# BAULKHAM HILLS HIGH SCHOOL

## YEAR 12

## YEAR 12 HALF YEARLY EXAMINATION

# 2004

# MATHEMATICS

*Time allowed - Three hours*
*(Plus five minutes reading time)*

### DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Start each of the 10 questions on a new page.
- All necessary working should be shown.
- Write your teacher's name and your name on the cover sheet provided.
- At the end of the exam, staple your answers in order behind the cover sheet provided.

**QUESTION 1**

(a) Evaluate correct to 3 significant figures

$$\frac{\sqrt{632.16}}{18.29+4.3}$$

(b) Factorize

$3x^3 - 81$

(c) Solve and graph the solution to

$|3x-4| < 8$

(d) Solve

$2^{2x} - 15.(2^x) - 16 = 0$

(e) Solve

$\sin\theta = -\frac{1}{2}$ for $0 \le \theta \le 360°$

(f) Solve

$x - \frac{x-5}{2} = 4$

**QUESTION 2 - (*Start a new page*)**

(a) If $\frac{\sqrt{2}}{3-2\sqrt{2}} = a + b\sqrt{2}$ calculate the values of $a$ and $b$.

(b) A function is defined such that

$$f(x) = \begin{cases} 2 \text{ for } x < 0 \\ x^2 + 2 \text{ for } 0 \le x < 3 \\ 17 - 2x \text{ for } x \ge 3 \end{cases}$$

(i) Calculate $f(1) + f(-3) - f(5)$

(ii) Sketch this function for $-2 \le x \le 4$

(c) Differentiate

(i) $3x(x+6)^3$

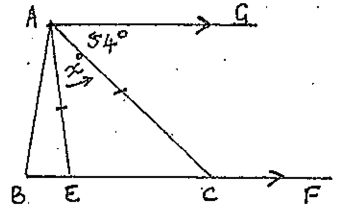
(ii) $\sqrt{2x+1}$

(iii) $\frac{4x+2}{x-1}$

**QUESTION 3 - (*Start a new page*)**

(a) In the diagram AE = AC, AG is parallel to BF. $G\hat{A}C = 54°$.

Find the value of $x$, giving reasons:




(b) On a number plane, plot the points A(-3, -1), B(1, 4) and C(4, 2). Join A to B and B to C.

(i) Find the midpoint of AC.

(ii) State the co-ordinates of D, the fourth vertex of parallelogram ABCD.

(iii) Find the equation of the line AB.

(iv) Calculate the perpendicular distance from the point D to the line AB.

(v) Find the area of the parallelogram ABCD.

**QUESTION 4 - (*Start a new page*)**

(a) The focus of a parabola is S(0, 2) and its directrix is the line y=4.

(i) Sketch the parabola and indicate the co-ordinates of the vertex V.

(ii) Write down the focal length of the parabola.

(iii) Find the equation of this parabola.

(b) Find

(i) $\int x^3 - 3x^{-2}\,dx$

(ii) $\int_4^{16} \frac{dx}{\sqrt{x}}$

(c) Find the equation of the normal to $y = \frac{1}{\sqrt{3x-5}}$ at the point where $x = 2$.

(d) Solve

$2\cos\theta = \sqrt{3}$ for $0 \le \theta \le 360°$.

**QUESTION 5 - (*Start a new page*)**

(a) Consider the equation

$x^2 + (k+2)x + 4 = 0$ For what values of $k$ does the equation have

(i) equal roots

(ii) distinct real roots?

(b) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44.

(i) Find the value of the common difference and the value of the first term.

(ii) Find the sum of the first 45 terms.

(c) Consider the curve $y = 3x^2 - x^3$.

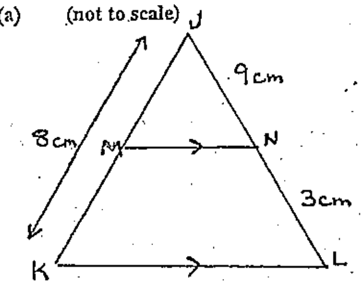
(i) Find the stationary points and determine their nature.

(ii) Sketch the curve over the domain $-3 \le x \le 5$ showing all relevant features.

(iii) For which values of $x$ is this function increasing.

**QUESTION 6 - (*Start a new page*)**

(a) (not to scale)



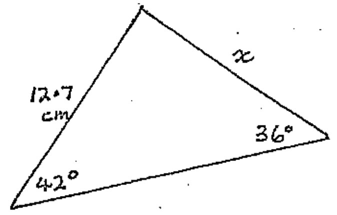

The diagram shows a triangle JKL. MN ∥ KL, JK = 8cm, JN = 9cm and NL = 3cm.

(i) Prove that $\triangle JMN$ is similar to $\triangle JKL$.

(ii) Find the length of MK.

(b) Find the value of $x$, correct to 2 decimal places.

(not to scale)




(c) The length of the line joining the points A(a, -2) and B(3, -7) is $5\sqrt{2}$ units. Find all possible values of a.

(d) Sketch the region where

$y \le \sqrt{9-x^2}$ and $x - 3y \le 3$ hold simultaneously

(e) Sketch the curve

$y = \cos 2x$ for $0 \le x \le \pi$

QUESTION 7 - (*Start a new page*)

(a) Given that $\cos\alpha = -\frac{5}{\sqrt{29}}$ and $\tan\alpha < 0$ find the value of $\sin\alpha$.

(b) The curve $y = ax^3 + bx$ passes through the point (1, 7). The tangent at this point is parallel to the line $y = 2x - b$. Find the values of a and b.

(c) (i) On the same diagram graph the functions $y = x^2$ and $y = 5 - 4x$ showing all intercepts with the $x$ and $y$ axis.

(ii) Show that the graphs intersect at $x = 1$ and $x = -5$.

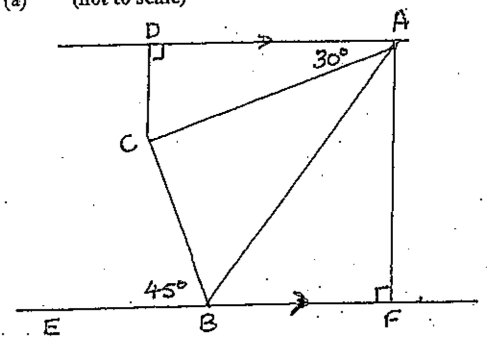
(iii) Hence find the exact area bounded by the two functions.

(d) Express $0.\dot{2}\dot{9}$ as a sum of an infinite series hence, find a simple equivalent fraction for $0.\dot{2}\dot{9}$.



QUESTION 8 - (*Start a new page*)

(a) (not to scale)




In the diagram AC = AB and DA is parallel to EF. $D\hat{A}C = 30°$ and $C\hat{B}E = 45°$. CD is perpendicular to DA and AF is perpendicular to EF.

(i) Copy or trace the diagram onto your working paper and show all information clearly.

(ii) Find the size of $A\hat{C}B$, giving reasons.

(iii) Hence find the size of $C\hat{A}B$.

(iv) Prove that $\Delta ACD \equiv \Delta ABF$.

(b) A set of packing crates has been designed each in the shape of a rectangular prism. When empty, each crate packs inside the next sized crate. The largest crate is 2 metres long by 2 metres wide by 1 metre high. The crate inside this is 1 metre by 1 metre by 0.5 metres. Each succeeding crate has dimensions which are half those of the preceding one.

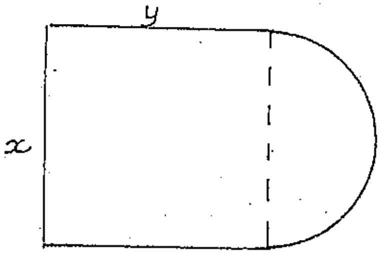
(i) Write down the dimensions of the third largest crate.

(ii) Show that the volumes of the first 3 crates form a G.P.

(iii) Calculate the maximum possible total volume for the complete set.

## QUESTION 9 - *(Start a new page)*

(a) A garden bed is to be constructed in the shape of a semi-circle along one side of a rectangle.




The width of the rectangle is $x$ and the length $y$ metres as in the diagram.

Forty metres of edging material is available to be laid around the garden bed.

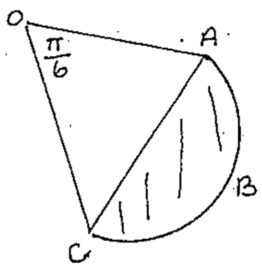
(i) Show that $y = 20 - \frac{1}{2}x - \frac{1}{4}\pi x$.

(ii) Show that the area of the garden bed can be expressed as $A = 20x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$.

(iii) Find the dimensions of the rectangle such that the area contained in the garden bed is a maximum.

(b) If $\alpha$ and $\beta$ are the roots of the equation $x^2 + x - 4 = 0$ find the equation whose roots are $\alpha^2$ and $\beta^2$.

(c) The diagram shows a sector OABC with angle $\frac{\pi^c}{6}$ and OA = 5 cm.

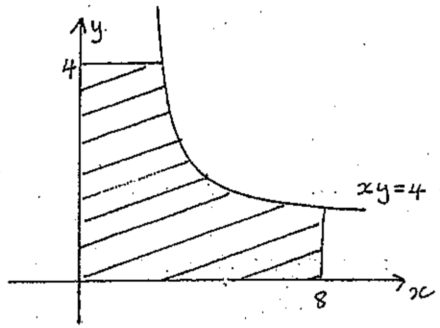



(i) Find the length of arc ABC.

(ii) Find the area of segment ABC

## QUESTION 10 - *(Start a new page)*

(a)




The area enclosed by the curve $xy=4$, the $x$ axis and $y$ axis, and the lines $y=4$ and $x=8$, is rotated about the $y$ axis.

(i) Show that the volume of the solid of revolution obtained is given by

$$V_y = 32\pi + \pi \int_{\frac{1}{2}}^{4} \frac{16}{y^2} dy$$

(ii) Hence find the volume of the solid of revolution.

(b) For the function $y = \frac{1}{1+x^2}$ complete the table below

| $x$ | 4 | 4.5 | 5 | 5.5 | 6 |
|---|---|---|---|---|---|
| $y$ | | 0.047 | | 0.032 | 0.027 |

Using Simpson's Rule with 5 functions values find the approximate area enclosed between the curve $y = \frac{1}{1+x^2}$ the $x$ axis and the lines $x=4$ and $x=6$.

(c) Prove that $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$.

# STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$