NORTH SYDNEY GIRLS HIGH SCHOOL

HSC Mathematics Extension 2

Assessment Task 3 Term 2 2013

Name: ___________________________ **Mathematics Class:** 12MZ___

Student Number: ________________

Time Allowed: **60 minutes + 2 minutes reading time**

Available Marks: **47**

Instructions:

- Questions are *not* of equal value.
- Start each question in a new booklet.
- Show all necessary working.
- Do not work in columns.
- Marks may be deducted for incomplete or poorly arranged work.

Section	I 1-2	I 3-5	II Q6	II Q7	II Q8a	II Q8b	Total
E2					/7		/7
E3		/3					/3
E4				/14			/14
E8	/2		/14				/16
E9						/7	/7
							/47

Section I: Multiple Choice

5 marks
Attempt all questions.
Answer on the multiple choice answer sheet provided for Section I.

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1. In evaluating the integral $\int f(x)\,dx$, the substitution $t = \tan 2x$ is made.

Which of the following is the correct expression for dx?

(A) $dx = \dfrac{8dt}{1+t^2}$ (B) $dx = \dfrac{2dt}{1+t^2}$

(C) $dx = \dfrac{dt}{1+t^2}$ (D) $dx = \dfrac{dt}{2(1+t^2)}$

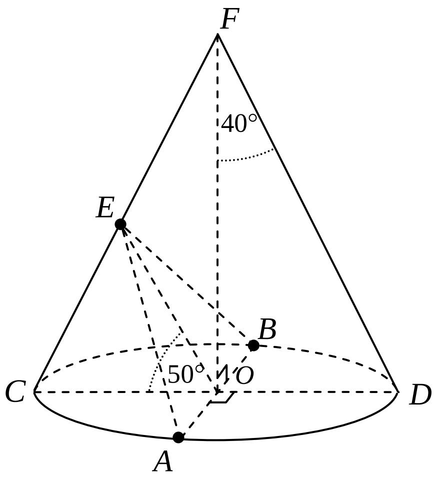
2. Four students come up with the following four answers to the same indefinite integral.

Given that three of the answers are correct, which is the incorrect answer?

(A) $2\cos^2 x + c$ (B) $-2\sin^2 x + c$

(C) $\cos 2x + c$ (D) $-\sin 2x + c$

3. The semi-vertical angle of a hollow cone is $40°$. The cone is sliced at an angle of $50°$ to the base, so that the slice includes the triangle ABE.



NOT TO SCALE

What is the shape of the cross-section formed?

(A) Circle (B) Ellipse

(C) Hyperbola (D) Parabola

4. The equation $|x-2|+|x+2|=6$ corresponds to an ellipse in the Argand diagram. What is the length of the semi-minor axis of this ellipse?

(A) 3 (B) $\sqrt{5}$

(C) $\sqrt{13}$ (D) 6

5. A conic curve (either an ellipse or a hyperbola) is centred on the origin. A focus is located at $(4,0)$, one of its directrices is $x=9$ and the point $(-6,\ 0)$ lies on the curve.

What is the eccentricity of this conic?

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(B) $\frac{3}{2}$ (D) 2

Section II

Attempt all questions.
Answer each question in a separate writing booklet.
Your responses should include relevant mathematical reasong and/or calculations.

Question 6 (14 marks)

(a) Find $\int 2x\sqrt{2x-3}\,dx$. **3**

(b) (i) Write $\dfrac{8}{(x+2)(x^2+4)}$ in the form $\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$ **2**

(ii) Hence evaluate $\int_0^2 \dfrac{8\,dx}{(x+2)(x^2+4)}$ **3**

(c) Find $\int \dfrac{dx}{(4-x^2)^{\frac{3}{2}}}$ **3**

(d) Evaluate $\int_0^{\frac{1}{2}} \sin^{-1} x\,dx$ **3**

Question 7 (14 marks)

(a) Consider the ellipse $12x^2 + 16y^2 = 192$.

(i) Find the eccentricity of this ellipse. **1**

(ii) Draw a neat sketch of this ellipse, showing the coordinates of the vertices and the foci, and the equations of the directrices. **3**

(iii) Derive the equation of the tangent to the ellipse at the point $P(2, 3)$. **2**

(iv) Consider the triangle PBS', where B is the y intercept of this tangent and S' is the focus furthest from P. Find the area of this triangle. **2**

(b) $P\left(2p, \dfrac{2}{p}\right)$ is a point on the hyperbola $xy = 4$.

(i) Show that the equation of the normal at P is $y = p^2 x - 2p^3 + \dfrac{2}{p}$. **3**

(ii) Find the coordinates of the midpoint M of PQ, where Q is the point where the normal at P meets the x-axis. **2**

(iii) Hence find the equation of the locus of M as P moves on the hyperbola. **1**

Question 8 (14 marks)

(a) (i) Show that the equation of the tangent to the hyperbola $x^2 - y^2 = 4$ at the point $P\left(2\sec\theta, 2\tan\theta\right)$ is $x\sec\theta - y\tan\theta = 2$. **2**

(ii) Show that this tangent intersects the asymptotes of the hyperbola at the points **2**

$$A\left(\frac{2\cos\theta}{1-\sin\theta}, \frac{2\cos\theta}{1-\sin\theta}\right) \text{ and } B\left(\frac{2\cos\theta}{1+\sin\theta}, \frac{-2\cos\theta}{1+\sin\theta}\right).$$

(iii) Hence find the ratio $PA:PB$ as a number that is independent of θ. **3**

(b) Let $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\, dx$, where $n \geq 2$.

(i) Show that $I_n = \dfrac{n-1}{n} I_{n-2}$. **3**

(ii) Show that for any continuous function $f\left(x\right)$, $\int_0^a f\left(x\right) dx = \int_0^a f\left(a-x\right) dx$. **1**

(iii) By first considering $\int_0^{\frac{\pi}{2}} x\sin^6 x\, dx$, use parts (i) and (ii) to evaluate **3**

$$\int_0^{\frac{\pi}{2}} x\left(\sin^6 x + \cos^6 x\right) dx.$$

End of paper

Extension 2 Assessment 3 2013 – Solutions

Section I

1. D

$$t = \tan 2x$$
$$x = \frac{1}{2}\tan^{-1} t$$
$$dx = \frac{dt}{2\left(1+t^2\right)}$$

2. D

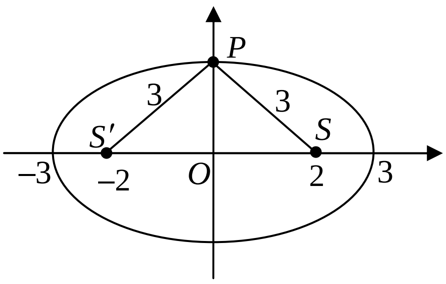
$$\frac{d}{dx}\left(2\cos^2 x\right) = 4\cos x \cdot \left(-\sin x\right) = -4\sin x\cos x = -2\sin 2x$$
$$\frac{d}{dx}\left(-2\sin^2 x\right) = -4\sin x \cdot \cos x = -4\sin x\cos x = -2\sin 2x$$
$$\frac{d}{dx}\left(\cos 2x\right) = -2\sin 2x$$
$$\frac{d}{dx}\left(-\sin 2x\right) = -2\cos 2x$$

3. D

Through simple geometrical arguments, $OE \parallel DF$, so it is a parabola.

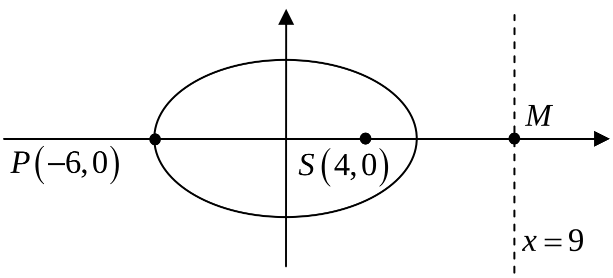
4. B

The equation corresponds to $PS + PS' = 2a$ (property of an ellipse) where a is the semi-major axis, S and S' are the foci, and P is an arbitrary point on the ellipse.



Using Pythagoras', $OP = \sqrt{5}$

5. B



$$
\begin{aligned}
e &= \frac{PS}{PM} \\
&= \frac{10}{15} \\
&= \frac{2}{3}
\end{aligned}
$$

Section II

Question 6

(a)	Find $\int 2x\sqrt{2x-3}\,dx$.	**3**

$$\int 2x\sqrt{2x-3}\,dx = \int (u+3)\cdot u^{\frac{1}{2}}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\int\left(u^{\frac{3}{2}}+3u^{\frac{1}{2}}\right)du$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{\frac{5}{2}}+2u^{\frac{3}{2}}\right)+c$$

$$\boldsymbol{= \frac{1}{5}(2x-3)^{\frac{5}{2}}+(2x-3)^{\frac{3}{2}}+c}$$

Let $u = 2x-3 \quad \Rightarrow \quad 2x = u+3$

$du = 2\,dx \quad \Rightarrow \quad dx = \frac{1}{2}\,du$

OR

$$\int 2x\sqrt{2x-3}\,dx = \int (u^2+3)\cdot u\cdot u\,du$$

$$= \int (u^4+3u^2)\,du$$

$$= \frac{1}{5}u^5+u^3+c$$

$$\boldsymbol{= \frac{1}{5}(2x-3)^{\frac{5}{2}}+(2x-3)^{\frac{3}{2}}+c}$$

Let $u^2 = 2x-3 \quad \Rightarrow \quad 2x = u^2+3$

$2u\,du = 2\,dx \quad \Rightarrow \quad dx = u\,du$

(b)	(i)	Write $\dfrac{8}{(x+2)(x^2+4)}$ in the form $\dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$	**2**

Let $\dfrac{8}{(x+2)(x^2+4)} = \dfrac{A}{x+2}+\dfrac{Bx+C}{x^2+4}$

$$8 = A(x^2+4)+(Bx+C)(x+2)$$

$(x=-2) \quad 8 = 8A \quad \Rightarrow \quad A = 1$

$(x=0) \quad 8 = 4A+2C \quad \Rightarrow \quad 8 = 4+2C \quad \Rightarrow \quad C = 2$

$(x=1) \quad 8 = 5A+3B+3C \quad \Rightarrow \quad 8 = 5+3B+6 \quad \Rightarrow \quad B = -1$

$$\therefore \quad \frac{8}{(x+2)(x^2+4)} = \boldsymbol{\frac{1}{x+2}+\frac{-x+2}{x^2+4}}$$

(ii) Hence evaluate $\displaystyle\int_0^2 \frac{8\,dx}{(x+2)(x^2+4)}$	**3**

$$\int_0^2 \frac{8\,dx}{(x+2)(x^2+4)} = \int_0^2 \left(\frac{1}{x+2} - \frac{x}{x^2+4} + \frac{2}{x^2+4}\right)dx$$

$$= \left[\ln(x+2) - \frac{1}{2}\ln(x^2+4) + \tan^{-1}\frac{x}{2}\right]_0^2$$

$$= \left(\ln 4 - \frac{1}{2}\ln 8 + \frac{\pi}{4}\right) - \left(\ln 2 - \frac{1}{2}\ln 4 + 0\right)$$

$$= 2\ln 2 - \frac{3}{2}\ln 2 + \frac{\pi}{4} - \ln 2 + \ln 2$$

$$\boldsymbol{= \frac{\pi}{4} + \frac{1}{2}\ln 2}$$

(c) Find $\displaystyle\int \frac{dx}{(4-x^2)^{\frac{3}{2}}}$	**3**

$$\int \frac{dx}{(4-x^2)^{\frac{3}{2}}} = \int \frac{2\cos\theta\,d\theta}{(4-4\sin^2\theta)^{\frac{3}{2}}}$$

Let $x = 2\sin\theta$

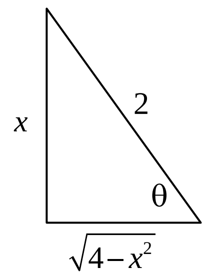
$dx = 2\cos\theta\,d\theta$

$$= \int \frac{2\cos\theta\,d\theta}{8\cos^3\theta}$$

$$= \frac{1}{4}\int \sec^2\theta\,d\theta$$

$$= \frac{1}{4}\tan\theta + c$$

$$\boldsymbol{= \frac{x}{4\sqrt{4-x^2}} + c}$$



OR

$$\int (4-x^2)^{-\frac{1}{2}}\,dx = \int (4-x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(x)\,dx$$

$$= x(4-x^2)^{-\frac{1}{2}} - \int x \cdot -\frac{1}{2}(4-x^2)^{-\frac{3}{2}} \cdot (-2x)\,dx$$

$$= \frac{x}{\sqrt{4-x^2}} - \int x^2(4-x^2)^{-\frac{3}{2}}\,dx$$

$$= \frac{x}{\sqrt{4-x^2}} - \int \left[4-(4-x^2)\right]\cdot(4-x^2)^{-\frac{3}{2}}\,dx$$

$$\int (4-x^2)^{-\frac{1}{2}}\,dx = \frac{x}{\sqrt{4-x^2}} - 4\int (4-x^2)^{-\frac{3}{2}}\,dx + \int (4-x^2)^{-\frac{1}{2}}\,dx$$

$$\int \frac{dx}{(4-x^2)^{\frac{3}{2}}} = \frac{x}{4\sqrt{4-x^2}} + c$$

(d)	Evaluate $\int_0^{\frac{1}{2}} \sin^{-1} x\, dx$	**3**

$$\int_0^{\frac{1}{2}} \sin^{-1} x\, dx = \int_0^{\frac{1}{2}} \sin^{-1} x \cdot \frac{d}{dx}(x) \cdot dx$$

$$= \left[x \sin^{-1} x\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} x \cdot \frac{d}{dx}\left(\sin^{-1} x\right) \cdot dx$$

$$= \frac{1}{2} \cdot \frac{\pi}{6} - 0 - \int_0^{\frac{1}{2}} x \cdot \frac{1}{\sqrt{1-x^2}} \cdot dx$$

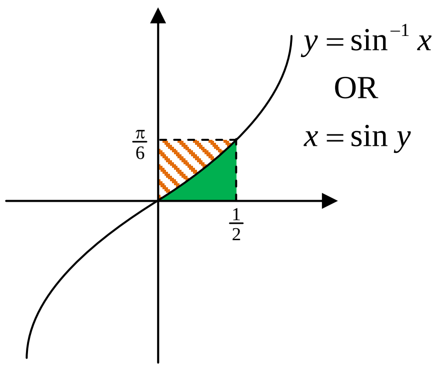
$$= \frac{\pi}{12} + \frac{1}{2} \int_0^{\frac{1}{2}} \left(1-x^2\right)^{-\frac{1}{2}} \cdot (-2x\, dx)$$

$$= \frac{\pi}{12} + \frac{1}{2} \cdot \left[\frac{\left(1-x^2\right)^{\frac{1}{2}}}{1/2}\right]_0^{\frac{1}{2}}$$

$$= \frac{\pi}{12} + \left[\sqrt{1-x^2}\right]_0^{\frac{1}{2}}$$

$$\mathbf{= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}$$

OR



$$\int_0^{\frac{1}{2}} \sin^{-1} x\, dx = \text{Area of solid region}$$

$$= \text{Area of rectangle} - \text{Area of hashed region}$$

$$= \frac{1}{2} \cdot \frac{\pi}{6} - \int_0^{\frac{\pi}{6}} \sin y\, dy$$

$$= \frac{\pi}{12} + \left[\cos y\right]_0^{\frac{\pi}{6}}$$

$$\mathbf{= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1}$$

Question 7

(a) Consider the ellipse $12x^2 + 16y^2 = 192$.

(i) Find the eccentricity of this ellipse. **1**

$$12x^2 + 16y^2 = 192$$

$$(\div 192) \quad \frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$b^2 = a^2\left(1 - e^2\right)$$

$$12 = 16\left(1 - e^2\right)$$

$$1 - e^2 = \frac{3}{4}$$

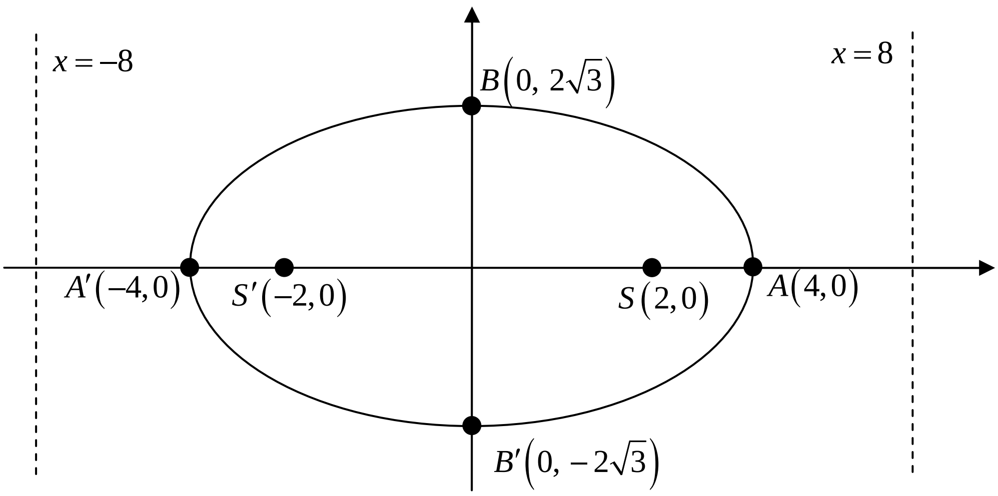
$$e^2 = \frac{1}{4}$$

$$\boldsymbol{e = \frac{1}{2}}$$

(ii) Draw a neat sketch of this ellipse, showing the coordinates of the vertices and the foci, and the equations of the directrices. **3**

$$ae = 4 \times \frac{1}{2} = 2 \quad \Rightarrow \quad S\left(\pm 2,\ 0\right)$$

$$\frac{a}{e} = \frac{4}{1/2} = 8 \quad \Rightarrow \quad \text{directrices: } x = \pm 8$$



(iii)	Derive the equation of the tangent to the ellipse at the point $P(2, 3)$.	**2**

$$12x^2 + 16y^2 = 192$$

$$24x + 32y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{3x}{4y}$$

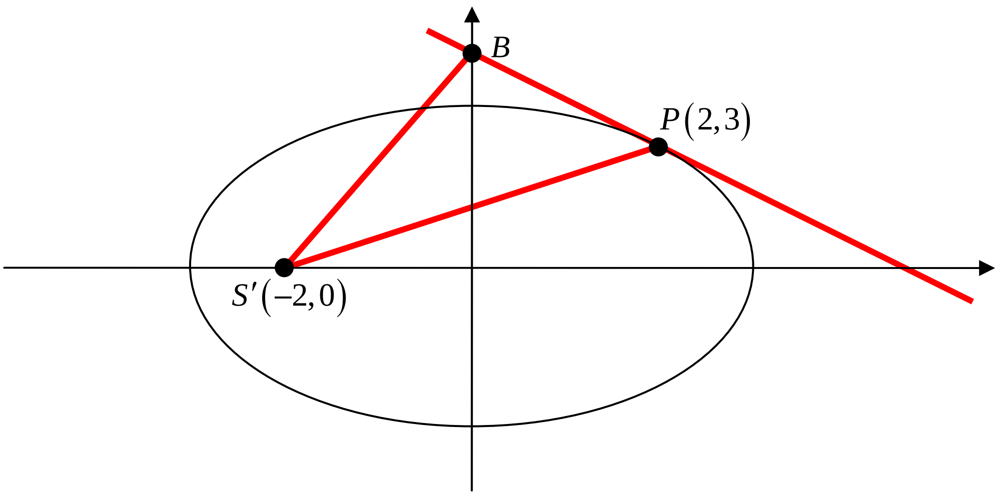
At $P(2,3)$, $m_T = -\frac{1}{2}$

Tangent: $y - 3 = -\frac{1}{2}(x-2)$

$$2y - 6 = -x + 2$$

$$\mathbf{x + 2y - 8 = 0}$$

(iv)	The vertices of a triangle are P; B, the y intercept of this tangent; and S', the focus furthest from P. Find the area of this triangle.	**3**



$B(0, 4)$

$\therefore\ PB = \sqrt{5}$

Perpendicular distance from S' to PB: $d = \frac{|1(-2)+2(0)-8|}{\sqrt{1^2+2^2}}$

$$= \frac{10}{\sqrt{5}}$$

$\therefore$ Area $\Delta PBS' = \frac{1}{2} \times \sqrt{5} \times \frac{10}{\sqrt{5}}$

$$\mathbf{Area = 5\ units^2}$$

(b) $P\left(2p, \frac{2}{p}\right)$ is a point on the hyperbola $xy = 4$.

(i) Show that the equation of the normal at P is $y = p^2x - 2p^3 + \frac{2}{p}$. **3**

$$x = 2p \qquad y = \frac{2}{p}$$

$$\frac{dx}{dp} = 2 \qquad \frac{dy}{dp} = -\frac{2}{p^2}$$

$$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dt}$$
$$= -\frac{2}{p^2} \cdot 2$$
$$= -\frac{4}{p^2}$$
$$\therefore \; m_N = p^2$$

Normal: $y - \frac{2}{p} = p^2\left(x - 2p\right)$

$$= p^2x - 2p^3$$

$$y = p^2x - 2p^3 + \frac{2}{p}$$

(ii) Find the coordinates of the midpoint M of PQ, where Q is the point where the normal at P meets the x-axis. **2**

x-intercepts $\left(y = 0\right)$: $p^2x = 2p^3 - \frac{2}{p}$

$$x = 2p - \frac{2}{p^3}$$

$$\therefore \; Q\left(2p - \frac{2}{p^3}, 0\right), \quad P\left(2p, \frac{2}{p}\right)$$

$$M\left(\frac{1}{2}\left[2p + \left(2p - \frac{2}{p^3}\right)\right], \frac{1}{2}\left(\frac{2}{p} + 0\right)\right) \Rightarrow \boldsymbol{M\left(2p - \frac{1}{p^3}, \frac{1}{p}\right)}$$

(iii) Hence find the equation of the locus of M as P moves on the hyperbola. **1**

$$y = \frac{1}{p} \Rightarrow p = \frac{1}{y}$$

$$x = 2p - \frac{1}{p^3}$$

$$\boldsymbol{x = \frac{2}{y} - y^3}$$

Question 8

(a)	(i)	Show that the equation of the tangent to the hyperbola $x^2 - y^2 = 4$ at the point $P(2\sec\theta,\ 2\tan\theta)$ is $x\sec\theta - y\tan\theta = 2$.	**2**

$$x = 2\sec\theta \qquad y = 2\tan\theta$$

$$\frac{dx}{d\theta} = 2\sec\theta\tan\theta \qquad \frac{dy}{d\theta} = 2\sec^2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dt}$$
$$= \frac{2\sec^2\theta}{2\sec\theta\tan\theta}$$
$$= \frac{\sec\theta}{\tan\theta}$$

Tangent: $y - 2\tan\theta = \dfrac{\sec\theta}{\tan\theta}(x - 2\sec\theta)$

$$y\tan\theta - 2\tan^2\theta = x\sec\theta - 2\sec^2\theta$$
$$x\sec\theta - y\tan\theta = 2(\sec^2\theta - \tan^2\theta)$$
$$x\sec\theta - y\tan\theta = 2$$

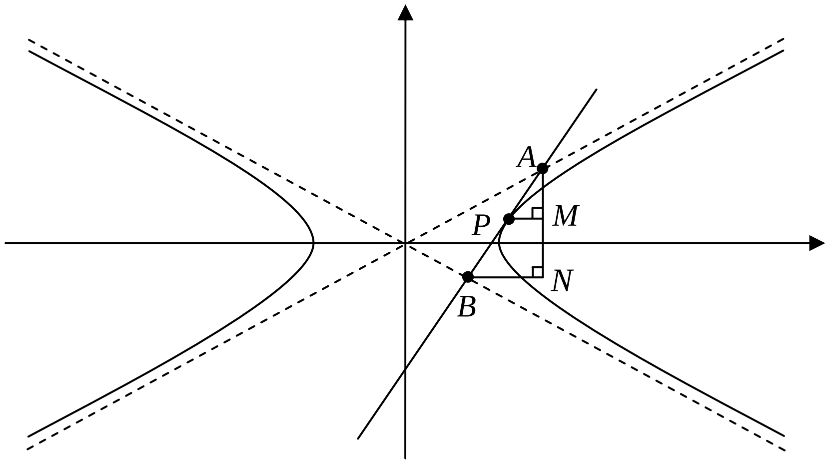
(ii)	Show that this tangent intersects the asymptotes of the hyperbola at the points $A\left(\dfrac{2\cos\theta}{1-\sin\theta}, \dfrac{2\cos\theta}{1-\sin\theta}\right)$ and $B\left(\dfrac{2\cos\theta}{1+\sin\theta}, \dfrac{-2\cos\theta}{1+\sin\theta}\right)$.	**2**

Asymptotes: $y = \pm x$

$$x\sec\theta - (\pm x)\tan\theta = 2$$
$$x(\sec\theta \mp \tan\theta) = 2$$
$$x = \frac{2}{\sec\theta \mp \tan\theta} \times \frac{\cos\theta}{\cos\theta}$$
$$= \frac{2\cos\theta}{1 \mp \sin\theta}$$
$$y = \pm\frac{2\cos\theta}{1 \mp \sin\theta}$$

ie. $A\left(\dfrac{2\cos\theta}{1-\sin\theta}, \dfrac{2\cos\theta}{1-\sin\theta}\right)$, $B\left(\dfrac{2\cos\theta}{1+\sin\theta}, \dfrac{-2\cos\theta}{1+\sin\theta}\right)$

(iii) Hence find the ratio $PA:PB$ as a number that is independent of θ.	**3**



$$\frac{PA}{PB}=\frac{MA}{MN} \quad \text{(parallel lines preserve ratio)}$$

$$=\frac{\dfrac{\cancel{2}\cos\theta}{1-\sin\theta}-\cancel{2}\tan\theta}{\cancel{2}\tan\theta+\dfrac{\cancel{2}\cos\theta}{1+\sin\theta}}\times\frac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=\frac{(1+\sin\theta)\left[\cos\theta-\tan\theta(1-\sin\theta)\right]}{(1-\sin\theta)\left[\tan\theta(1+\sin\theta)+\cos\theta\right]}\times\frac{\cos\theta}{\cos\theta}$$

$$=\frac{(1+\sin\theta)\left(\cos^2\theta-\sin\theta+\sin^2\theta\right)}{(1-\sin\theta)\left(\sin\theta+\sin^2\theta+\cos^2\theta\right)}$$

$$=\frac{(1+\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$=1$$

ie. $PA:PB=1:1$

(b) Let $I_n = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$, where $n \geq 2$.

(i) Show that $I_n = \frac{n-1}{n} I_{n-2}$. **3**

$$\begin{aligned}
I_n &= \int_0^{\frac{\pi}{2}} \cos^n x\,dx \\
&= \int_0^{\frac{\pi}{2}} \cos^{n-1} x \cdot \cos x \cdot dx \\
&= \int_0^{\frac{\pi}{2}} \cos^{n-1} x \cdot d\left(\sin x\right) \\
&= \left[\cos^{n-1} x \cdot \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \cdot d\left(\cos^{n-1} x\right) \\
&= 0 - 0 - \int_0^{\frac{\pi}{2}} \sin x \cdot \left(n-1\right)\cos^{n-2} x \cdot \left(-\sin x\right) \cdot dx \\
&= \left(n-1\right)\int_0^{\frac{\pi}{2}} \cos^{n-2} x \cdot \sin^2 x \cdot dx \\
&= \left(n-1\right)\int_0^{\frac{\pi}{2}} \cos^{n-2} x \cdot \left(1-\cos^2 x\right) \cdot dx \\
&= \left(n-1\right)\int_0^{\frac{\pi}{2}} \cos^{n-2} x\,dx - \left(n-1\right)\int_0^{\frac{\pi}{2}} \cos^n x\,dx \\
&= \left(n-1\right)I_{n-1} - \left(n-1\right)I_n \\
\left[1+\left(n-1\right)\right]I_n &= \left(n-1\right)I_{n-1} \\
I_n &= \frac{n-1}{n} I_{n-1}
\end{aligned}$$

(ii) Show that for any continuous function $f\left(x\right)$, $\int_0^a f\left(x\right)dx = \int_0^a f\left(a-x\right)dx$. **1**

$$\begin{aligned}
\int_0^a f\left(x\right)dx &= \int_a^0 f\left(a-u\right)\cdot\left(-du\right) \\
&= \int_0^a f\left(a-u\right)du \\
&= \int_0^a f\left(a-x\right)dx \quad \text{(indef integ indept of choice of var)}
\end{aligned}$$

Let $x = a-u$

$dx = -du$

$\left(x=0\right) \quad u = a$

$\left(x=a\right) \quad u = 0$

(iii) By first considering $\int_0^{\frac{\pi}{2}} x\sin^6 x\,dx$, use parts (i) and (ii) to evaluate **3**

$\int_0^{\frac{\pi}{2}} x\left(\sin^6 x+\cos^6 x\right)dx$.

$$\int_0^{\frac{\pi}{2}} x\sin^6 x\,dx = \int_0^{\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\sin^6\left(\frac{\pi}{2}-x\right)dx \qquad \text{(from part ii)}$$

$$= \int_0^{\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\cos^6 x\,dx$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}}\cos^6 x\,dx - \int_0^{\frac{\pi}{2}} x\cos^6 x\,dx$$

$$\int_0^{\frac{\pi}{2}} x\sin^6 x\,dx + \int_0^{\frac{\pi}{2}} x\cos^6 x\,dx = \frac{\pi}{2}\int_0^{\frac{\pi}{2}}\cos^6 x\,dx$$

$$\int_0^{\frac{\pi}{2}} x\left(\sin^6 x+\cos^6 x\right)dx = \frac{\pi}{2}I_6 \qquad \text{(where } I_n \text{ is defined as in part i)}$$

$$I_2 = \int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}}(1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x+\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(\frac{\pi}{2}+0-0-0\right)$$

$$= \frac{\pi}{4}$$

$$I_4 = \frac{3}{4}I_2 = \frac{3\pi}{16}$$

$$I_6 = \frac{5}{6}I_4 = \frac{15\pi}{96}$$

$$\therefore \int_0^{\frac{\pi}{2}} x\left(\sin^6 x+\cos^6 x\right)dx = \frac{15\pi^2}{192}$$