NORTH SYDNEY GIRLS' HIGH SCHOOL

HSC Mathematics Extension 2

Assessment Task 3 Term 2 2016

Name: ___________________________ **Teacher:** ____________________

Student Number: ________________

Time Allowed: 60 minutes + 2 minutes reading time

Available Marks: 42

Instructions:

- Start each question in a new booklet.
- Show all necessary working.
- Do not work in columns.
- Marks may be deducted for incomplete or poorly arranged work.

Question	1-2	3-4	5	6a	6b	7a	7b	8ai-ii	8aiii,b	Total
E3	/2			/5						/7
E4						/4		/4		/8
E8		/2	/9		/5		/6			/22
E9									/5	/5
		MC /4	5 /9		6 /10		7 /10		8 /9	/42

Section I

4 marks
Attempt Questions 1–4
Allow about 6 minutes for this section

Use the multiple-choice answer sheet for Questions 1–4.

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1. What is the eccentricity of the ellipse $16x^2 + 25y^2 = 400$?

(A) $\frac{9}{25}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{\sqrt{41}}{5}$

2. What is the equation of the chord of contact from the point $(2,-1)$ to the hyperbola $x^2 - y^2 = 9$?

(A) $2x - y = 9$

(B) $2x + y = 9$

(C) $2x - y = 1$

(D) $2x + y = 1$

3. Consider the integral $\int \frac{dx}{x^2 + bx + c}$.

What is the precise condition under which the primitive will be an inverse tan function?

(A) $b^2 - 4c < 0$

(B) $b^2 - 4c \leq 0$

(C) $b^2 - 4c > 0$

(D) $b^2 - 4c \geq 0$

4. Consider the statement $\int_1^4 f(x)\,dx = \int_1^4 f(a-x)\,dx$.

For which value of a is this statement true for all continuous functions f ?

(A) 0

(B) 1

(C) 4

(D) 5

Section II

38 marks
Attempt Questions 5–8
Allow about 54 minutes for this section
Answer each question in a SEPARATE writing booklet. Extra writing paper is available.

In Questions 5–8, your responses should include relevant mathematical reasoning and/ or calculations.

Question 5 (9 marks)

(a) Evaluate $\displaystyle\int_0^3 \frac{x\,dx}{\sqrt{x^2+16}}$ **2**

(b) Find $\displaystyle\int \cos^3 x \sin^4 x\,dx$ **2**

(c) Find $\displaystyle\int \frac{dx}{\sqrt{6x-x^2-5}}$ **2**

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin x}$ **3**

Question 6 starts on page 5

Question 6 (10 marks) Start a new booklet

(a) (i) Sketch the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, showing the x-intercepts, coordinates of the foci C and D (where C lies on the positive x-axis), equations of the directrices and equations of the asymptotes. **3**

(ii) P is a point on the branch of the hyperbola where $x>0$. **2**
Use the focus-directrix definition of the hyperbola to show that $PD-PC=8$.

(b) (i) Find A and B such that $\frac{1}{25u^2-9}=\frac{A}{5u-3}+\frac{B}{5u+3}$. **2**

(ii) Hence find $\int \frac{\sin x \, dx}{25\cos^2 x-9}$. **3**

Question 7 starts on page 6

Question 7 (10 marks) Start a new booklet

(a) You are given that the equation of the tangent to the hyperbola $xy = c^2$ at the point $\left(ct, \frac{c}{t}\right)$ is $x + t^2 y = 2ct$. (DO NOT PROVE THIS)

(i) Show that the tangents at the distinct points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ intersect at the point $T\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$. **2**

(ii) Find the equation of the locus of T if P and Q vary such that PQ is parallel to the line $y = mx$. **2**

(There is no need to consider any restrictions on the locus.)

(b) (i) Given $I_n = \int x^n \cos x \, dx$, show that $I_n = x^{n-1}\left(x \sin x + n \cos x\right) - n(n-1) I_{n-2}$. **3**

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} x^3 \cos x \, dx$. **3**

Question 8 starts on page 7

Question 8 (9 marks) Start a new booklet

(a) Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b$, and let $P(a\cos\theta,\, b\sin\theta)$ be a point on the ellipse in the first quadrant.

Let O be the origin, and let T and N respectively be the points where the tangent and normal to the ellipse at P meet the y-axis.

(i) Show that the equation of the normal to the ellipse at P is **2**

$$by\cos\theta - ax\sin\theta = \left(b^2-a^2\right)\cos\theta\sin\theta.$$

You are given that the equation of the tangent to the ellipse at P is $ay\sin\theta + bx\cos\theta = ab$.

(ii) Show that $OT\times ON$ is independent of θ. **2**

(iii) A circle with diameter TN is drawn. Use the result of part (ii) to show that the points of intersection of this circle with the x-axis are independent of the location of P. **2**

(b) By using a substitution defined by $x+u=\frac{\pi}{4}$, evaluate $\int_0^{\frac{\pi}{4}} \ln\left(1+\tan x\right)\,dx$. **3**

End of paper

Extension 2 Mathematics Assessment Task 3 Solutions

Section I

Answers Only

1.	C	**2.**	B	**3.**	A	**4.**	D

Worked Solutions

1.

$$16x^2 + 25y^2 = 400$$
$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\begin{aligned} 16 &= 25\left(1-e^2\right) \\ 1-e^2 &= \frac{16}{25} \\ e^2 &= \frac{9}{25} \\ e &= \frac{3}{5} \end{aligned}$$

2.

$$\begin{aligned} \frac{xx_0}{a^2} - \frac{yy_0}{b^2} &= 1 \\ \frac{2x}{9} - \frac{-1y}{9} &= 1 \\ 2x + y &= 9 \end{aligned}$$

3. Require $x^2 + bx + c$ to be in form $(x-B)^2 + C$ where $C > 0$

ie. $\Delta < 0$

$$b^2 - 4c < 0$$

4. Let

$$\begin{aligned} x &= a-u \\ dx &= -du \\ (x=1) \quad u &= a-1 \\ (x=4) \quad u &= a-4 \end{aligned}$$

$$\begin{aligned} \int_1^4 f(x)\,dx &= \int_{a-1}^{a-4} f(a-u)(-du) \\ &= \int_{a-4}^{a-1} f(a-x)\,dx \end{aligned}$$

$$a-4=1 \quad \Rightarrow \quad a=5$$
$$a-1=4 \quad \Rightarrow \quad a=5$$

Section II

Question 5

Overall comments:

Students need to avoid making costly careless errors on questions they know how to do.

Too many of the responses are basically illegible. It is not for the marker to guess what you know. Students are taking a huge risk if they do not make the effort to write clearly. Watch out in the TRIAL.

(a) Evaluate $\int_0^3 \frac{x\,dx}{\sqrt{x^2+16}}$ **2**

$$\int_0^3 \frac{x\,dx}{\sqrt{x^2+16}} = \frac{1}{2}\int_0^3 \left(x^2+16\right)^{-\frac{1}{2}} \cdot 2x\,dx$$

$$= \frac{1}{2}\left[\frac{\left(x^2+16\right)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^3$$

$$= \left[\sqrt{x^2+16}\right]_0^3$$

$$= 5-4$$

$$= 1$$

Many students used substitution rather that recognising that they can immediately use reverse chain rule. This meant more lines of working for these students.

.(b) Find $\int \cos^3 x \sin^4 x\,dx$ **2**

$$\int \cos^3 x \sin^4 x\,dx = \int \cos x\left(1-\sin^2 x\right)\sin^4 x\,dx$$

$$= \int \left(\sin^4 x - \sin^6 x\right)\cdot \cos x\,dx$$

$$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$$

Very well done.

(c) Find $\int \frac{dx}{\sqrt{6x-x^2-5}}$ **2**

$$\int \frac{dx}{\sqrt{6x-x^2-5}} = \int \frac{dx}{\sqrt{-\left(x^2-6x+9\right)-5+9}}$$

$$= \int \frac{dx}{\sqrt{4-(x-3)^3}}$$

$$= \sin^{-1}\frac{x-3}{2} + c \qquad \left[\text{or } -\cos^{-1}\frac{x-3}{2} + c\right]$$

Generally well done. A number of students used the incorrect inverse trig result despite it being available on the reference sheet.

(d) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin x}$ **3**

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2+\sin x} = \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{2+\frac{2t}{1+t^2}}$$

$$= \int_0^1 \frac{2dt}{2\left(1+t^2\right)+2t}$$

$$= \int_0^1 \frac{dt}{t^2+t+1}$$

$$= \int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{2t+1}{\sqrt{3}}\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

Let $t = \tan\frac{x}{2}$

$x = 2\tan^{-1} t$

$dx = \frac{2\,dt}{1+t^2}$

$(x=0)$ $t = 0$

$\left(x=\frac{\pi}{2}\right)$ $t = 1$

Same as part (c). You have a reference sheet. If you need it, USE IT.

Question 6

(a) (i) Sketch the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, showing the x-intercepts, coordinates of the foci C and D (where C lies on the positive x-axis), equations of the directrices and equations of the asymptotes. **3**

$9 = 16\left(e^2-1\right)$

$e^2-1 = \frac{9}{16}$

$e^2 = \frac{25}{16}$

$e = \frac{5}{4}$

$ae = 4\times\frac{5}{4} = 5$

$\therefore$ Foci $(\pm 5,\ 0)$

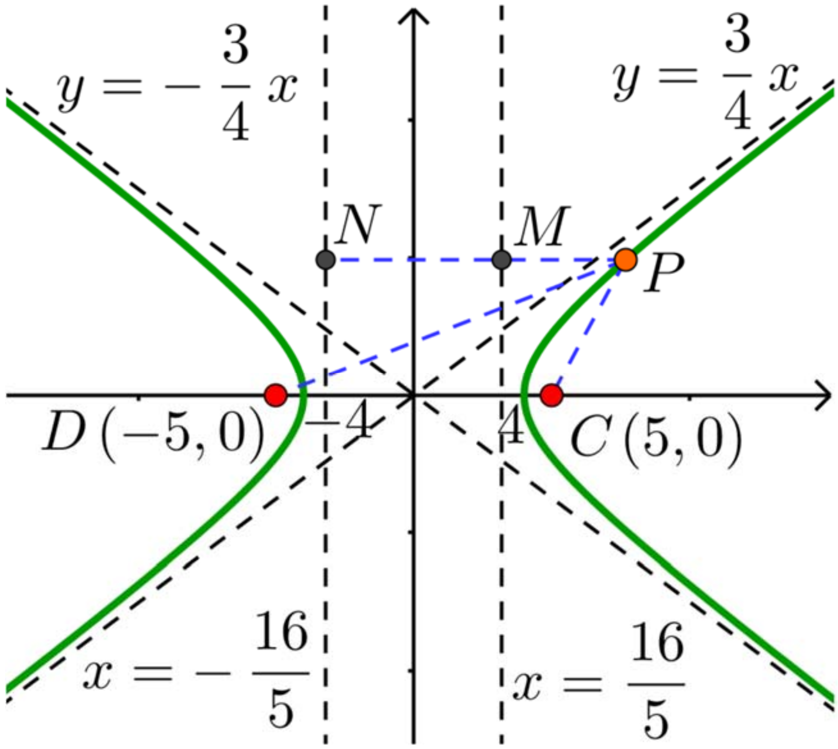
$\frac{a}{e} = \frac{4}{5/4} = \frac{16}{5}$

$\therefore$ Direct: $x = \pm\frac{16}{5}$

$\frac{b}{a} = \frac{3}{4}$

$\therefore$ Asympt: $y = \pm\frac{3}{4}x$

Diagram for parts (i) and (ii):



Generally well done. Most errors were careless numerical ones.

Please make your graph asymptotic to the as ymptotes.

(ii) P is a point on the branch of the hyperbola where $x > 0$. **2**
Use the focus-directrix definition of the hyperbola to show that $PD - PC = 8$.

$$\begin{aligned} PD - PC &= e \cdot PN - e \cdot PM && \text{(focus-directrix definition)} \\ &= \frac{5}{4}(PN - PM) \\ &= \frac{5}{4} \cdot MN && (\text{positive for } x > 0) \\ &= \frac{5}{4} \cdot \frac{32}{5} \\ &= 8 \end{aligned}$$

The focus-directrix definition is **NOT** $|PS - PS'| = 2a$.
There is no mention of the directrix in that statement.
The aim was to **prove** that statement for this specific example.
Students who merely substituted into this result could not be awarded any marks.

As this was a **SHOW** question, all steps had to be shown. Students who stated that $PN - PM = \frac{16}{5} - \left(-\frac{16}{5}\right)$ without either stating that $PN - PM = MN$ or doing a calculation involving the x coordinate of P were not awarded full marks, because neither PN nor PM are equal to those values.

(b) (i) Find A and B such that $\dfrac{1}{25u^2-9} = \dfrac{A}{5u-3}+\dfrac{B}{5u+3}$. **2**

$$\frac{1}{25u^2-9} = \frac{A}{5u-3}+\frac{B}{5u+3}$$
$$1 = A(5u+3)+B(5u-3)$$

$$\left(u=\frac{3}{5}\right) \quad 1 = 6A \quad \Rightarrow \quad A=\frac{1}{6}$$
$$\left(u=-\frac{3}{5}\right) \quad 1 = -6B \quad \Rightarrow \quad B=-\frac{1}{6}$$

Only careless errors arose .

(ii) Hence find $\displaystyle\int \frac{\sin x\, dx}{25\cos^2 x - 9}$. **3**

Let $u = \cos x$

$du = -\sin x\, dx$

$$\int \frac{\sin x\, dx}{25\cos^2 x-9} = -\int \frac{du}{25u^2-9}$$
$$= \frac{1}{6}\int\left(\frac{1}{5u+3}-\frac{1}{5u-3}\right)du$$
$$= \frac{1}{30}\{\ln|5u+3| - \ln|5u-3|\}$$
$$= \frac{1}{30}\ln\left|\frac{5\cos x+3}{5\cos x-3}\right| + c$$

A very large number of students left their answer in terms of u

Question 7

(a) You are given that the equation of the tangent to the hyperbola $xy=c^2$ at the point $\left(ct, \dfrac{c}{t}\right)$ is $x+t^2y=2ct$. (DO NOT PROVE THIS)

(i) Show that the tangents at the distinct points $P\left(cp, \dfrac{c}{p}\right)$ and $Q\left(cq, \dfrac{c}{q}\right)$ intersect at the point $T\left(\dfrac{2cpq}{p+q}, \dfrac{2c}{p+q}\right)$. **2**

Tangent at P: $\quad x+p^2y = 2cp \quad$ ①

Tangent at Q: $\quad x+q^2y = 2cq \quad$ ②

①−②:

$$\begin{aligned}\left(p^2-q^2\right)y &= 2c\left(p-q\right)\\ \cancel{\left(p-q\right)}\left(p+q\right)y &= 2c\cancel{\left(p-q\right)}\\ y &= \frac{2c}{p+q}\end{aligned}$$

Sub in ①:

$$\begin{aligned}x+\frac{2cp^2}{p+q} &= 2cp\\ x &= 2cp-\frac{2cp^2}{p+q}\\ &= \frac{2cp\left(p+q\right)-2cp^2}{p+q}\\ &= \frac{2cp^2+2cpq-2cp^2}{p+q}\\ &= \frac{2cpq}{p+q}\end{aligned}$$

$$\therefore\ T\left(\frac{2cpq}{p+q},\frac{2c}{p+q}\right)$$

Again, this was a **SHOW** question – the answer was given. So you had to show all algebraic steps, in particular the factorisation of p^2-q^2 before cancelling.

The question specifically said DO NOT PROVE THIS, yet many students wasted time deriving the equation of the tangent.

Using the chord of contact formula is the reverse of the logic required here.

(ii) Find the equation of the locus of T if P and Q vary such that PQ is parallel to the line $y = mx$. **2**

(There is no need to consider any restrictions on the locus.)

Restriction:

$$m_{PQ} = m$$

$$\frac{\frac{c}{p}-\frac{c}{q}}{cp-cq} = m$$

$$\left[\times\frac{pq}{pq},\ \div\frac{c}{c}\right]\quad \frac{q-p}{pq\left(p-q\right)} = m$$

$$pq = -\frac{1}{m}$$

$$\frac{y}{x} = \frac{\frac{2c}{p+q}}{\frac{2cpq}{p+q}} = -\frac{1}{pq} = -m$$

$$y = -mx$$

Most students could derive $pq=-\frac{1}{m}$, but a significant number could not substitute this correctly to get the required answer. As usual, omission of negative signs was a common issue.

(b) (i) Given $I_n = \int x^n \cos x \, dx$, show that $I_n = x^{n-1}(x\sin x + n\cos x) - n(n-1)I_{n-2}$. **3**

$$
\begin{aligned}
I_n &= \int x^n \cos x \, dx \\
&= \int x^n \cdot d(\sin x) \\
&= x^n \sin x - \int \sin x \cdot d(x^n) \\
&= x^n \sin x - \int \sin x \cdot nx^{n-1} \, dx \\
&= x^n \sin x + n\int x^{n-1} \cdot d(\cos x) \\
&= x^n \sin x + nx^{n-1} \cos x - n\int \cos x \cdot d(x^{n-1}) \\
&= x^n \sin x + nx^{n-1} \cos x - n\int \cos x \cdot (n-1)x^{n-2} \, dx \\
&= x^{n-1}(x\sin x + n\cos x) - n(n-1)I_{n-2}
\end{aligned}
$$

Not well done by a significant portion of the grade.

Some students did not know the IBP rule correctly.

Others did not get their signs correct, either because they did not know the correct integrals for $\sin x$ and $\cos x$, or because care was not taken in the algebraic manipulation.

Many did not make the correct choice for u and v − use **LIATE**.

(ii) Hence evaluate $\int_0^{\frac{\pi}{2}} x^3 \cos x \, dx$. **3**

Let $J_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx$

$$
\begin{aligned}
&= \left[x^n \sin x + nx^{n-1}\cos x\right]_0^{\frac{\pi}{2}} - n(n-1)J_{n-2} \\
&= \left(\frac{\pi}{2}\right)^n - n(n-1)J_{n-2}
\end{aligned}
$$

$$
\begin{aligned}
J_1 &= \int_0^{\frac{\pi}{2}} x\cos x \, dx \\
&= \int_0^{\frac{\pi}{2}} x \cdot d(\sin x) \\
&= \left[x\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx \\
&= \left[x\sin x + \cos x\right]_0^{\frac{\pi}{2}} \\
&= \frac{\pi}{2} - 1
\end{aligned}
$$

$$
\begin{aligned}
J_3 &= \left(\frac{\pi}{2}\right)^3 - 3\cdot 2\cdot\left(\frac{\pi}{2} - 1\right) \\
&= \left(\frac{\pi}{2}\right)^3 - 3\pi + 6
\end{aligned}
$$

The integral for I_1 must be evaluated. You cannot use the rule from part (i) to infer it.

Many students did not interpret the rule correctly. I_3 depends on I_1, **NOT** on I_2.

Other students evaluated the **definite** integral only for the I_1 component, and did not substitute into x for the rest of the expression.

As usual, there were careless errors involving negative signs and expanding brackets.

Question 8

(a) Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $a>b$, and let $P(a\cos\theta,\, b\sin\theta)$ be a point on the ellipse in the first quadrant.

Let O be the origin, and let T and N respectively be the points where the tangent and normal to the ellipse at P meet the y-axis.

(i) Show that the equation of the normal to the ellipse at P is **2**

$$by\cos\theta - ax\sin\theta = \left(b^2-a^2\right)\cos\theta\sin\theta.$$

$$x = a\cos\theta \qquad\qquad y = b\sin\theta$$
$$dx = -a\sin\theta\, d\theta \qquad\qquad dy = b\cos\theta\, d\theta$$

$$\frac{dy}{dx} = \frac{b\cos\theta\, d\theta}{-a\sin\theta\, d\theta} = -\frac{b\cos\theta}{a\sin\theta}$$

$$\therefore m_N = \frac{a\sin\theta}{b\cos\theta}$$

Normal:

$$\begin{aligned} y-b\sin\theta &= \frac{a\sin\theta}{b\cos\theta}(x-a\cos\theta) \\ by\cos\theta - b^2\sin\theta\cos\theta &= ax\sin\theta - a^2\sin\theta\cos\theta \\ by\cos\theta - ax\sin\theta &= \left(b^2-a^2\cos\theta\sin\theta\right) \end{aligned}$$

Very well done. Students who did not <u>show</u> how they obtained $\frac{dy}{dx}$ lost $\frac{1}{2}$ a mark.

You are given that the equation of the tangent to the ellipse at P is $ay\sin\theta + bx\cos\theta = ab$.

(ii) Show that $OT\times ON$ is independent of θ. **2**

Normal: $by\cos\theta - ax\sin\theta = \left(b^2 - a^2\right)\cos\theta\sin\theta$

$(x = 0)$ $by\cos\theta = \left(b^2 - a^2\right)\cos\theta\sin\theta$

$$y = \frac{b^2 - a^2}{b}\sin\theta \quad < 0 \text{ since } a > b \text{ and } \sin\theta > 0 \text{ (1st quadrant)}$$

$$\text{So } ON = \frac{a^2 - b^2}{b}\sin\theta$$

Tangent: $ay\sin\theta + bx\cos\theta = ab$

$(x = 0)$ $ay\sin\theta = ab$

$$y = \frac{b}{\sin\theta} \quad > \quad 0 \quad \text{since} \quad \sin\theta > 0 \text{ (1st quadrant)}$$

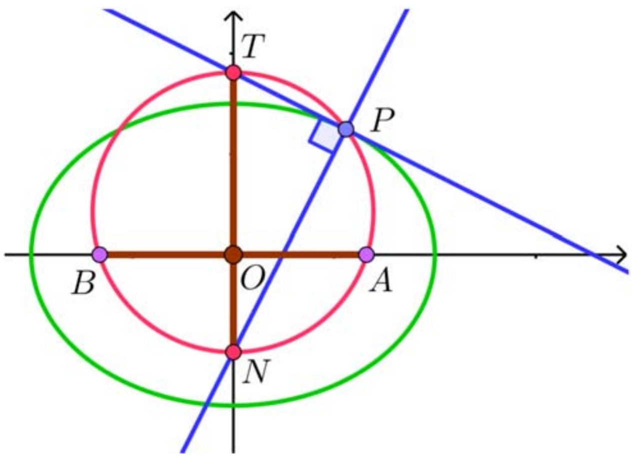
$$\text{So } OT = \frac{b}{\sin\theta}$$

$OT \cdot ON = a^2 - b^2$ which is independent of θ

Almost all students found the $y-$intercepts correctly. Most students then did not consider whether the intercepts were positive or negative values when determining the lengths of *OT* and *ON*. Students who at least used absolute value signs were awarded full marks.

This error most likely occurred because students failed to draw a useful diagram. Many of the sketches that were done were very small and rough and so did not really provide the student any benefit when developing an answer.

(iii) A circle with diameter TN is drawn. Use the result of part (ii) to show that the points of intersection of this circle with the x-axis are independent of the location of P. **2**



Let the x-intercepts of the circle be A and B.
$OA \cdot OB = OT \cdot ON$ (products of intercepts of chords)
Since $OT \cdot ON$ is a constant then $OA \cdot OB$ is also a constant.

But $OA = OB$ (axes are perpendicular, and diameter perpendicular to chord bisects the chord)

$\therefore$ OA and OB are constants

ie. the x-intercepts of the circle are independent of the choice of P

Draw a diagram to help with your response.

Some students recognised that they needed to use circle geometry but could not progress any further.

Some students managed to show the x intercepts were independent of P but did not use the result in part (ii). These students could only earn a maximum of 1 mark if they were able to develop an alternate method correctly.

(b) (i) Show that $\dfrac{\ln x}{(1+\ln x)^2} = \dfrac{1}{1+\ln x} - \dfrac{1}{(1+\ln x)^2}$. **1**

$$\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2} = \frac{(1+\ln x)-1}{(1+\ln x)^2}$$

$$= \frac{\ln x}{(1+\ln x)^2}$$

This was an easy mark for most students although quite a few used partial fractions where basic algebra would suffice.

(ii) Hence find $\displaystyle\int \frac{\ln x}{(1+\ln x)^2}\,dx$. **2**

$$\int \frac{\ln x}{(1+\ln x)^2}\,dx = \int \left(\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2}\right) dx$$

$$= \int \left(\frac{1}{1+\ln x} \times 1 \quad + \quad x \times \frac{-1}{(1+\ln x)^2}\cdot\frac{1}{x}\right) dx$$

$$= \int \left[\frac{1}{1+\ln x}\cdot\frac{d}{dx}(x) \quad + \quad x\cdot\frac{d}{dx}\left(\frac{1}{1+\ln x}\right)\right] dx$$

$$= \int \frac{d}{dx}\left[x\cdot\frac{1}{1+\ln x}\right] dx \qquad \text{(product rule for differentiation)}$$

$$= \frac{x}{1+\ln x} + c$$

Alternatively:

$$
\begin{aligned}
\int \frac{\ln x}{(1+\ln x)^2}\,dx &= \int \left(\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2} \right) dx \\
&= \int \frac{1}{1+\ln x} \cdot d(x) - \int \frac{dx}{(1+\ln x)^2} \\
&= x \cdot \frac{1}{1+\ln x} - \int x \cdot d\left(\frac{1}{1+\ln x} \right) - \int \frac{dx}{(1+\ln x)^2} \qquad \text{[IBP]} \\
&= \frac{x}{1+\ln x} - \int \cancel{x} \cdot -\frac{1}{(1+\ln x)^2} \cdot \frac{1}{\cancel{x}}\,dx - \int \frac{dx}{(1+\ln x)^2} \\
&= \frac{x}{1+\ln x} + \int \frac{dx}{(1+\ln x)^2} - \int \frac{dx}{(1+\ln x)^2} \\
&= \frac{x}{1+\ln x}
\end{aligned}
$$

This question proved difficult for the majority of students. Students need to recognise the word ‘hence’ in this question and use the hint that was given to them in (i).