SAINT IGNATIUS' COLLEGE RIVERVIEW
YEAR 12

EXTENSION 2

ASSESSMENT TASK

2004

MATHEMATICS

Time allowed – 50 minutes
(plus 5 minutes reading time)

Directions to Candidates

1. Attempt **ALL** questions.
2. All questions are of equal value.
3. All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
4. Board-approved calculators may be used.
5. Each question attempted is to be returned on a **SEPARATE SHEET** clearly marked Question 1, Question 2, ...etc.
6. Each answer sheet must show your **NAME** and your **TEACHER'S NAME**.

QUESTION 1 – INTEGRATION (15 marks)

(a) Find $\int \sin^3 x.dx$ (2)

(b) Find $\int \frac{dx}{x^2 - 4x + 13}$ (2)

(c) Find $\int \frac{3x}{\sqrt{16 + x^2}} dx$ (2)

(d) Use Integration by parts to find $\int 2xe^{-2x}\, dx$ (2)

(e) Using the substitution $t = \tan\frac{x}{2}$, show that (3)

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4\cos x} = \frac{2}{3}\tan^{-1}\frac{1}{3}$$

(f) Given that $I_n = \int_0^1 \frac{x^n}{x^2 + 1} dx$, for $n = 0,1,2,3,........$

(i) Show that $I_n + I_{n-2} = \frac{1}{n-1}$, for $n \ge 2$ (2)

(ii) Hence evaluate I_4 (2)

QUESTION 2 – POLYNOMIALS (15 marks)

(a) Find the polynomial in x with degree 3, such that two zeros are $x=1$ and $x=-2$, also given that $P(-1)=4$ and $P(2)=28$ (2)

(b) Consider the polynomial $P(x)=(x-\alpha)^3.Q(x)$, where $Q(x)$ is also a polynomial and α is a real zero of $P(x)$.

(i) Show that $P(\alpha)=P'(\alpha)=P''(\alpha)=0$ (2)

(ii) Hence or otherwise, solve the equation (2)

$8x^4-25x^3+27x^2-11x+1=0$

given that it has a triple root.

(c) If $3-i$ is a root of the polynomial $P(z)=z^3+rz^2+sz+20$, and r and s are real numbers,

(i) state why $3+i$ is also a root of $P(z)$. (1)

(ii) hence factorise $P(z)$ over the complex field. (2)

(d) The equation $x^3+2x^2+bx-16=0$ has roots α,β and γ such that $\alpha\beta=4$.

(i) Show that $b=-20$. (2)

(ii) Find the equation with roots given by α^2,β^2 and γ^2 (2)

(iii) Find the value of $\alpha^3+\beta^3+\gamma^3$. (2)