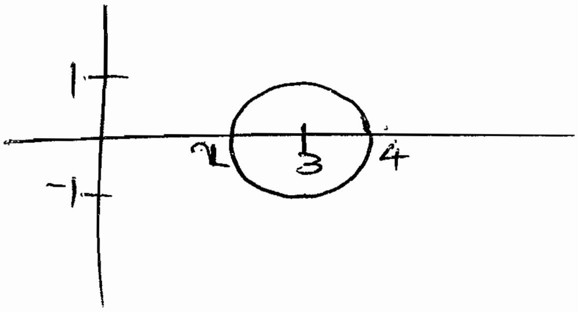
1 for correct shape

/2

b) $y^2 = -x^2 + 6x - 8$

$x^2 + y^2 - 6x = -8$

$(x-3)^2 + y^2 = 1$

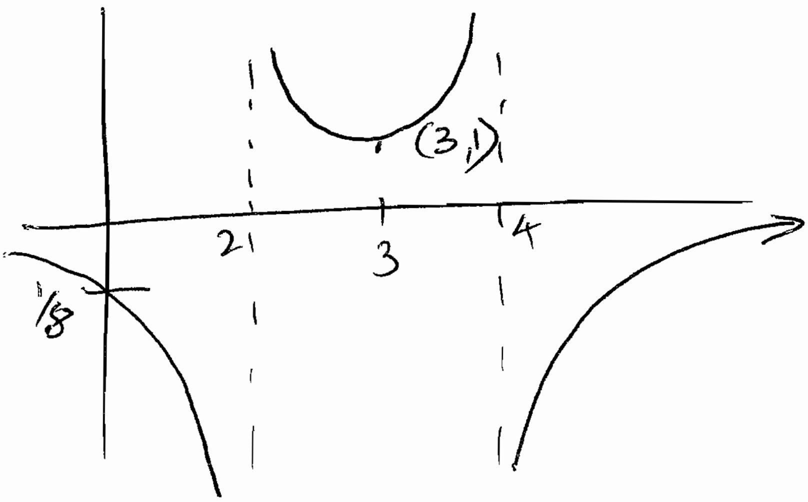


1 for circle

2 for intercepts

/3

c) $y = \dfrac{-1}{x^2 - 6x + 8} = \dfrac{-1}{(x-2)(x-4)}$



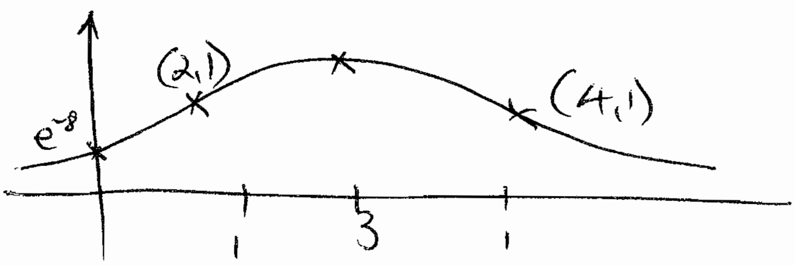
1 Shape

1 Asymptotes

1 intercepts etc

/3

d)



1 – Shape

1 – Nos

/2

10

Q4

a) $I_n = \int x(\ln x)^n dx$

$= \left[\frac{x^2}{2}(\ln x)^n\right] - \frac{n}{2}\int x^2 \times \frac{1}{x}(\ln x)^{n-1} dx$

$= \frac{x^2(\ln x)^n}{2} - \frac{n}{2}\int x(\ln x)^{n-1} dx$

$I_n = \frac{x^2(\ln x)^n}{2} - \frac{n}{2} I_{n-1}$

b) $\int_1^3 x(\ln x)^2 dx = \left[\frac{x^2 \ln x^2}{2}\right]_1^3 - 1 \times \int_1^3 x \ln x \, dx$

$= \frac{9(\ln 3)^2}{2} - 0 - \left\{\left[\frac{x^2 \ln x}{2}\right]_1^3 - \frac{1}{2}\int_1^3 x(\ln x)^0 dx\right\}$

$= 9\frac{(\ln 3)^2}{2} - \frac{9\ln 3}{2} + \frac{1}{2}\left[\frac{x^2}{2}\right]_1^3$

$= 9\frac{(\ln 3)^2}{2} - \frac{9\ln 3}{2} + 2$

$= \frac{9(\ln 3)}{2}\left[\ln 3 - 1\right] + 2$

1 method shown.

must have the expanded form showing understanding of the use of 'parts'

1

1/2

1

1

1/3

$\boxed{5}$

Q5

a) $y^2 = (x-1)(x-3)^2$

$2y\frac{dy}{dx} = (x-1)\times 2(x-3) + (x-3)^2 \times 1$

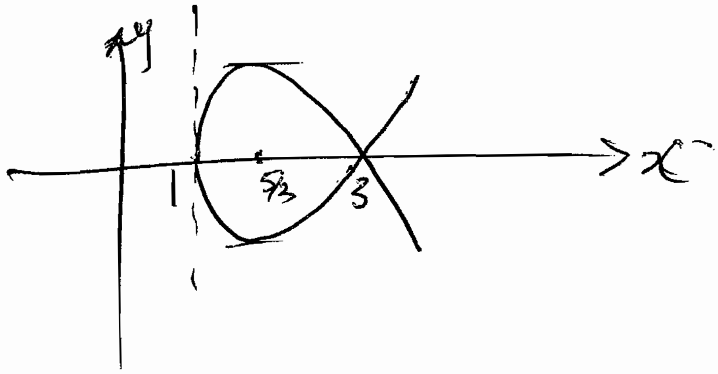
$= (x-3)\left(2(x-1) + (x-3)\right)$

$= (x-3)(3x-5)$ 1

$\therefore \frac{dy}{dx} = \frac{(x-3)(3x-5)}{(x-3)^2(x-1)}$

$\Rightarrow$ Vertical tgt at $x=-1$ 1

$\Rightarrow$ critical point at $x=3$



1 /3

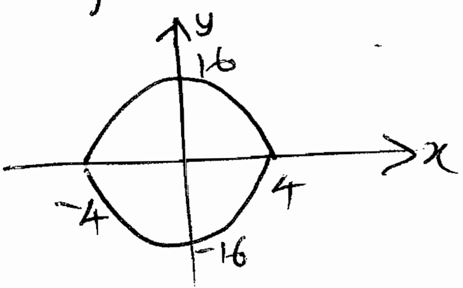
or

D: $x \geq 1$

$y = \pm(x-1)\sqrt{x-1}$ etc

b) $|y| = 16 - x^2$ D: $-4 \leq x \leq 4$

(Note: 2 parabolic a
Not rounded at ± 4



1 shape

1 intercepts

/2.

c) $y = \sin(\cos^{-1} x)$

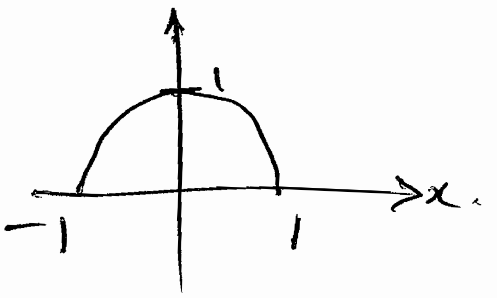
$\cos^{-1} x = \alpha \Rightarrow y = \sin\alpha.$ 1

$\Rightarrow x = \cos\alpha$

$x^2 + y^2 = \sin^2\alpha + \cos^2\alpha = 1$ 1

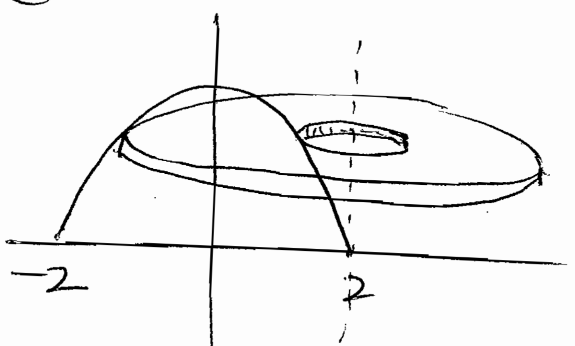
D: $-1 \leq x \leq 1$

R: $0 \leq y \leq 1$



1/3 $\boxed{8}$

Q6



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$ 1

$\delta V = [\pi(2+x)^2 - \pi(2-x)^2]\delta y$

$\delta V = \pi[2+x-(2-x)][2+x+2-x]\delta y$

$= \pi \times 2x \times 4\ \delta y$

$= 8\pi x\,\delta y = 8\pi\sqrt{4-x}\ \delta y$ 1

$V = \lim_{\delta y \to 0} \sum_0^4 8\pi\sqrt{4-x}\ \delta y$

$= 8\pi \int_0^4 \sqrt{4-x}\ dx$ 1

$= 8\pi\left[-\frac{2}{3}(4-y)^{3/4}\right]_0^4$

$= 8\pi\left[0 + \frac{2}{3}4^{3/2}\right]$

$= \frac{128\pi}{3} u^3$ 1

/5

$\boxed{5}$