SYDNEY GIRLS HIGH SCHOOL

2006 HSC Assessment Task 3

June 7, 2006

MATHEMATICS Extension 2

Year 12

Time allowed: 90 minutes

Topics: Polynomials, Integration

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Questions are not of equal value
- There are 19 questions with part marks shown in brackets
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work
- Board approved calculators may be used
- Write on one side of the paper only

1. Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$ given the roots are in Arithmetic Progression   [4]

2. Given $1 + i$ is a root of $P(x) = x^3 + x^2 - 4x + 6$, factorise $P(x)$ over the real field.   [4]

3. a) If $P(x)$ has a root α of multiplicity m, show that $P'(x)$ has a root α of multiplicity (m-1).

   b) Given $P(x) = 8x^4 - 20x^3 - 18x^2 + 81x - 54$ has a triple root, solve for x over the real field and sketch the polynomial.   [8]

4. Given $P(x) = x^3 - 7x^2 + 18x - 7$ has roots α, β, γ, find

   a) The values of $i) \alpha^2 + \beta^2 + \gamma^2$

      $ii) \alpha^3 + \beta^3 + \gamma^3$

      $iii) \alpha^4 + \beta^4 + \gamma^4$

   b) The polynomial equation with roots

      $i) \frac{2}{\alpha}, \frac{2}{\beta}, \frac{2}{\gamma}$

      $ii) \alpha^2, \beta^2, \gamma^2$ [7]

5. If one root of the equation $x^3 + ax^2 + bx + c = 0$ is equal to the sum of the other two roots, show that $a^3 - 4ab + 8c = 0$ [6]

In Questions 6 – 14, find the following integrals

6. $\int \frac{2x+4}{x^2+4} . dx$ [3]

7. $\int \frac{\tan^2 x}{\sec^2 x} . dx$ [3]

8. $\int \frac{dx}{x(\log_e x)^3}$ [3]

9. $\int \frac{x dx}{\sqrt{6x - x^2}}$ [6]

10. $\int \cos^5 x . \sin^2 x . dx$ [4]

11. $\int \dfrac{dx}{5 + 4\cos x}$ [4]

12. $\int \dfrac{dx}{x^2 \sqrt{x^2 + 9}}$ [6]

13. $\int \dfrac{e^{2x} . dx}{e^x + 1}$ [4]

14. $\int \sqrt{\dfrac{4+x}{4-x}} . dx$ [6]

15. a) Simplify $\sin(A+B) + \sin(A-B)$

    b) Use the result in part a) to find $\int \sin 5x . \cos 3x . dx$ [4]

16. a) Show $\displaystyle\int_0^a f(x) . dx = \int_0^a f(a-x) . dx$

    b) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{\sin 2x}{\cos 2x + \sin 2x} . dx$ [5]

17. a) Express $\dfrac{10}{(x^2 - 1)(x^2 + 4)}$ in partial fractions

    b) Find $\int \dfrac{10 . dx}{(x^2 - 1)(x^2 + 4)}$ [7]

18. When a monic cubic polynomial $A(x)$ is divided by $x^2 + x + 1$, the remainder is $2x + 3$. When $A(x)$ is divided by $x(x+3)$, the remainder is $5(x+1)$.
Find the equation of $A(x)$.

[6]

19. Given $I_n = \int \left(1 - \sqrt{x}\right)^n . dx$,

    a) Show that $I_n = \frac{2}{n+2}[x(1-\sqrt{x})^n] + \frac{n}{n+2} I_{n-1}$

    b) Hence evaluate $\displaystyle\int_0^1 \left(1 - \sqrt{x}\right)^6 dx$ [10]

------------------------------end of exam------------------------------