Ravenswood

16 June 2016

Assessment 3/ HSC Assessment 2
Conics and Integration

4 Copies	**Year 12**	**Mrs Kim**

Mathematics Extension 2

PART I: CONICS

General Instructions

- Working time **40 minutes**
- Write using blue or black pen
- Draw diagrams in pencil
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks will be deducted for careless or badly arranged work.

Total marks (28)

- Use the Multiple Choice answer sheet provided for Section I
- Answer the questions in answer booklets marked with your student number and the Question Number for Section II
- If you have not attempted a question or part of a question, write the question number with "NO ATTEMPT" beside it.

Section I Multiple Choice

There are 4 questions in this section. Use the multiple choice sheet provided to record your answers.

1. What are the coordinates of the foci of the ellipse with equation $\frac{x^2}{64}+\frac{y^2}{36}=1$?

   (A) $(-10, 0)$ and $(10, 0)$

   (B) $\left(-2\sqrt{7},0\right)$ and $\left(2\sqrt{7},0\right)$

   (C) $\left(0,-2\sqrt{7}\right)$ and $\left(0,2\sqrt{7}\right)$

   (D) $(-28, 0)$ and $(28, 0)$

2. What is the equation of the directrices of the hyperbola with parametric coordinates $\left(10\sec\theta, 5\tan\theta\right)$?

   (A) $x=\pm 5\sqrt{3}$

   (B) $x=\pm 5\sqrt{5}$

   (C) $x=\pm 4\sqrt{3}$

   (D) $x=\pm 4\sqrt{5}$

3. The ellipse with a focus at $(4, 0)$ and directrix $x=8$ has equation:

   (A) $\frac{x^2}{16}+\frac{y^2}{32}=1$

   (B) $\frac{x^2}{32}+\frac{y^2}{16}=1$

   (C) $\frac{x^2}{16}-\frac{y^2}{32}=1$

   (D) $\frac{x^2}{32}-\frac{y^2}{16}=1$

4. What is the equation of the chord of contact from the point $\left(x_0, y_0\right)$ to the hyperbola $\frac{x^2}{10}-\frac{y^2}{5}=1$?

(A) $\frac{xx_0}{10}-\frac{yy_0}{5}=1$

(B) $\frac{xx_0}{5}-\frac{2yy_0}{5}=1$

(C) $\frac{x_0^2}{10}-\frac{y_0^2}{5}=1$

(D) $\frac{x_0 \sec\theta}{10}-\frac{y_0 \tan\theta}{5}=1$

End of Section I

Section II.

There are 2 questions in this section.

Complete your solutions in the booklets provided. Please start each question in a new booklet.

Question 5 (12 Marks)

(a) Draw a neat sketch of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$. **3**

On your diagram show the coordinates of the foci, the equations of the directrices and asymptotes.

(b) A hyperbola has the equation: $\frac{x^2}{3}-\frac{y^2}{2}=1$.

(i) Find the coordinates of the foci. **2**

(ii) The equation of the tangent to the hyperbola at $P(\sqrt{3}\sec\theta, \sqrt{2}\tan\theta)$ is:

$$\sqrt{2}x\sec\theta-\sqrt{3}y\tan\theta=\sqrt{6}.$$

(Do not prove this equation)

Show that the equation of the normal at P is: **2**

$$\sqrt{3}x\tan\theta+\sqrt{2}y\sec\theta=5\sec\theta\tan\theta.$$

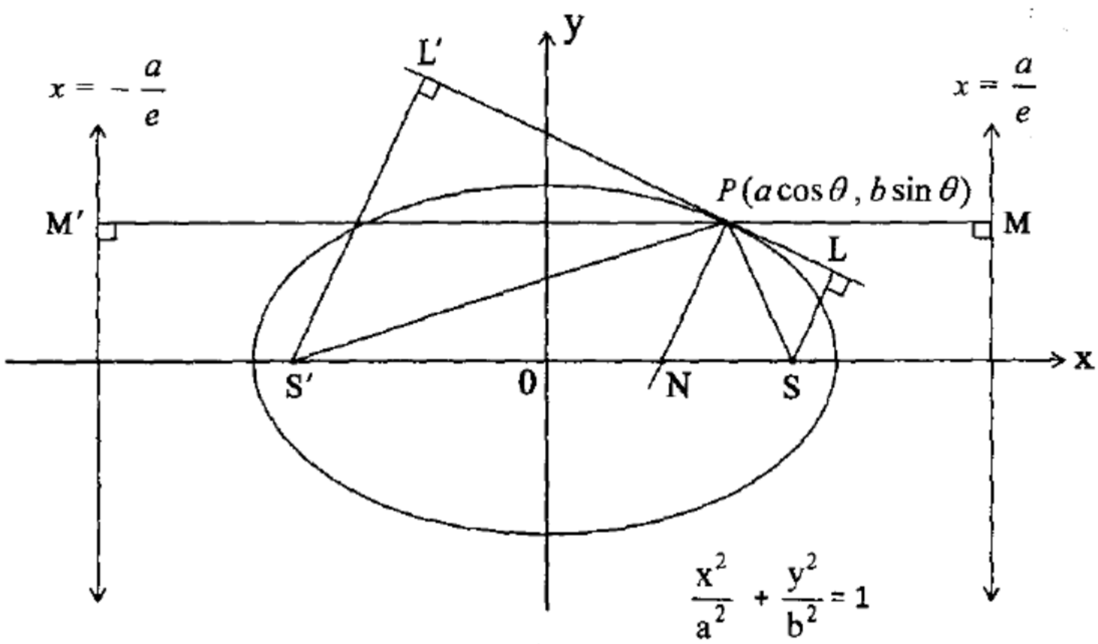
(iii) The tangent and normal to the hyperbola at P cut the y axis at T and N respectively.

Show that the circle with TN as diameter passes through the foci of the hyperbola. **5**

Question 6 (12 Marks)

Lines drawn from the foci S and S' of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, are perpendicular to the tangent drawn at $P(a\cos\theta, b\sin\theta)$. They meet this tangent at L and L' respectively.

The line parallel to the x – axis passing though P intersects the directrices at M and M' and the normal at P meets the x −axis at N.



(i)	Show that $PS = a(1 - e\cos\theta)$	**2**
(ii)	Write down a similar expression for PS'	**1**
(iii)	Show that the equation of the tangent at P is $bx\cos\theta + ay\sin\theta - ab = 0$.	**2**
(iv)	Find the distances SL and $S'L'$ from the foci S and S' to the tangent at P.	**2**
(v)	Hence, or otherwise, show that PN bisects $\angle SPS'$.	**3**
(vi)	Show that $\frac{PS}{NS} = \frac{PS'}{NS'}$	**2**

End of Section II

Ravenswood

Student Number

MULTIPLE CHOICE ANSWER SHEET
Section I – Questions 1 – 4

Sample
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word correct and drawing an arrow as follows:

correct

A ☒ B ☒ ← C ○ D ○

SECTION 1: Colour in the appropriate circle.

1. A ◯ B ◯ C ◯ D ◯

2. A ◯ B ◯ C ◯ D ◯

3. A ◯ B ◯ C ◯ D ◯

4. A ◯ B ◯ C ◯ D ◯

Raveswood

16 June 2016

Assessment 3/ HSC Assessment 2
Conics and Integration

4 Copies	**Year 12**	**Mrs Kim**

Mathematics Extension 2

<u>PART II: Integration</u>

General Instructions

- Working time **40 minutes**
- Write using blue or black pen
- Draw diagrams in pencil
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks will be deducted for careless or badly arranged work.

Total marks (24)

- Use the Multiple Choice answer sheet provided for Section I
- Answer the questions in answer booklets marked with your student number and the Question Number for Section II
- If you have not attempted a question or part of a question, write the question number with "NO ATTEMPT" beside it.

Section I Multiple Choice

There are 4 questions in this section. Use the multiple choice sheet provided to record your answers.

1. To solve $\int_{\sqrt{3}}^{2} \sqrt{4-x^2}$, which of the following methods would you apply?

   (A) Integration by parts

   (B) Partial fractions

   (C) Substitution with $x = \cos\theta$

   (D) Substitution with $x = 2\sin\theta$

2. The integral of $x^2e^x + 2xe^x$ is:

   (A) $x^2 + e^x + c$

   (B) $x^2e^x + c$

   (C) $xe^x + c$

   (D) $2xe^x + c$

3. Which of the following is an expression for $\int \frac{dx}{\sqrt{7-6x-x^2}}$?

   (A) $\sin^{-1}\left(\frac{x-3}{2}\right)+c$

   (B) $\sin^{-1}\left(\frac{x+3}{2}\right)+c$

   (C) $\sin^{-1}\left(\frac{x-3}{4}\right)+c$

   (D) $\sin^{-1}\left(\frac{x+3}{4}\right)+c$

4. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx$ where n is a positive integer, and $I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x \, dx$ for $n \geq 2$, then $I_n = \frac{n-1}{n} I_{n-2}$, for $n \geq 2$. What is the value of I_4?

(A) $\frac{\pi}{16}$

(B) $\frac{3\pi}{16}$

(C) $\frac{3}{8}$

(D) $\frac{3\pi}{8}$

End of Section I

Section II.

There are 2 questions in this section.

Complete your solutions in the booklets provided. Please start each question in a new booklet.

Question 5 (10 Marks)

(a) Find $\int \frac{x^2}{\sqrt{x^3-1}}\,dx$ **2**

(b) Evaluate $\int_0^{\frac{1}{2}} \sin^{-1} x\,dx$ **3**

(c) (i) Find A, B and C if $\frac{x^2-4x+2}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4}$ **3**

(ii) Hence, evaluate $\int_0^2 \frac{x^2-4x+2}{(2x+1)(x^2+4)}\,dx$. **2**

Leave your answer in simplest exact form.

Question 6 (10 Marks)

(a) Let $t = \tan\frac{\theta}{2}$

(i) Show that $d\theta = \frac{2}{1+t^2}\,dt$ **2**

(ii) Use the substitution $t = \tan\frac{\theta}{2}$ to evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\operatorname{cosec}^2\theta \tan\frac{\theta}{2}\,d\theta$ **3**

Question 6 continues on the following page.

Question 6 (continued)

(b) (i) Given that $I_{2n+1} = \int_0^1 x^{2n+1} e^{x^2}\, dx$ where n is a positive integer, **2**

show that $I_{2n+1} = \frac{1}{2} e - nI_{2n-1}$.

(ii) Hence, or otherwise, evaluate $\int_0^1 x^5 e^{x^2}\, dx$. **3**

End of Section II

Ravenswood

Student Number

MULTIPLE CHOICE ANSWER SHEET

Section I – Questions 1 – 4

Sample

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word correct and drawing an arrow as follows:

A ☒ B ☒ ← *correct* C ○ D ○

SECTION 1: Colour in the appropriate circle.

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

2016 AT3 - CONICS /28

CONICS - Please Learn!

Ellipse

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- $a^2 = b^2(1-e^2)$
- $e = \sqrt{1 - \frac{b^2}{a^2}}$
- Focus : $(\pm ae, 0)$
- Equation of Directrix : $x = \pm \frac{a}{e}$
- Tangent Equation :
  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$
- Normal equation :
  $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$
- chord of contact :
  $\frac{x_0x}{a^2} + \frac{y_0y}{b^2} = 1$

Hyperbola

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- asymptotes : $y = \pm \frac{bx}{a}$
- $b^2 = a^2(e^2 - 1)$
- $e = \sqrt{1 + \frac{b^2}{a^2}}$
- focus : $(\pm ae, 0)$
- directrix : $x = \pm \frac{a}{e}$
- Tangent equation :
  $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
- Normal equation
  $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$
- Chord of contact :
  $\frac{x_0x}{a^2} - \frac{y_0y}{b^2} = 1.$

2016 AT3 – CONICS

Section I (MC)

Q1. $S = (\pm ae, 0)$.

$a = 8, \quad b = 6$

$\therefore S(\pm 8e, 0)$.

$e = \sqrt{1 - \frac{36}{64}}$

$= \frac{2\sqrt{7}}{8}$

$\therefore S(\pm 2\sqrt{7}, 0) \Rightarrow$ Answer $\boxed{B}$

Q2. $a = 10, \quad b = 5$.

$x = \pm \frac{a}{e}$

$= \pm \frac{10}{e}$.

$e = \sqrt{1 + \frac{25}{100}}$

$= \frac{\sqrt{5}}{2}$.

$\therefore x = \pm \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \pm 4\sqrt{5} \Rightarrow \boxed{D}$

Q3. Focus (4, 0).

directrix $x = 8$.

$\therefore 4 = ae$

$x \pm 8 = \frac{a}{e}$.

$e = \sqrt{1 - \frac{16}{32}}$

$= \sqrt{\frac{1}{2}}$

$= \frac{1}{\sqrt{2}}$

$\therefore a = \frac{4}{e}$

$= \frac{4}{\frac{1}{\sqrt{2}}}$

$= 4\sqrt{2} \Rightarrow \boxed{B}$ since ellipse

Q4. chord of contact:

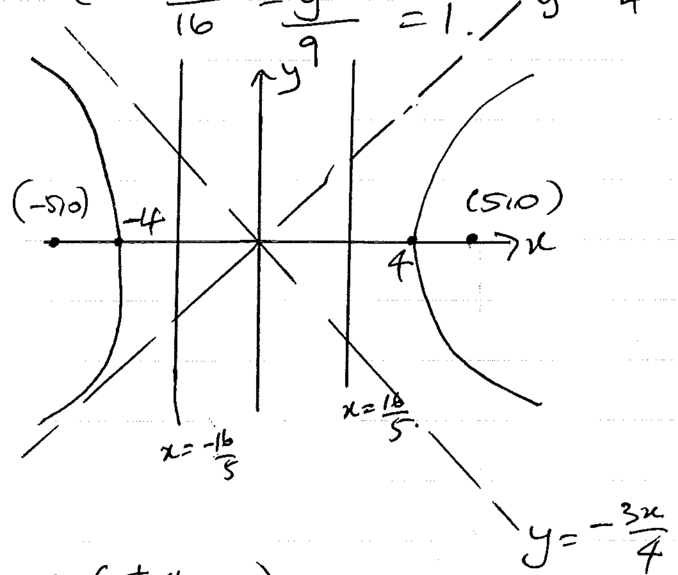
$\frac{x x_0}{a^2} - \frac{y y_0}{b^2} = 1$.

$a^2 = 10, \quad b^2 = 5$.

$\therefore \boxed{A}$.

SECTION II

5. (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



$S(\pm 4, 0)$

$e = \frac{5}{4}$.

(1) shape with vertex & focus.

(2) as above + directrices

(3) as above + asymptotes

5(b) $\frac{x^2}{3} - \frac{y^2}{2} = 1$

$a = \sqrt{3}$, $b = \sqrt{2}$

$e = \sqrt{1 + \frac{2}{3}}$
$= \sqrt{\frac{5}{3}}$ ✓

(i) $S = (\pm ae, 0)$
$= (\pm \sqrt{3} \times \frac{\sqrt{5}}{\sqrt{3}}, 0)$
$= (\pm\sqrt{5}, 0)$ ✓

---

(2) correct answer
(1) Focus $(\pm\sqrt{3}e, 0)$

Tangent:
(ii) $\sqrt{2}\, x \sec\theta - \sqrt{3}\, y \tan\theta = \sqrt{6}$

Normal:
$\sqrt{3}\, x \tan\theta + \sqrt{2}\, y \sec\theta = k$.
sub (point P)

$\therefore$ $k = 3\tan\theta\sec\theta + 2\tan\theta\sec\theta$
$k = 5\tan\theta\sec\theta$

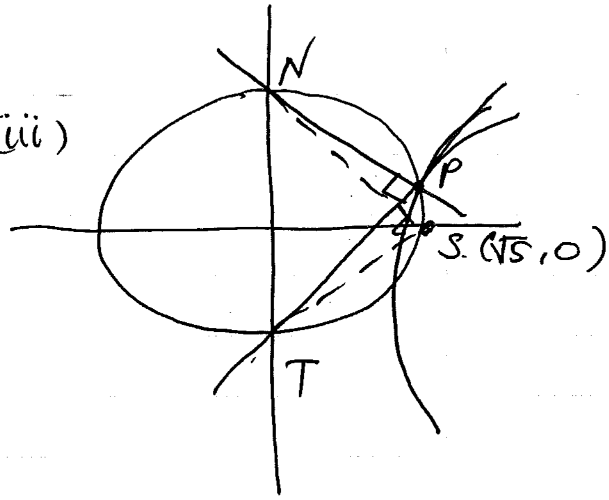
Normal is:
$\sqrt{3}\, x \tan\theta + \sqrt{2}\, y \sec\theta$
$= 5\tan\theta\sec\theta$

---

(2) Full correct equation

(1) only finds k.

(iii)



Know $\angle NPT = 90°$
trying to prove $\angle NST = 90°$

$T\left(0, \frac{-\sqrt{2}}{\tan\theta}\right)$ ✓

$N\left(0, \frac{5\tan\theta}{\sqrt{2}}\right)$ ✓

$m_{TS} = -\frac{\sqrt{2}}{\tan\theta} \div \sqrt{5}$
$= \frac{-\sqrt{2}}{\sqrt{5}\tan\theta}$

$m_{SN} = \frac{5\tan\theta}{\sqrt{2}} \div \sqrt{5}$
$= \frac{5\tan\theta}{\sqrt{10}}$
$= \frac{\sqrt{5}\tan\theta}{\sqrt{2}}$ ✓

$m_{TS} \times m_{SN}$
$= \frac{-\sqrt{2}}{\sqrt{5}\tan\theta} \times \frac{\sqrt{5}\tan\theta}{\sqrt{2}}$
$= -1$

$\therefore \angle NST = 90°$ ✓
similarly $\angle NS'T = 90°$ ✓
$\therefore$ circle passes through foci

Q6 (i) By definition:

$PS = e\,PM.$

$PM = \frac{a}{e} - a\cos\theta$

$= \frac{a}{e}(1 - e\cos\theta)$ ✓

$\therefore PS = e \times \frac{a}{e}(1 - e\cos\theta)$ ✓

$= a(1 - e\cos\theta)$

(ii) $PS' = a(1 + e\cos\theta)$ ✓

(iii) $P(a\cos\theta, b\sin\theta)$

eqn of tangent:

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ ✓

$\therefore \frac{ax\cos\theta}{a^2} + \frac{by\sin\theta}{b^2} = 1$

$\therefore \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

✓

$bx\cos\theta + ay\sin\theta = ab$

$\therefore bx\cos\theta + ya\sin\theta - ab = 0.$

as required

(iv) $S(ae, 0)$

Tangent: $b\cos\theta x + ay\sin\theta - ab = 0.$

$d_{SL} = \frac{|b\cos\theta(ae) + a\sin\theta(0) - ab|}{\sqrt{(b\cos\theta)^2 + (a\sin\theta)^2}}$

$= \frac{|abe\cos\theta - ab|}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$ ✓

similarly

$d_{S'L'} = \frac{|-abe\cos\theta - ab|}{\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$ ✓

(v) $\frac{SL}{S'L'} = \frac{|abe\cos\theta - ab|}{|-abe\cos\theta - ab|}$

$= \frac{ab|e\cos\theta - 1|}{-ab|e\cos\theta + 1|}$

$= -\frac{e\cos\theta - 1}{e\cos\theta + 1}.$

$\frac{PS}{PS'} = \frac{a(1 - e\cos\theta)}{a(1 + e\cos\theta)}$

$= \frac{1 - e\cos\theta}{1 + e\cos\theta}$

$\therefore \frac{SL}{S'L'} = \frac{PS}{PS'}$ ✓

Hence $\frac{PL}{PL'} = \frac{PS}{PS'}$

(By Pythagoras' theorem

$\therefore \triangle PSL \,|||\, \triangle PS'L'$

$\therefore \angle LPS = \angle L'PS'$ ✓

$\therefore \angle NPL = 90°$

$\angle NPS = 90° - \angle LPS$

✓

similarly $\angle NPS' = 90° - \angle L'PS'$

$\therefore \angle NPS = \angle NPS'$

$\therefore$ PN bisects $\angle S'PS$

[The tangent at P on the ellipse is equally inclined to the focal chords through P.]

✓

(vi) Since $\triangle PSN \,|||\, \triangle PS'N$ (2 sides in proportion about an equal angle). ✓

$\therefore \frac{PS}{NS} = \frac{PS'}{NS'}$

Integration /28

1. [D]

$$\int_{\sqrt{3}}^{2} \sqrt{4-x^2}\, dx$$

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

when $x = 2$, $\theta = \frac{\pi}{2}$

$x = \sqrt{3}$, $\theta = \frac{\pi}{3}$

$$\therefore \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\, dx$$

$$= 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{4(1-\sin^2\theta)}\,.\cos\theta\, d\theta$$

$$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$

etc.

2. [B]

$$\frac{d}{dx}(x^2e^x + c)$$

$$= x^2(e^x) + e^x(2x)$$

$$= e^x x^2 + 2xe^x$$

3. [C]

$$\int \frac{dx}{\sqrt{7-6x-x^2}}$$

$$= \int \frac{dx}{\sqrt{16-(x+3)^2}}$$

* use $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

4. [B]. $I_4 = \frac{4-1}{4} I_2$

$I_2 = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$= \frac{\pi}{4}$

$\therefore I_4 = \frac{3}{4} \times \frac{\pi}{4}$

$= \frac{3\pi}{16}$,

5(a) $\int \frac{x^2}{\sqrt{x^3+1}} \, dx$

$u = x^3 + 1$

$du = 3x^2 \, dx$

$= \frac{1}{3} \int \frac{1}{\sqrt{u}} \, du$ ✓

$= \frac{1}{3} [2u^{\frac{1}{2}}] + c$

$= \frac{2}{3} \sqrt{x^3+1} + c.$ ✓

b) $\int_0^{\frac{1}{2}} \sin^{-1} x \, dx$

$u = \sin^{-1} x \qquad v = x$

$du = \frac{1}{\sqrt{1-x^2}} dx \qquad dv = dx.$

$\therefore \int_0^{\frac{1}{2}} \sin^{-1} x \, dx$ ✓

$= [x \sin^{-1} x]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx$

$= \frac{1}{2} \times \sin^{-1}(\frac{1}{2}) - \int_0^{\frac{1}{2}} \frac{x \, dx}{\sqrt{1-x^2}}$

let $w = 1 - x^2$

$dw = -2x \, dx$ ✓

$= \frac{1}{2} \times \frac{\pi}{6} - \int_0^{\frac{3}{4}} \frac{x}{\sqrt{w}} \times \frac{-1}{2x} dw$

$= \frac{\pi}{12} + \frac{1}{2} \int_0^{\frac{3}{4}} w^{-\frac{1}{2}} dw$

$= \frac{\pi}{12} + [\sqrt{w}]_0^{\frac{3}{4}}$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2}$ ✓

(c)(i) $\frac{x^2 - 4x + 2}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4}$

$x^2 - 4x + 2 = A(x^2+4) + (Bx+C)(2x+1)$

when $x = 0$.

$2 = 4A + C$

$x = 1$: $-1 = 5A + 3B + 3C$

$x = -\frac{1}{2}$: $A = 1$ ✓

$B = 0$ ✓ $C = -2$ ✓

(ii) $\int_0^2 \frac{x^2-4x+2}{(2x+1)(x^2+4)}\,dx$

$= \int_0^2 \frac{1}{2x+1} - \frac{2}{x^2+4}\,dx$ ✓

$= \frac{1}{2}\ln(2x+1)\Big]_0^2 - 2\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{1}{2}\ln 5 - \tan^{-1}(1)$

$= \frac{1}{2}\ln 5 - \frac{\pi}{4}.$

or $\ln\sqrt{5} - \frac{\pi}{4}$ ✓

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{1}{t} + t\,dt$

$= \left[\ln(t) + \frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$ ✓

$= \ln 1 - \frac{1}{2} - \left(\ln\left(\frac{1}{\sqrt{3}}\right) + \frac{\frac{1}{3}}{2}\right)$

$= -\frac{1}{2} + \ln\sqrt{3} + \frac{2}{3}.$

$= \frac{1}{6} + \ln\sqrt{3}.$ ✓

6(a) $t = \tan\left(\frac{\theta}{2}\right)$

$\frac{\theta}{2} = \tan^{-1} t$ ✓

$\frac{d\theta}{2} = \frac{1\,dt}{1+t^2}$ ✓

$d\theta = \frac{2\,dt}{1+t^2}$

Now $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\,\mathrm{cosec}^2\theta \tan\left(\frac{\theta}{2}\right)d\theta$

$t_1 = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$t_2 = \tan\left(\frac{\pi}{4}\right) = 1$

$\mathrm{cosec}^2\theta = \frac{1}{\sin^2\theta}$

$= \frac{(1+t^2)^2}{4t^2}$

$2\int_{\frac{1}{\sqrt{3}}}^{1} \cdot \frac{(1+t^2)^2}{4t^2} \cdot t \cdot \frac{2\,dt}{1+t^2}$

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{1+t^2}{t}\,dt$ ✓

(b) (i)

$I_{2n+1} = \int_0^1 x^{2n+1} \cdot e^{x^2}\,dx$

$u = x^{2n}$ $\quad v' = x e^{x^2}$

$du = 2n\,x^{2n-1}$ $\quad v = \frac{e^{x^2}}{2}.$

IBP:

$\int_0^1 x^{2n} \cdot x \cdot e^{x^2}\,dx.$ ✓ (1)

$= \left[x^{2n} \cdot \frac{e^{x^2}}{2}\right]_0^1 - \int_0^1 \frac{e^{x^2}}{2} \cdot 2n x^{2n-1}\,dx$ (1)

$= \frac{e}{2} - n\int_0^1 x^{2n-1} \cdot e^{x^2}\,dx.$ ✓

$= \frac{e}{2} - n\,I_{2n-1}$ ■

(ii) $I_5 = \frac{e}{2} - 2I_3$.

$= \frac{e}{2} - 2\left[\frac{e}{2} - I_1\right]$.

$= -\frac{e}{2} + 2I_1$. ✓ ①

$I_1 = \int_0^1 x e^{x^2} dx$

$= \frac{1}{2}\int_0^1 2x e^{x^2} dx$

$= \left.\frac{e^{x^2}}{2}\right]_0^1$

$= \frac{1}{2}(e-1)$ ✓ ①

$\therefore I_5 = -\frac{e}{2} + 2\left(\frac{1}{2}(e-1)\right)$

$= -\frac{e}{2} + e - 1$

$= \frac{e}{2} - 1$ ✓ ①.