SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2003
HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK #2

Mathematics Extension 2

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 Hours
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded
- Marks may **NOT** be awarded for messy or badly arranged work.
- Hand in your answer booklets in three sections. Section A (Questions 1 and 2), Section B (Questions 3 and 4), Section C (Questions 5 and 6).
- Write each **NEW** section in a separate answer booklet.

Total Marks – 90 Marks

- Attempt Sections A - C
- All questions are of equal value.

Examiner: *F.Jordan*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

SECTION A

Question 1 (Use a separate Writing Booklet) **Marks**

(a) Find $\int x \log x \, dx$ **2**

(b) Find $\int \frac{x^2}{x+1} \, dx$ **2**

(c) Find $\int \sin^3 x \, dx$ **2**

(d) Find $\int \frac{x+4}{x\left(x^2+2\right)} dx$ **2**

(e) By making the substitution $t = \tan\left(\frac{x}{2}\right)$ show that **3**
$\int \operatorname{cosec} x \, dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$ where C is a constant.

(f) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$ (where $n \geq 3$) show that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence **4**
evaluate I_7.

Question 2 **Marks**

(a) (i) Find the modulus and the argument of $\dfrac{1+2i}{1-3i}$ **2**

(ii) Hence find the value of $\left(\dfrac{1+2i}{1-3i}\right)^9$ **2**

(b) Solve the equation $z^2+(z+1)^2=0$ where z is a complex number. **1**

(c) Sketch the following loci on separate Argand diagrams:

(i) $z^2-\overline{z}^2=16i$ **2**

(ii) $\arg\left(\dfrac{z-i}{z-2}\right)=\dfrac{\pi}{2}$ **2**

(d) (i) Suppose z is any nonzero complex number. Explain why $\dfrac{z}{\overline{z}}$ has modulus 1 and argument twice the argument of z. **2**

(ii) Find all complex numbers z so that $\dfrac{z}{\overline{z}}=i$. Give your answer in the form $a+ib$, where a and b are real. **4**

SECTION B

Question 3 (Start a new answer booklet) **Marks**

(a) Sketch $y = 1 + x^2$ and hence sketch on separate diagrams (do not use calculus): **1**

(i) $y = \dfrac{1}{x^2 + 1}$ **1**

(ii) $y = \dfrac{x}{x^2 + 1}$ **2**

(iii) $y = \left|\dfrac{x}{x^2 + 1}\right|$ **2**

(iv) $y = \pm\sqrt{\dfrac{x}{x^2 + 1}}$ **2**

(b) For the rational function $F(x) = \dfrac{x^4}{x^2 - 1}$

(i) Find if $F(x)$ is odd or even or neither. **1**

(ii) Show algebraically that the range of $F(x)$ is : $y \le 0$ or $y \ge 4$. **2**
Hence calculate the coordinates of its three turning points.

(c) (i) Using the same axes, sketch the graphs $y = x + 1$ and $y = |x| - x - 1$ **2**

(ii) Hence find all values of x for which $\dfrac{|x|}{x + 1} - 1 \le 0$ **2**

Question 4 **Marks**

(a) A quiz consists of twenty True-False questions. Find the chance that someone who knows the correct answers to ten of the questions, but answers the remaining ones by tossing a coin, will obtain a score of at least 85% on the quiz. **3**

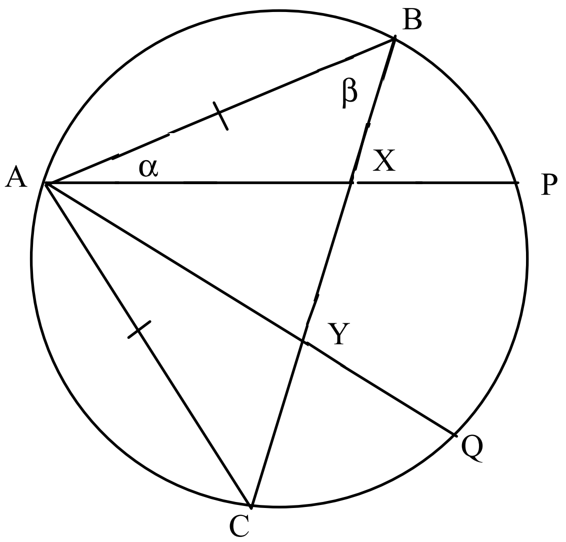
(b) Find real numbers A and B such that $\dfrac{1-3x}{x^2-3x+2} = \dfrac{A}{x-1} + \dfrac{B}{x-2}$ **2**

(c) (i) Find the square roots of the complex number $-3+4i$. **2**

(ii) Find the roots of the quadratic equation **2**
$x^2-(4-2i)x+(6-8i)=0$

(d) Use integration by parts to evaluate $\displaystyle\int_0^{\frac{1}{2}} \cos^{-1} x\, dx$. **3**

(e) **3**



Let $ABPQC$ be a circle that $AB = AC$, AP meets BC at X, and AQ meets BC at Y, as in the diagram.
Let $\angle BAP = \alpha$ and $\angle ABC = \beta$

(i) Copy the diagram onto your answer page and state why $\angle AXC = \alpha + \beta$

(ii) Prove that $\angle BQP = \alpha$ (giving all reasons)

SECTION C

Mark

Question 5 (Start a new answer booklet)

(a) Consider the equation: $y = \dfrac{x^2 + 3}{x(x+3)}$ **8**

(i) Show that the curve represented by this equation has stationary points at $\left(3, \dfrac{2}{3}\right)$ and $(-1, -2)$.

(ii) Determine the nature of these stationary points

(iii) Find any horizontal or vertical asymptotes

(iv) Sketch the curve

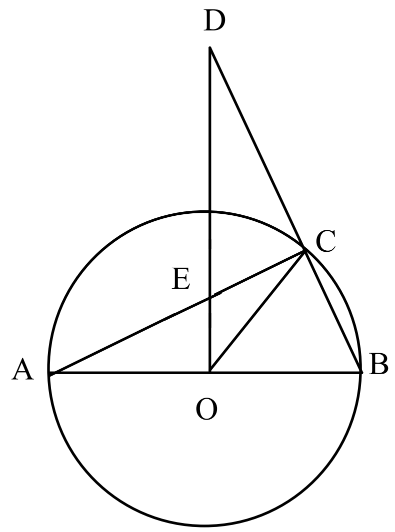
(b) Two people, John and Kim, play a dice game. John throws two dice and Kim throws one, Kim wins if John fails to beat his number with either one of his two dice. (Clearly, if all the numbers are the same, Kim wins.) What is the probability that Kim wins the game? **4**

(c) *O* is the centre of a circle *ABC*. *DEO* is drawn perpendicular to the diameter *AOB*. **3**

Prove that:

(i) $\angle ACO = \angle ODB$

(ii) *AOCD* is a cyclic quadrilateral.



Question 6 **Marks**

(a) Find the simplest polynomial with integer coefficients which includes $1+\sqrt{2}$, 3+4i, and 5 amongst its zeros. **3**

(b) α, β, γ are the roots of $x^3 + 2x^2 - 3x + 4 = 0$

(i) Evaluate $\alpha^2 + \beta^2 + \gamma^2$ **2**

(ii) Evaluate $a^3 + \beta^3 + \gamma^3$ **2**

8

(c) The quartic polynomial $f(x) = x^4 + px^3 + qx^2 + rx + s$ has four zeroes, α, β, γ and δ such that the sum of α and β equals the sum of γ and δ.

Let $C = \alpha + \beta = \gamma + \delta$, $P = \alpha\beta$ and $Q = \gamma\delta$.

(i) Find p, q, r and s in terms of C, P and Q.

(ii) Show that the coefficients of $f(x)$ satisfy the condition $p^3 + 8r = 4pq$.

(iii) It is given that the polynomial $g(x) = x^4 - 18x^3 + 79x^2 + 18x - 440$ has the property that the sum of two of the zeroes equals the sum of the other two zeroes. Using the identities of part (i) or otherwise, find all four zeroes of $g(x)$.

This is the end of the paper.

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x,\ x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

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Sample Solutions

① (a) $\int x \log x \, dx$

$u = \log x \quad u' = \frac{1}{x}$
$v = \frac{1}{2}x^2 \quad v' = x$

$= \frac{1}{2}x^2 \log x - \int \frac{1}{2}x^2 \times \frac{1}{x} \, dx$

$= \frac{1}{2}x^2 \log x - \frac{1}{2}\int x \, dx$ ②

$= \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + c.$

(b) $\int \frac{x^2}{x+1} dx$

$$\begin{array}{r} x - 1 \\ x+1 \,\overline{)\, x^2 \qquad} \\ x^2 + x \\ \hline -x \\ -x - 1 \\ \hline 1 \end{array}$$

$= \int \left(x - 1 + \frac{1}{x+1}\right) dx$

$= \frac{1}{2}x^2 - x + \log(x+1) + c$ ②

OR $\frac{(x+1)^2}{2} - 2(x+1) + \ln|x+1| + c$

(c) $\int \sin^3 x \, dx$

$= \int \sin^2 x . \sin x \, dx$

$= \int (1 - \cos^2 x). \sin x \, dx$ ②

$= \int \sin x \, dx - \int \cos^2 x . \sin x \, dx$

$= -\cos x + \frac{1}{3}\cos^3 x + c.$

d) $\frac{x+4}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$

$\therefore x + 4 = A(x^2+2) + Bx^2 + Cx$

$\therefore x + 4 = (A+B)x^2 + Cx + 2A.$

$\therefore A + B = 0 \quad C = 1 \quad 2A = 4$

$\therefore A = 2$

$\therefore B = -2$

$\therefore \int \frac{x+4}{x(x^2+2)} dx$ ②

$= \int \left(\frac{2}{x} + \frac{-2x+1}{x^2+2}\right) dx$

$= \int \left(\frac{2}{x} - \frac{2x}{x^2+2} + \frac{1}{x^2+2}\right) dx$

$= 2 \ln x - \ln(x^2+2) + \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$

OR $\ln\left(\frac{x^2}{x^2+2}\right) + \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c.$

(e) $\int \operatorname{cosec} x \, dx$

$= \int \frac{1}{\sin x} dx$

$= \int \frac{1+t^2}{2t} \cdot \frac{2 \, dt}{1+t^2}$

$= \int \frac{1}{t} dt$ ③

$= \ln|t| + c$

$= \ln\left|\tan \frac{x}{2}\right| + c.$

(f) $I_n = \int_0^{\pi/4} \tan^n x \, dx$

$= \int_0^{\pi/4} \tan^{n-2} x . \tan^2 x \, dx$

$= \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1) \, dx$

$= \int_0^{\pi/4} \tan^{n-2} x . \sec^2 x \, dx - \int_0^{\pi/4} \tan^{n-2} x$

$= \left[\frac{1}{n-1} \tan^{n-1} x\right]_0^{\pi/4} - I_{n-2}$

$= \frac{1}{n-1} \cdot 1 - \frac{1}{n-1} \cdot 0 - I_{n-2}$

$= \frac{1}{n-1} - I_{n-2}$

$\therefore I_7 = \frac{1}{6} - I_5$ ④

$= \frac{1}{6} - \left(\frac{1}{4} - I_3\right)$

$= -\frac{1}{12} + \frac{1}{2} - I_1$

$= \frac{5}{12} - \int_0^{\pi/4} \frac{\sin x}{\cos x} dx$

$= \frac{5}{12} + \left[\ln|\cos x|\right]_0^{\pi/4}$

$= \frac{5}{12} + \ln\left(\frac{1}{\sqrt{2}}\right) - \ln 1$

$= \frac{5}{12} - \ln\sqrt{2}$

$= \frac{5}{12} - \frac{1}{2}\ln 2$

(2)

(a) (i) $\frac{1+2i}{1-3i} = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$

$= \frac{1+3i+2i-6}{1+9}$

$= \frac{-5+5i}{10}$

$= \frac{1}{2}(-1+i)$

$= \frac{\sqrt{2}}{2} \text{ cis } \frac{3\pi}{4}$ (2)

Modulus $\frac{\sqrt{2}}{2}$ argument $\frac{3\pi}{4}$

ii) $\left(\frac{1+2i}{1-3i}\right)^9 = \left(\frac{\sqrt{2}}{2}\right)^9 \left(\text{cis } \frac{3\pi}{4}\right)^9$

$= \frac{1}{16\sqrt{2}} \text{ cis } \frac{27\pi}{4}$

$= \frac{1}{16\sqrt{2}} \text{ cis } \frac{3\pi}{4}$ (2)

$= \frac{1}{16\sqrt{2}}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

$= \frac{1}{32}(-1+i)$

(b) $z^2 + (z+1)^2 = 0$

$z^2 + z^2 + 2z + 1 = 0$

$2z^2 + 2z + 1 = 0$

$\therefore z = \frac{-2 \pm \sqrt{4 - 4\times2\times1}}{2\times2}$

$= \frac{-2 \pm \sqrt{-4}}{4}$ (1)

$= \frac{-2 \pm 2i}{4}$

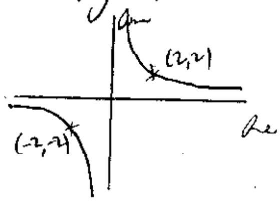
$= \frac{-1 \pm i}{2}$

c) (i) $z^2 - \bar{z}^2 = 16i$

$\therefore x^2 - y^2 + 2ixy - (x^2 - y^2 - 2ixy) = 16i$

$\therefore 4ixy = 16i$

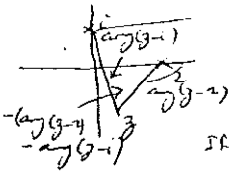
$\therefore xy = 4$



(2)

(ii) $\arg\left(\frac{z-i}{z-2}\right) = \frac{\pi}{2}$

$\therefore \arg(z-i) - \arg(z-2) = \frac{\pi}{2}$.



$\therefore$ Semicircle

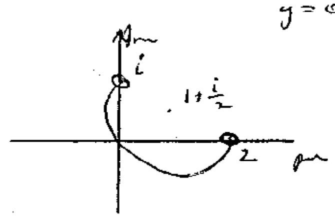
centre $\frac{1}{2}(1+2i)$

$(x-1)^2 + (y-\frac{1}{2})^2 = \frac{5}{4}$

If $x=0$ $1 + (y-\frac{1}{2})^2 = \frac{5}{4}$

$(y-\frac{1}{2})^2 = \frac{1}{4}$

$y = 0$ or 1



(2)

(d)(i) $\frac{z}{\bar{z}} = \frac{z^2}{z\bar{z}}$

$= \frac{z^2}{|z|^2}$

$= \frac{(r \text{ cis}\theta)^2}{r^2}$ (2)

$= \text{cis } 2\theta$

$\therefore$ Modulus 1, argument 2θ
$= 2 \times \arg z$.

(ii) $\frac{z}{\bar{z}} = i$

$\therefore \text{cis } 2\theta = i$

$\therefore \text{cis } 2\theta = \text{cis } \frac{\pi}{2}$

$\therefore 2\theta = \frac{\pi}{2} + 2k\pi$ (4)

$\therefore \theta = \frac{\pi}{4} + k\pi$

$\therefore z = r \text{ cis}\left(\frac{\pi}{4} + k\pi\right)$

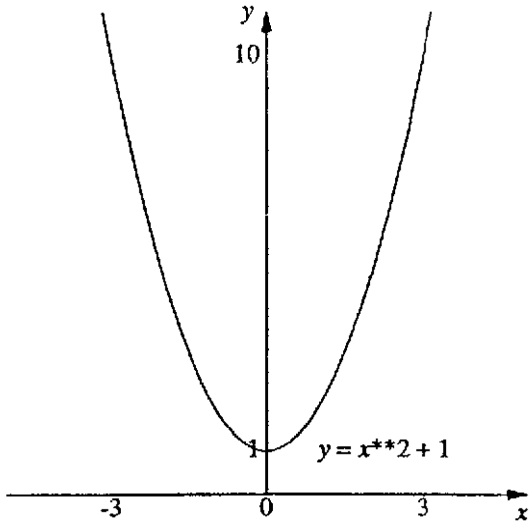
$= \pm r\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

$= \pm(a + ai)$

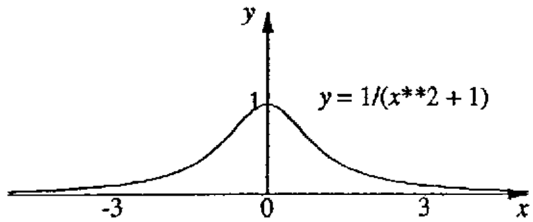
$= a + ia$ or $-a - ia$

$a > 0$

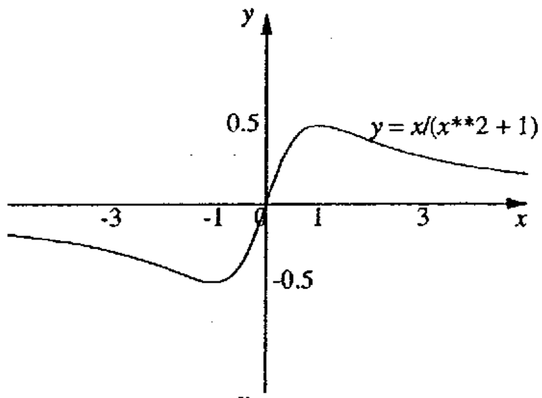
[1] 3.



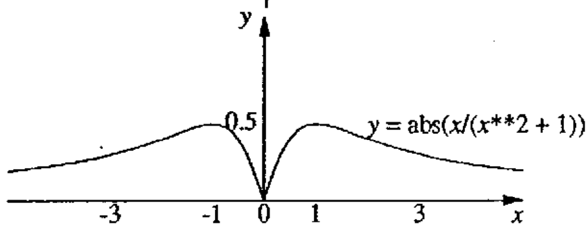
[1] (a) (i)



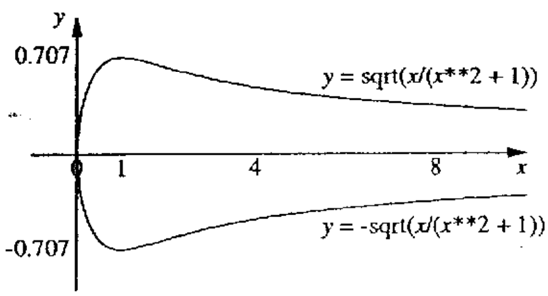
[2] (ii)



[2] (iii)



[2] (iv)



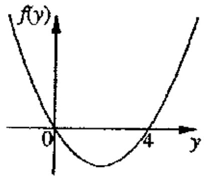
[1] (b) (i)

$$\begin{aligned} F(-x) &= \frac{(-x)^4}{(-x)^2-1}, \\ &= \frac{x^4}{x^2-1}, \\ &= F(x). \end{aligned}$$

$\therefore F(x)$ is even.

[2] (ii) Putting

$$\begin{aligned} y &= \frac{x^4}{x^2-1}, \\ y(x^2-1) &= x^4, \\ x^4 - yx^2 + y &= 0, \\ x^2 &= \frac{y \pm \sqrt{y^2-4y}}{2}. \end{aligned}$$



Now, for $x \in \Re$, (but of course $x \neq \pm 1$)

$\Delta \geq 0$,

i.e., $y(y-4) \geq 0$ (See rough sketch above right).

So the range of F(x) is : $y \leq 0$ or $y \geq 4$.

When $-1 < x < 1$ then $x^2 - 1 < 0$ and $y \leq 0$ (noting that, when $x = 0$, $y = 0$).

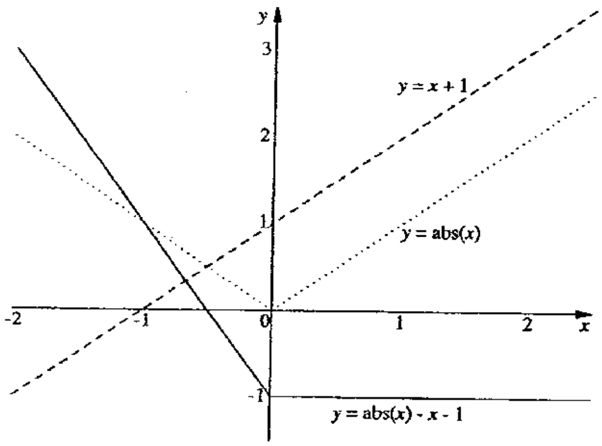
Thus we have a maximum at (0, 0).

When $y = 4$ then

$$\begin{aligned} x^2 &= \frac{4 \pm \sqrt{16-16}}{2}, \\ &= 2. \\ \textit{i.e.},\ x &= \pm\sqrt{2}. \end{aligned}$$

Thus we have minima at ($\sqrt{2}$, 4) and ($-\sqrt{2}$, 4).

[2] (c) (i)



[2] (ii) If $x + 1 > 0$ (*i.e.*, $x > -1$), rearranging the inequation gives $|x| - x - 1 \leq 0$.
From the graph, this is true when $x \geq -\frac{1}{2}$.
But if $x + 1 < 0$ (*i.e.*, $x < -1$), rearranging the inequation gives $|x| - x - 1 \geq 0$.
From the graph, this is true when $x < -1$.
Thus the statement is true for $\{x : x \geq -\frac{1}{2} \ \cup \ x < -1\}$.

[3] 4. (a) The person needs to get at least 7 out of the remaining 10 questions.

$$I.e., \binom{10}{7}+\binom{10}{8}+\binom{10}{9}+\binom{10}{10} = 120+45+10+1, \\ = 176 \text{ ways.}$$

Total ways of choosing is $2^{10} = 1024$.

$\therefore$ Probability is $\dfrac{176}{1024} = \dfrac{11}{64}$.

[2] (b) $1-3x = (A+B)x - 2A - B$.

Equating coefficients,

$$A+B = -3 \cdots (1),$$
$$2A+B = -1 \cdots (2),$$
$$\therefore A = 2 \cdots (2)-(1),$$
$$B = -5.$$

or

$$1-3x = (x-2)A + (x-1)B.$$

Let $x = 1$,
$$-A = -2,$$
$$A = 2.$$
Let $x = 2$,
$$B = -5.$$

[2] (c) (i) Let $(a+bi)^2 = -3+4i$,

$$a^2+2abi-b^2 = -3+4i.$$

So $a^2-b^2 = -3$,

$$2ab = 4,$$
$$a^2+b^2 = 5.$$
$$\therefore 2a^2 = 2,$$
$$a = \pm 1,$$
$$2b^2 = 8,$$
$$b = \pm 2.$$

$\therefore$ Square roots are $\pm(1+2i)$.

[2] (ii)
$$x = \frac{4-2i \pm \sqrt{16-16i-4-24+32i}}{2},$$
$$= \frac{4-2i \pm \sqrt{-12+16i}}{2},$$
$$= 2-i \pm \sqrt{-3+4i},$$
$$= 2-i+1+2i \text{ or } 2-i-1-2i,$$
$$= 3+i \text{ or } 1-3i.$$

[3] (d)
$$\text{I} = \int_0^{\frac{1}{2}} 1 \times \cos^{-1}x.dx,$$
$$= x\cos^{-1}x\Big]_0^{\frac{1}{2}} + \int_0^{\frac{1}{2}} \frac{x.dx}{\sqrt{1-x^2}},$$
$$= \frac{1}{2}\times\frac{\pi}{3} + \frac{-1}{2}\int_1^{\frac{3}{4}} \frac{dt}{t^{\frac{1}{2}}},$$
$$= \frac{\pi}{6} + \frac{1}{2}\int_{\frac{3}{4}}^{1} t^{-\frac{1}{2}}.dt,$$
$$= \frac{\pi}{6} + \frac{1}{2}\left[2t^{\frac{1}{2}}\right]_{\frac{3}{4}}^{1},$$
$$= \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}.$$

$u = \cos^{-1}$ $v' = 1$

$u' = \frac{-1}{\sqrt{1-x^2}}$ $v = x$

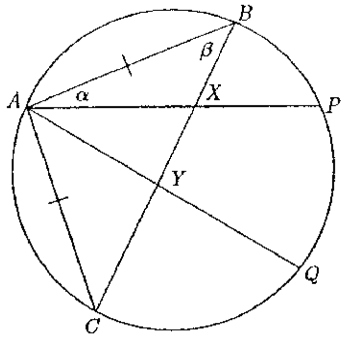
put $t = 1-x^2$

$dt = -2xdx$

when $x = 0$, $t = 1$

$x = \frac{1}{2}$, $t = \frac{3}{4}$

[2] (e) (i)



The exterior angle, $A\widehat{X}C$, of $\triangle ABX$ is equal to the sum of the interior opposite angles, $\alpha + \beta$.

[1] (ii) $B\widehat{Q}P = \alpha$ (angles standing on the same arc, BP)

QUESTION 5.

a) $y = \frac{x^2+3}{x(x+3)}$

$y' = \frac{x(x+3).2x - (x^2+3)(2x+3)}{x^2(x+3)^2}$

$= \frac{3x^2-6x-9}{x^2(x+3)^2}$

For st. pts. $y' = 0$.

ie $3x^2-6x-9 = 0$

$3(x-3)(x+1) = 0$

$x = -1, 3$

$\therefore y = -2, \frac{2}{3}$

$\therefore$ st. pts are $(-1,-2)$ + $(3, \frac{2}{3})$ (3)

Test

-2	-1	$-\frac{1}{2}$
$15/4$	0	$-1\frac{2}{5}$

$\therefore$ MAX. TURNING PT.

2	3	4
$\frac{-9}{100}$	0	$15/784$

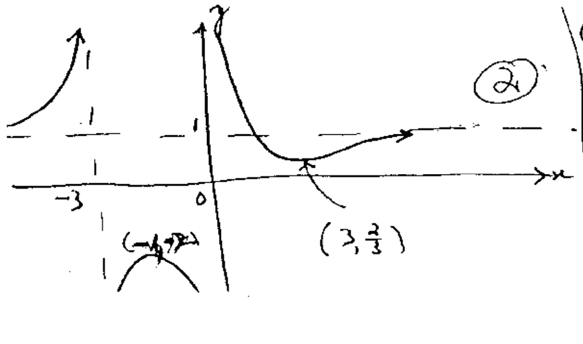
$\therefore$ MIN. TURNING PT.

(1)

VERTICAL ASYMPTOTES at $x=0$ and $x=-3$

HORIZONTAL ASYMPTOTE at $y=1$

$\lim_{x\to\infty} \frac{x^2+3}{x^2+3x} = \lim_{x\to\infty} \frac{1+\frac{3}{x^2}}{1+\frac{3}{x}} = 1.$ (2)



(2)

(b) P(K wins)

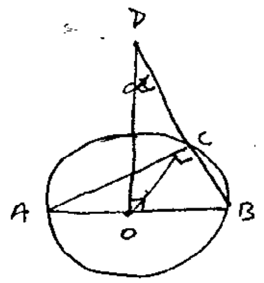
= P(throws 1 then wins OR throws 2 then wins - - - - - - OR throws 6 then wins)

$= \frac{1}{6}\times\frac{1}{36} + \frac{1}{6}\times\frac{4}{36} + \frac{1}{6}\times\frac{9}{36} + \frac{1}{6}\times\frac{16}{36} + \frac{1}{6}\times\frac{25}{36} + \frac{1}{6}\times\frac{36}{36})$

$= \frac{1}{216}[1+4+9+25+36]$

$= \frac{91}{216}$ (4)

(c)



(i) Let $\angle ODB = \alpha$. — (1)

now $\alpha + \angle B = \angle CAO + \angle B = 90°$ — (2)

From (1) + (2)

$\angle CAO = \angle ODB$. $(1\frac{1}{2})$

now $\angle ACO = \angle CAO$ (equal radii $\therefore$ isosceles △)

$\therefore \angle ACO = \angle ODB$

(ii) $\angle DCA = \angle DOA = 90°$

$\therefore$ AOCD is a cyclic quadrilateral (angles subtended by the same interval AD on the same side of the interval form a cyclic quadrilateral) $(1\frac{1}{2})$

Question 6.

a) The required polynomial is

$(x-(1+\sqrt{2}))(x-(1-\sqrt{2}))(x-(3+4i))$

$\times(x-(3-4i))(x-5)$

$(x^2-2x-1)(x^2-6x+25)(x-5)$. (3)

b) (i)

$\alpha^2+\beta^2+\gamma^2=(\sum\alpha)^2-2\sum\alpha\beta$

$= 4-2\times-3$

$= \underline{10}$. (2)

ii) $\sum\alpha^3+2\sum\alpha^2-3\sum\alpha+3\times4=0$

$\therefore \alpha^3+\beta^3+\gamma^3=-2(\alpha^2+\beta^2+\gamma^2)$
$+3(\alpha+\beta+\gamma)-12$

$=-2\times10-3\times-2-12$

$=-\underline{38}$. (2)

c) (i)
$S_1=\alpha+\beta+\gamma+\delta=-p \therefore \boxed{p=-2c}$

$S_2=q \Rightarrow (\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta$
$\therefore \boxed{q=c^2+P+Q}$ (4)

$S_3=-r \Rightarrow Pc+Qc=-r$
$\Rightarrow \boxed{r=-c(P+Q)}$

$S_4=s \Rightarrow \boxed{s=PQ}$

(ii) $LHS=p^3+8r$
$=-8c^3-8c(P+Q)$
$=-8c(c^2+P+Q)$
$=-8\times\frac{p}{-2}\times q$
$=4pq$
$=RHS.$ (2)

(iii) $-18=-2c \Rightarrow \boxed{c=9.}$

$79=c^2+P+Q \Rightarrow \boxed{P+Q=-2.}$ (a)

$18=-c(P+Q)$

$\boxed{-440=PQ.}$ (b)

Solving (a) & (b)

Since $c=9$ $P=\alpha\beta$ and ...

from (a) & (b) $\underline{P=-22}$ & $\underline{Q=20}$.

(or in reverse)

$\therefore$ Solving $\alpha+\beta=\gamma+\delta=9$

with $\alpha\beta=-22$ and $\beta\gamma=20$ (2)

leads to the roots.

$\boxed{-2, 4, 5 \;\&\; 11.}$

(NB otherwise use factor theorem & long division.)