SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2012

HSC ASSESSMENT TASK #2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes.
- Working time – 120 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Answers should be in simplest exact form unless specified otherwise.
- Start each **NEW** section in a separate answer booklet.
- Each section is to be returned in a separate bundle.

Total Marks - 89

- Attempt Questions 1 - 6
- All questions are NOT of equal value.

Examiner: *A. Fuller*

Section A

Question 1 (16 marks)

(a) $\int \frac{\cos x}{1+\sin x} dx$ **1**

(b) $\int \frac{\cos^2 x}{1+\sin x} dx$ **2**

(c) Given $a = 3 - 4i$ and $b = 1 + i$. **5**

Express the following in the form $x + iy$ where x and y are real numbers:

(i) $b - a$

(ii) $\overline{ab}$

(iii) $\frac{a}{b}$

(d) $P(x) = x^4 - x^3 - 2x^2 + 6x - 4$. **3**

(i) Given that $1 + i$ is a zero of $P(x)$, explain why $1 - i$ is also a zero of $P(x)$.

(ii) Hence, find all the zeros of $P(x)$.

(e) When $(1 + ax)^5 + (1 + bx)^5$ is expanded in ascending powers of x, the expansion begins $2 + 40x + 260x^2 + \ldots$ **5**

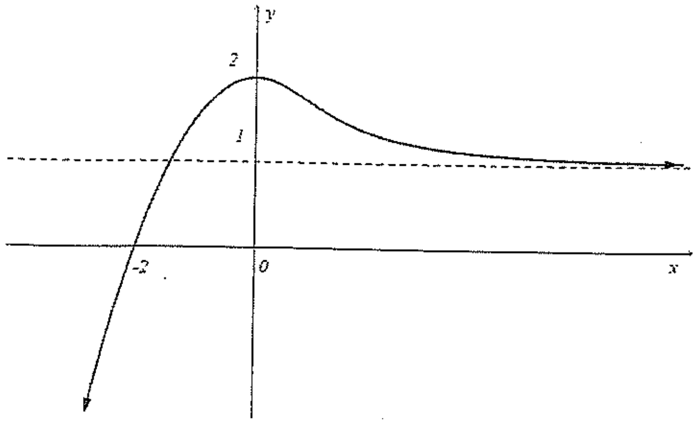
(i) Show that $a + b = 8$ and $a^2 + b^2 = 26$.

(ii) Deduce the value of ab.

(iii) Find the coefficient of x^3.

Question 2 (16 marks)

(a) Below is a sketch of $y = f(x)$. **6**



Sketch the following on separate diagrams:

(i) $y = f(|x|)$

(ii) $|y| = f(x)$

(iii) $y = [f(x)]^{-1}$

(iv) $y = 2^{f(x)}$.

(b) Plot a point A which represents the complex number z on an argand diagram given that $Re(z) < 0$, $Im(z) > 0$ and $|z| > 1$. **4**

On the same argand diagram plot the point:

(i) B representing the complex number $\bar{z}$

(ii) C representing the complex number $\frac{1}{z}$

(iii) Drepresenting the complex number $z(\cos\pi + i\sin\pi)$.

(c) $\int \ln(1 + x^2)dx$. **3**

(d) Evaluate $\int_{-1}^{1}(1 + x^3)^3 dx$ **3**

Section B **(Use a SEPARATE writing booklet)**

Question 3 (14 marks)

(a) Sketch $y^2 = (x-2)^2(x-1)$ without using calculus. **2**

(b) If $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$. **3**

Write z in Cartesian form $(x+iy)$.

(c) The equation $x^3 - 3x^2 + ax + 8 = 0$ has roots that are in arithmetic sequence. **3**

(i) Show that one of the roots is 1.

(ii) Find the value of a and solve the equation.

(d) A particle of mass 10 kg is projected vertically upwards with a velocity of $u\ m/s$. The resistive force is one-tenth of the square of its velocity. **6**

Assuming that $g = 10\ m/s^2$.

(i) Show that the particle takes $\sqrt{10}\tan^{-1}\frac{u}{10\sqrt{10}}$ seconds to reach its greatest height.

(ii) Show that the greatest height is $50\log_e\frac{1000+u^2}{1000}$ metres.

Question 4 (16 marks)

(a) (i) Show that $\int_{-a}^{a} f(x)dx = \int_{0}^{a}(f(x) + f(-x))dx$. **4**

(ii) Hence, or otherwise, evaluate

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\sin x}$$

(b) The complex number $z = x + iy$, where x and y are real, is such that **5**

$|z - i| = Im(z)$.

(i) Show that the locus of z is a parabola.

(ii) Hence, find the range of possible values for $\arg(z)$.

(c) (i) Find the values of A, B and C if **7**

$\frac{-1}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+1}$.

(ii) A particle of unit mass moves on the x-axis against a resistance numerically equal to $v^2 + v^3$, where v is its velocity. Initially the particle is travelling with velocity u, where $u > 0$.

It can be proven that when the velocity is $\frac{u}{2}$ the distance X travelled by the particle is given by $\boldsymbol{X = \ln\left(\frac{2+u}{1+u}\right)}$ (Do not prove this)

(α) Prove that if T is the time taken to travel the distance X then

$u(T + X) = 1$.

(β) It is alleged that if the particle started at the origin then the velocity v, displacement x, and time t are related by the equation $v = \frac{u}{ux+ut+1}$.

By finding a suitable derivative, show that this is in fact correct.

Section C **(Use a SEPARATE writing booklet)**

Question 5 (15 marks)

(a) (i) Write $(1+i)^n$ in modulus argument form. **6**

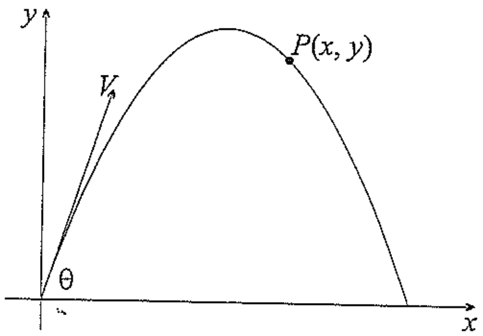
(ii) By considering the binomial expansion of $(1+i)^n$.

Find an expression for $1-\binom{n}{2}+\binom{n}{4}-\binom{n}{6}+\ldots$

(iii) If n is a multiple of 8.

Show that $\binom{n}{1}+\binom{n}{5}+\binom{n}{9}+\cdots=\binom{n}{3}+\binom{n}{7}+\binom{n}{11}+\cdots$

(b) **9**



A particle is projected at an angle of θ to the horizontal with velocity V. (Assume that there is no air resistance) and take gravity to be g. At time t, let x and y be the horizontal and vertical displacements respectively.

(i) Derive the equations of motion in the horizontal and vertical directions in terms of t.

(ii) It is known that at some time t during its flight, the x and y displacements of the particle are equal and the direction of motion is inclined at 45° to the downward vertical. The position of the particle at this time is marked P in the diagram above. Use this information to show that $\tan\theta = 3$.

(iii) Hence, find the range of the particle in terms of V and g.

(iv) If the speed of projection may be varied but the particle must not rise more than H above the ground. Find the maximum range in terms of H.

Question 6 (12 marks)

(a) Show that the polynomial $P(x) = x^n - x^{n-1} - 1$, where $n > 1$ **3**

cannot have a repeated real root.

(b) "Words" are to be formed from the letters of the word **9**

F U N D A M E N T A L

(i) Using all of the eleven letters. How many different "words"

are possible if:

(α) there is no restriction

(β) it must start and end with the same letter

(γ) **F U N** must appear together in that order

(δ) the same letter must not appear next to itself?

(ii) If I am to select five letters to form a "word". How many different five letter "words" are possible?

End of Examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a} \cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a} \tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a} \sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

2012 Assessment Task 2
Ext 2

Section A

Q1. (a) $\int \frac{\cos x}{1+\sin x}\,dx$ Let $u = \sin x$, $du = \cos x\,dx$

$= \int \frac{du}{1+u}$

$= \ln(1+u) + c$

$= \ln(1+\sin x) + C$

1

(b) $\int \frac{\cos^2 x}{1+\sin x}\,dx$

$= \int \frac{1-\sin^2 x}{1+\sin x}\,dx = \int \frac{(1-\sin x)(1+\sin x)}{1+\sin x}\,dx$.

$= \int (1-\sin x)\,dx$

$= x + \cos x + C$.

2

(c) $a = 3-4i$, $b = 1+i$.

(i) $b - a = -2+5i$

(ii) $ab = 7-i$ $\therefore \overline{ab} = 7+i$

5

(iii) $\frac{a}{b} = \frac{3-4i}{1+i}$

$= \frac{3-4i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{-1-7i}{1+1} = \frac{-1-7i}{2} = -\frac{1}{2} - \frac{7}{2}i$

1.(d) $P(x) = x^4 - x^3 - 2x^2 + 6x - 4$

(i) By the conjugate root theorem, if $P(x)$ has real coefficients and $(a+bi)$ is a root, then so is $(a-bi)$

$\therefore$ If $(1+i)$ is a zero, so is $(1-i)$ since $P(x)$ has real coefficients.

(ii) $(x-1-i)(x-1+i)$ is a factor of $P(x)$

$\Rightarrow (x^2-2x+2)$ is a factor

By division

$$\begin{array}{r} x^2 + x - 2 \\ x^2-2x+2 \,\big)\, x^4 - x^3 - 2x^2 + 6x - 4 \\ x^4 - 2x^3 + 2x^2 \\ \hline x^3 - 4x^2 + 6x \\ x^3 - 2x^2 + 2x \\ \hline -2x^2 + 4x - 4 \\ -2x^2 + 4x - 4 \\ \hline 0 \end{array}$$

3

for $q(x) = x^2 + x - 2 = (x+2)(x-1)$

$x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}$

$x = -2$ or 1

$\therefore$ zeros of $P(x)$ are $x = -2, 1, (1+i), (1-i)$.

1(e) $(1+ax)^5 + (1+bx)^5 = 2 + 40x + 260x^2 + \ldots$

For $(1+ax)^5$, $T_{r+1} = {}^5C_r\,(ax)^r$

For $(1+bx)^5$, $T_{r+1} = {}^5C_r\,(bx)^r$

$\therefore\ T_{r+1}$ (of sum) $= {}^5C_r\,(a^r + b^r)\,x^r$

coefficient of term in $x^1 = {}^5C_1(a+b) = 40$

$\therefore a+b = 8$ ①

coefficient of term in $x^2 = {}^5C_2(a^2+b^2) = 260$

$10(a^2+b^2) = 260$

$a^2+b^2 = 26$ ②

(ii) Now $(a+b)^2 = (a^2+b^2) + 2ab = 8^2$

$26 + 2ab = 64$ from ② and ①

$2ab = 38$

$ab = 19$

(iii) coefficient of $x^3 = {}^5C_3(a^3+b^3)$

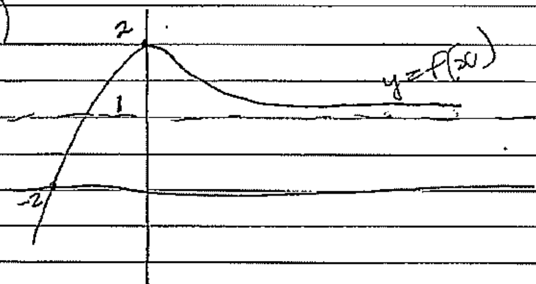
$= 10(a+b)(a^2-ab+b^2)$

$= 10 \times 8 \times (26-19)$

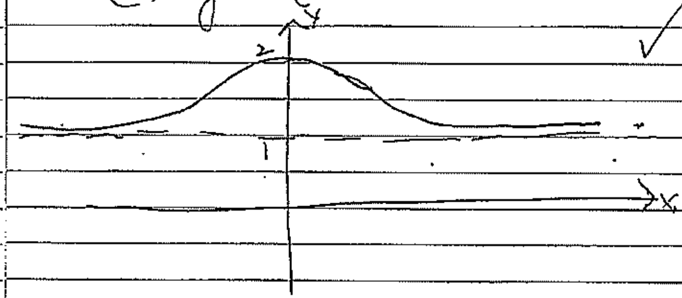
$= 560$

5

Q2. (a)



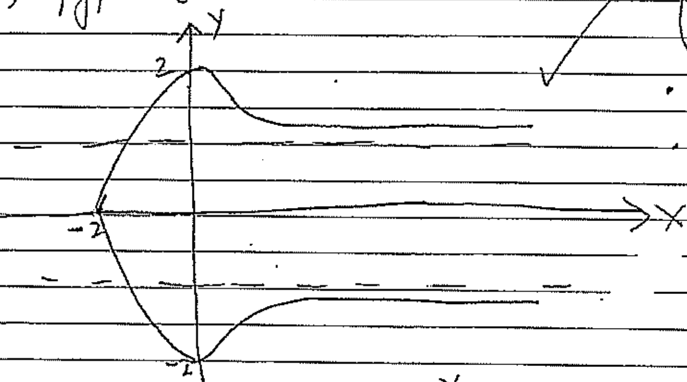
(i) $y = f(|x|)$



$1\frac{1}{2}$

Sketch $y = f(x)$ for $x \geq 0$ only. Then reflect in y-

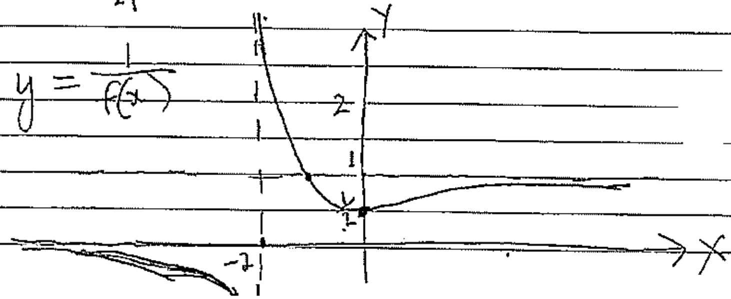
(ii) $|y| = f(x)$



$1\frac{1}{2}$

• Discard section of f(x) below X-axi
• Reflect leftover section in X-a

(iii) $y = \dfrac{1}{f(x)}$

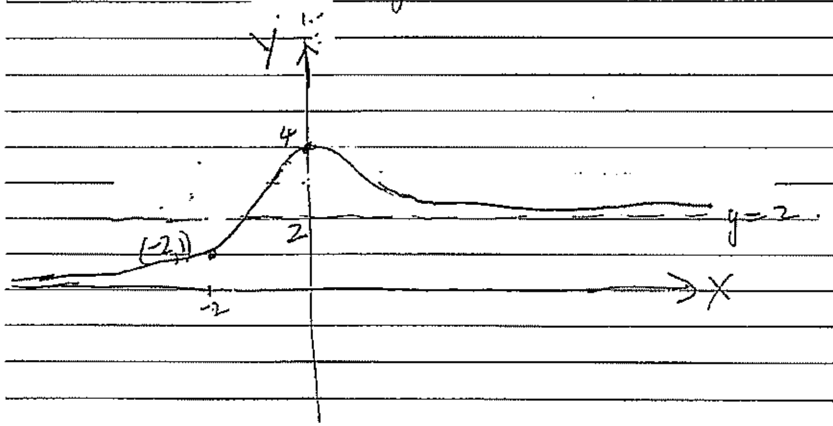


$1\frac{1}{2}$

2(a)

(iv) $y = 2^{f(x)}$

x	-2	0	∞	$-\infty$
$f(x)$	0	2	1^+	$-\infty$
$y=2^{f(x)}$	1	4	2^+	0^+



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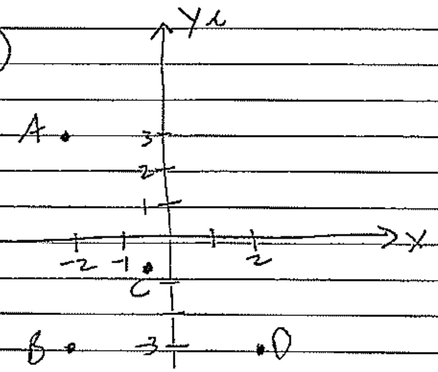
(b)



eg $A \to -2+3i = z$

$|z| = \sqrt{4+9} = \sqrt{13}$

$B \to \bar{z} = -2-3i$

$C = \frac{1}{z} = \frac{-2-3i}{\sqrt{13}}$

$= \frac{-2}{\sqrt{13}} - \frac{3}{\sqrt{13}} i$

$(\doteqdot -.55 - .83i)$

In general

If $A = -a+bi$

$B = -a-bi$

$C = \frac{-a-bi}{\sqrt{a^2+b^2}}$

$D = -z = a-bi$

4

$D = z \operatorname{cis} \pi$ $(-1,0)$

$= z(-1+0i)$

$= -z$

$= 2-3i$

Q2(c) $\int \ln(1+x^2)\,dx$

$= \int 1 \cdot \ln(1+x^2)\,dx$

Parts

$\int u\,dv = uv - \int v\,du$ where $u = \ln(1+x^2)$, $dv = 1$

$du = \frac{2x}{1+x^2}$, $v = x$

$I = x\ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2}\,dx$

$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2}\,dx$.

$= x\ln(1+x^2) - 2I_2$

Now $I_2 = \int \frac{x^2+1-1}{x^2+1}\,dx$

$= \int 1\,dx - \int \frac{1}{x^2+1}\,dx$.

$= x - \tan^{-1}x$.

3

$\therefore I = x\ln(1+x^2) - 2(x - \tan^{-1}x) + C$

$I = x\ln(1+x^2) - 2x + 2\tan^{-1}x + C$

(d) $\int_{-1}^{1} (1+x^3)^3\,dx$

$= \int_{-1}^{1} (1+3x^3+3x^6+x^9)\,dx$

$= \left[x + \frac{3x^4}{4} + \frac{3x^7}{7} + \frac{x^{10}}{10}\right]_{-1}^{1}$

$= \left(1+\frac{3}{4}+\frac{3}{7}+\frac{1}{10}\right) - \left(-1+\frac{3}{4}-\frac{3}{7}+\frac{1}{10}\right)$

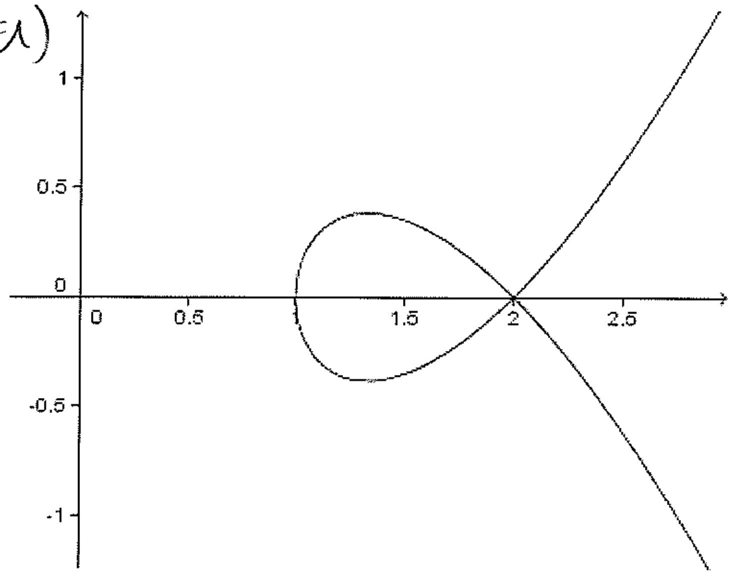
$= 2\frac{6}{7} = \frac{20}{7}$

3

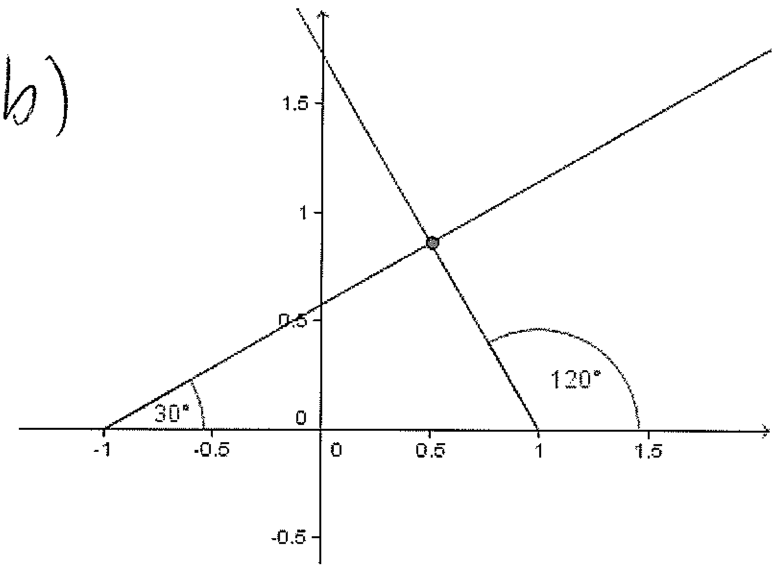
SECTION B

Q4

(a)



(b)



$m_1 = \tan \frac{2\pi}{3}$

$= -\sqrt{3}$

point $(1,0)$

$y = -\sqrt{3}(x-1)$

$m_2 = \tan \frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}$

point $(-1,0)$

$y = \frac{1}{\sqrt{3}}(x+1)$.

intersection

$$-\sqrt{3}(x-1) = \frac{1}{\sqrt{3}}(x+1)$$

$$-3x+3 = x+1$$

$$-4x = -2$$

$$x = \frac{1}{2}.$$

$$y = -\sqrt{3}\left(\frac{1}{2}-1\right).$$

$$y = \frac{\sqrt{3}}{2}$$

$$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(c) (i) $\alpha+\beta,\ \alpha,\ \alpha-\beta.$

$$3\alpha = -\frac{b}{a}$$

$$3\alpha = -3$$

$$\alpha = 1$$

(ii) $1+\beta,\ 1,\ 1-\beta.$

$$p(1) = 1^3 - 3 + a + 8 = 0$$

$$a = -6$$

$$\sum \alpha_i \alpha_j = \frac{c}{a}$$

$$1+\beta + 1-\beta + 1-\beta^2 = -6$$

$$\beta = \pm 3$$

$$x = 4, 1, -2.$$

(d) $\downarrow mg \quad \downarrow \frac{v^2}{10}$

$$10\ddot{x} = -100 - \frac{v^2}{10}$$

$$100\frac{dv}{dt} = -1000 - v^2$$

$$\frac{100}{1000+v^2}\frac{dv}{dt} = -1$$

$$100\int_u^0 \frac{dv}{(10\sqrt{10})^2+v^2} = -\int_0^t dt$$

$$100\left[\frac{1}{10\sqrt{10}}\tan^{-1}\frac{v}{10\sqrt{10}}\right]_u^0 = -t$$

$$t = \sqrt{10}\tan^{-1}\left(\frac{u}{10\sqrt{10}}\right).$$

(ii) $$10\ddot{x} = -100 - \frac{v^2}{10}$$

$$100\ddot{x} = -1000 - v^2$$

$$100\frac{dv}{dx}\frac{dx}{dt} = -1000 - v^2$$

$$\frac{100v}{1000+v^2}\frac{dv}{dx} = -1$$

$$50\int_u^0 \frac{2v}{1000+v^2}\,dv = -\int_0^x dx$$

$$50\left[\ln(1000+v^2)\right]_u^0 = -x$$

$$x = 50\ln\left(\frac{1000+u^2}{1000}\right)$$

Q4
(a) (i) LHS $= F(a) - F(-a)$

where $F' = f$

RHS $= \int_0^a f(x)\,dx + \int_0^a f(-x)\,dx.$

$= F(a) - F(0) - F(-a) + F(0)$

$= F(a) - F(-a)$

$=$ LHS.

(ii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\sin x} = \int_0^{\frac{\pi}{4}} \frac{1}{1+\sin x} + \frac{1}{1-\sin x}\,dx$

since $\sin(-x) = -\sin x$

$= \int_0^{\frac{\pi}{4}} \frac{2}{1-\sin^2 x}\,dx$

$= \int_0^{\pi/4} \frac{1}{\cos^2 x}\,dx.$

$= \int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

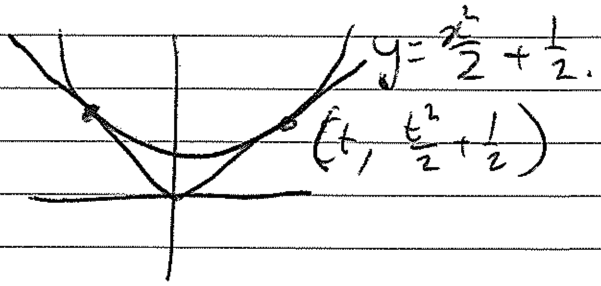
$= 2.$

(b)(i) $|x + i(y-1)| = y$

$$x^2 + (y-1)^2 = y^2$$

$$y = \frac{x^2}{2} + \frac{1}{2}.$$

(ii)



$$\frac{dy}{dx} = x.$$

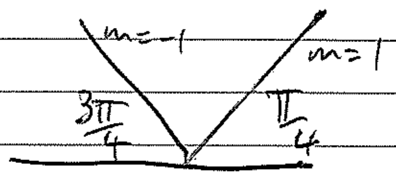
let $m = t$. point $(0,0)$.

$$y - \left(\frac{t^2}{2} + \frac{1}{2}\right) = t(x - t)$$

$$-\frac{t^2}{2} - \frac{1}{2} = -t^2$$

$$t^2 = 1$$

$$t = \pm 1$$



$$\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}.$$

(c) (i)

$$-1 \equiv A(x+1) + Bx(x+1) + Cx^2$$

When $x=-1$

$-1 = C$

When $x=0$

$A=-1$

$\underline{x^2}$

$Bx^2 + Cx^2 = 0x^2$

$B+C=0$

$B-1=0$

$B=1$

$$\frac{-1}{x^2(x+1)} = \frac{-1}{x^2} + \frac{1}{x} + \frac{1}{x+1}$$

(ii) $\dot{x} = -v^2 - v^3$

$$\frac{dv}{dt} = -(v^2+v^3)$$

$$\frac{dt}{dv} = -\frac{1}{v^2(v+1)}$$

$$= \frac{-1}{v^2} + \frac{1}{v} + \frac{-1}{v+1}$$

$$t = \frac{1}{v} + \ln v - \ln(v+1) + C$$

when $t=0$ $v=u$

$$C = -\frac{1}{u} + \ln\left(\frac{u+1}{u}\right)$$

$$t = \frac{1}{v} + \ln v - \ln(v+1) - \frac{1}{u} + \ln\left(\frac{u+1}{u}\right)$$

when $t = T$ $v = \frac{u}{2}$

$$T = \frac{2}{u} + \ln\left(\frac{\frac{u}{2}}{\frac{u}{2}+1}\right) - \frac{1}{u} + \ln\left(\frac{u+1}{u}\right)$$

$$T = \frac{1}{u} + \ln\left(\frac{u+1}{u+2}\right) \quad \text{But } X = \ln\left(\frac{2+u}{1+u}\right)$$

$$T = \frac{1}{u} - \ln\left(\frac{2+u}{1+u}\right)$$

$$T = \frac{1}{u} - X$$

$$u(T+X) = 1$$

(ii) $v = u(ux + ut + 1)^{-1}$

$$\frac{dv}{dt} = -u(ux+ut+1)^{-2}\left(u\frac{dx}{dt} + u\right)$$

$$\ddot{x} = \frac{-u}{(ux+ut+1)^2}(uv+u)$$

$$= \frac{-u^2(v+1)}{\left(\frac{u}{v}\right)^2}$$

$$= -v^2(v+1)$$

$$= -v^3 - v^2$$

SECTION C Q5

(a) (i) $(1+i)^n = \left(\sqrt{2}\operatorname{cis}\frac{\pi}{4}\right)^n$

$$= (\sqrt{2})^n \operatorname{cis}\frac{n\pi}{4} \qquad (A)$$

(ii) $(1+i)^n = \binom{n}{0} + \binom{n}{1}i + \binom{n}{2}i^2 + \binom{n}{3}i^3 + \binom{n}{4}i^4 + \cdots$

$$= \binom{n}{0} + \binom{n}{1}i - \binom{n}{2} - \binom{n}{3}i + \binom{n}{4} - \cdots$$

$$= \binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} - \cdots + i\left(\binom{n}{1} - \binom{n}{3}\right) - \cdots$$

now real part is $\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \cdots$

which is $\boxed{(\sqrt{2})^n \cos\frac{n\pi}{4}}$ from (A)

(iii) If n is a multiple of 8. say 8m

In (A) $(\sqrt{2})^n \operatorname{cis}\frac{8m\pi}{4}$

$$= (\sqrt{2})^n\left[\cos 2m\pi + i\sin 2m\pi\right]$$

$$= (\sqrt{2})^n\left[1 + 0\right]$$

$\therefore$ imaginary part is zero.

ie. $\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \binom{n}{7} + \binom{n}{9} - \binom{n}{11} + \cdots = 0$

ie. $\boxed{\binom{n}{1} + \binom{n}{5} + \binom{n}{9} \cdots = \binom{n}{3} + \binom{n}{7} + \binom{n}{11} + \cdots}$

(b) (i)

$\boxed{\ddot{x} = 0}$

$\dot{x} = c_1$

$\therefore \boxed{\dot{x} = V\cos\theta}$

$x = Vt\cos\theta + c_3$

clearly $c_3 = 0$

$\therefore \boxed{x = Vt\cos\theta}$

$\boxed{\ddot{y} = -g}$

$\dot{y} = -gt + c_2$

$\therefore \boxed{\dot{y} = -gt + V\sin\theta}$

$y = -\frac{1}{2}gt^2 + Vt\sin\theta + c_4$

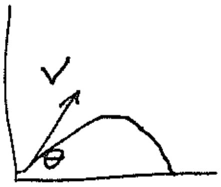
$c_4 = 0$

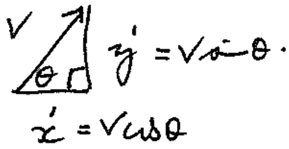
$\therefore \boxed{y = -\frac{1}{2}gt^2 + Vt\sin\theta}$

$t = 0$

$x = 0$

$y = 0$





(ii). now $x = y$

$\therefore \quad Vt\cos\theta = -\frac{1}{2}gt^2 + Vt\sin\theta$

OR $\boxed{V\cos\theta = -\frac{1}{2}gt + V\sin\theta}$ (A) (NB $t \neq 0$)

& $\dot{x} = -\dot{y}$

$\therefore \boxed{V\cos\theta = gt - V\sin\theta.}$ (B)

(A) − (B)

$\boxed{0 = -\frac{3}{2}gt + 2V\sin\theta}$ (C) $\Rightarrow \boxed{V\sin\theta = \frac{3}{4}gt}$

(A) + (B)

$2V\cos\theta = \frac{1}{2}gt$

ie $\boxed{V\cos\theta = \frac{1}{4}gt}$ (D)

From (C) & (D)

$$\frac{V\sin\theta}{V\cos\theta} = \frac{\frac{3}{4}gt}{\frac{1}{4}gt}.$$

$\therefore \boxed{\tan\theta = 3}$

Q6. (a) $P(x) = x^n - x^{n-1} - 1 \quad ; n > 1.$

& $P'(x) = nx^{n-1} - (n-1)x^{n-2}$

For a repeated root α $\quad P(\alpha) = P'(\alpha) = 0$

Consider $P'(\alpha) = n\alpha^{n-1} - (n-1)\alpha^{n-2} = 0.$

ie $\alpha^{n-2}\left[n\alpha - (n-1)\right] = 0$

$\alpha \neq 0.$ as $P(\alpha) = -1$

$\therefore \quad \alpha = \frac{n-1}{n}.$

Assume $P(\alpha) = 0$

ie. $\left(\frac{n-1}{n}\right)^n - \left(\frac{n-1}{n}\right)^{n-1} - 1 = 0$

ie. $\left(\frac{n-1}{n}\right)^{n-1}\left[\frac{n-1}{n} - 1\right] = 1$

$\left(\frac{n-1}{n}\right)^{n-1}\left(-\frac{1}{n}\right) = 1$

This is impossible

since $\left(\frac{n-1}{n}\right)^{n-1} > 0 \quad [n > 1]$

& $-\frac{1}{n} < 0$

$\therefore$ LHS is negative

$\therefore$ contradiction

$P(\alpha) \neq 0$

$\therefore$ no repeated roots.

Q6 (b) (i) (α) $\dfrac{11!}{2! \times 2!} = \boxed{9,979,200}$

(β) $\dfrac{9!}{2} \times 2 = \boxed{362,880}$

(γ) $\dfrac{9!}{2} = \boxed{181,440}$

(δ) $\dfrac{11!}{2! \times 2!} - \dfrac{10!}{2} - \dfrac{10!}{2} + 9! = \boxed{6,713,280}$

(ii) Look at cases.

$$7 \times \frac{5!}{2! \times 2!} + \binom{7}{2} \times \frac{5!}{2!} \times 2 + \binom{7}{3} \times \frac{5!}{2!} \times 2$$

$$+ \binom{7}{3} \times 5! + \binom{7}{4} \times 5! \times 2 + {}^7P_5$$

$$= 210 + 2520 + 420 + 420 + 840 + 2520$$

$$= \boxed{22\,050}$$