STUDENT'S NAME: ______________________

TEACHER'S NAME: ______________________

BAULKHAM HILLS HIGH SCHOOL

Year 12

MATHEMATICS ADVANCED ASSESSMENT

HALF-YEARLY

March 2008

Time allowed – 3 hours + 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Start each question on a new page.
- Show all relevant working.
- Use black or blue pen.
- **NO** liquid paper is to be used.
- Approved Maths aids and calculators may be used.

QUESTION 1 [12 marks] **Marks**

(a) Evaluate, correct to three significant figures **2**

$$1-\sqrt{\frac{(2.044)^3}{34.5-1.2^2}}$$

(b) Factorise x^3-5x^2+6x **2**

(c) A function $f(x)$ is defined as $f(x)=\begin{cases}|x-1| & for\ x\le 0\\ 4-x^2 & for\ x>0\end{cases}$ **2**

Evaluate $f(2)+f(0)$

(d) Evaluate $\sum_{r=1}^{3} r^2$ **2**

(e) $\lim_{x->3}\frac{x^2-9}{x-3}$ **2**

(f) Evaluate $\int_0^3 e^{2x}dx$ in terms of e **2**

QUESTION 2 [12 marks]

(a) Find $\int\frac{3x^2}{x^3+5}dx$ **2**

(b) Differentiate

i) $x^2\ln 3x$ **2**

ii) $\frac{x}{3x-2}$ **2**

(c) Given $\log_a 2=0.36$ and $\log_a 5=0.83$,

Show working to find the value of

(i) $\log_a 10$ **2**

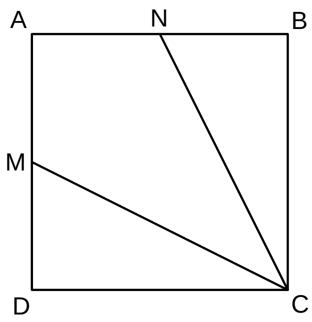
(ii) $\log_2 5$ **2**

(d) By rationalising the denominators, **2**

express $\frac{1}{3-\sqrt{2}}+\frac{1}{3+\sqrt{2}}$ in simplest form

QUESTION 3 [12 marks] **Marks**

(a)



ABCD is a square. Lines are drawn from *C* to *M* and *N*, the midpoints of *AD* and *AB* respectively.

(i) Show that $\Delta CBN \equiv \Delta CDM$ **3**

(ii) Prove that $MC = NC$ **1**

(b) Let $T_n =$ the nth term of a series, and $S_n =$ the sum of n terms of a series. For a particular arithmetic series, $S_n = n(2n-1)$

(i) Find S_1 and S_2 **2**

(ii) Find the expression for T_n for this series **2**

(c) For the curve $y = \dfrac{1}{x+1} + 4$

(i) State the domain and range **2**

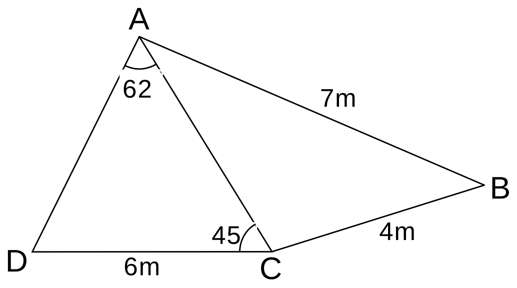
(ii) Sketch the graph showing all important features **2**

QUESTION 4 [12 marks] **Marks**

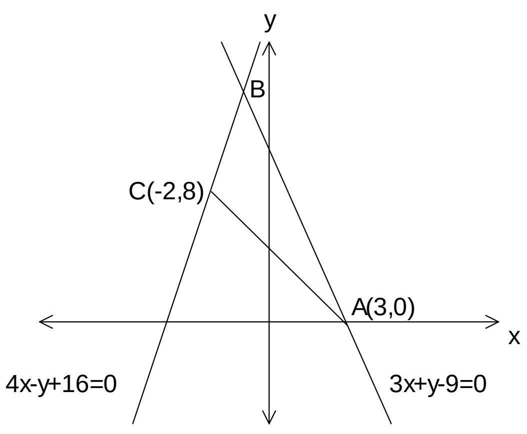
(a) For the following diagram, find

(i) find AC **2**

(ii) find the size of $\angle ABC$ (correct to the nearest degree) **2**



(b) In the diagram below the lines $3x+y-9=0$ and $4x-y+16=0$ intersect at the point B. The point A has the coordinates A(3,0) and the point C has the coordinates C(-2,8).



(i) Show that the line AC has the equation $8x+5y-24=0$ **2**

(ii) B is the intersection point of the lines $3x+y-9=0$ and $4x-y+16=0$, show that the co-ordinates of B is (-1,12) **1**

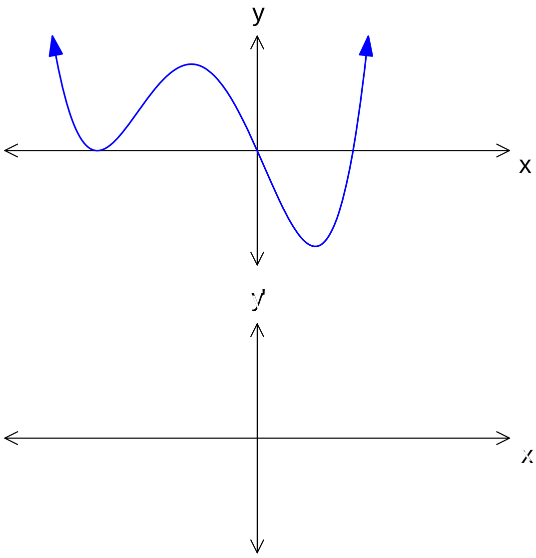
(iii) Find the length of AC. Leave your answer as a surd. **2**

(iv) Find the perpendicular distance from point B to AC. Leave your answer as a surd. **2**

(v) Hence find the area of ΔABC **1**

QUESTION 5 [12 marks] **Marks**

(a) Copy the diagram and below the axis sketch the derivative function **1**



(b) Consider the function $f(x) = 3x^4 - 4x^3$

(i) Show that $f'(x) = 12x^2(x-1)$ **1**

(ii) Find the coordinates of the stationary points of the curve $y = f(x)$ and determines their nature. **4**

(iii) Sketch the graph of the curve $y = f(x)$, showing stationary points, **1**

(iv) Find the coordinates of any points of inflection. **2**

(v) Find the values of x for which the graph of $y = f(x)$ is concave down. **2**

(vi) Find the greatest values of $y = f(x)$ in the domain $-1 \leq x \leq 2$ **1**

QUESTION 6 [12 marks] **Marks**

(a) If α , β are roots of the quadratic equation $6x^2 - x + 5 = 0$, find:

(**i**) $\alpha + \beta$ **1**

(**ii**) $\alpha \times \beta$ **1**

(**iii**) $\alpha^2 + \beta^2$ **2**

(iv) $\alpha^3 + \beta^3$ **2**

(b) Given the parabola $y = x^2 + 6x + 7$, find:

(**i**) focal length **1**

(**ii**) vertex **1**

(iii) Find the equation of the normal at $x = -1$, in general form. **2**

(c) Solve for $2\cos\theta = 1 \quad for\ 0^o \le \theta \le 360^o$ **2**

QUESTION 7 [12 marks]

(a) Show that $\dfrac{3x+3}{x^2-9} = \dfrac{1}{x+3} + \dfrac{2}{x-3}$ **2**

Hence find $\int \dfrac{3x+3}{x^2-9}\,dx$.

(b) Find the volume of the solid of revolution formed by rotating **4**
the curve $y = x - \dfrac{1}{x}$ about the x-axis between $x = 1$ and $x = 4$
Answer in terms of π .

(c) Find $\dfrac{d}{dx}(x^2 - 3)^5$ and **2**

Hence find $\int x(x^2-3)^4\,dx$

(d) Find the largest term less than 10 000 in the series $1 + 4 + 7 + 10$ **2**

(e) Find the values of K for which the equation **2**
$x^2 - Kx + 4 = 0$, has two real distinct roots

QUESTION 8 [12 marks] **Marks**

(a) Differentiate:

(i) 4^x **1**

(ii) $\log_{10} x$ **2**

(b) Solve the equation $2\ln x = \ln(6 - x)$ **3**

(c) By expressing $0.\overline{25}$ as the sum of an infinite geometric series, find the simple equivalent fraction for $0.\overline{25}$. **2**

(d) The gradient of the curve $y = f(x)$ is given by $f'(x) = (2x+1)^5$. Find the equation of the curve if it passes through the point (1, 5) **2**

(e) Find the values A ,B and C if **2**
$x^2 + 5 \equiv A(x-1)^2 + B(x-1) + C$

QUESTION 9 [12 marks]

(a) Solve for *x:* $3^{2x} - 4(3^x) + 3 = 0$ **2**

(b) **(i)** Sketch the graph $y = |2x| - 4$ for $-2 \le x \le 4$. **2**
Showing all features of the graph.

(ii) Hence or otherwise, evaluate **2**

$$\int_{-2}^{4} |2x| - 4 \, dx$$

(c) A farmer is building a wheat silo in the shape of a closed cylinder of radius r metres. The silo is to be made from galvanised iron sheeting and is to have a capacity of $300m^3$.

(i) Find an expression for the height of the silo in terms of r. **1**

(ii) Show that the surface area A, of the silo is given by the equation **2**

$$A = 2\pi r^2 + \frac{600}{r}$$

(iii) Hence find the minimum area of galvanised iron sheeting needed to make the silo, leaving your answer in one decimal place. **3**

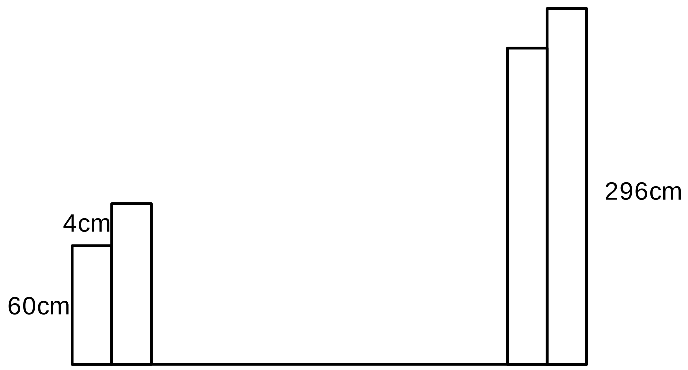
QUESTION 10 [12 marks] **Marks**

(a) Find the area enclosed between the curve $y = x^2$ and the line $y = x + 2$ **3**

(b) **(i)** Shade the region bounded by the curve $y = \log_e x$, the y-axis the x-axis and the line $y = 2$. **1**

(ii) Calculate the volume of the solid of revolution formed when this region is rotated about the y-axis, in terms of π. **3**

(c) The following diagram represents a retaining wall in a nursery made from timber. Each length of the timber is 4*cm* longer than the proceeding one. The last length of timber is 296*cm* long and the first is 60*cm* long.



Find:

(i) How many lengths of timber make up the wall **2**

(ii) The total length of the timber needed to make the retaining wall. **2**

(iii) If each of timber is *10*cm wide, find the area of the wall in square metres **1**

End of Paper

The third term and the tenth term of an arithmetic series are 10 and 31 respectively. Find the:

i) first term and the common difference **2**

ii) sum of the first ten terms of the series **1**

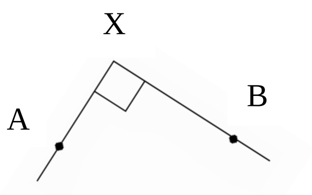
(c) Using Simpson's Rule with three function values, find an approximate value for the area represented by the definite integral **3**

$$\int_2^3 \cos^2 x \, dx$$

(d) For what values of k does the quadratic equation $kx^2 + kx + 1 = 0$ have no real roots? **1**

Find the equation of the tangent to the curve $y = \ln(x^2 + 2)$ at the point where $x = 1$. Answer in general form **4**

(b)
(c)



Solve $\log_{27} 32 = x \log_3 2$ without the aid of a calculator. Show all working **2**

(f)

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Q1 a) $0.49175\ldots$ ✓

$= 0.492$ (3 sig fig) ✓

b) $x^3 - 5x^2 + 6x$

$= x(x^2 - 5x + 6)$ ✓

$= x(x-3)(x-2)$ ✓

c) $f(2) = 4 - 2^2 = 0$

$f(0) = |0-1| = 1$ ✓

$f(2) + f(0) = 0 + 1 = 1$ ✓

d) $\sum_{r=1}^{3} r^2 = 1^2 + 2^2 + 3^2$ ✓

$= 1 + 4 + 9$

$= 14$ ✓

e) $\lim_{x\to 3} \frac{(x+3)(x-3)}{(x-3)} = \lim_{x\to 3} x+3$ ✓

$= 6$ ✓

f) $\int_0^3 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_0^3$ ✓

$= \frac{1}{2}e^6 - \frac{1}{2}e^0$

$= \frac{1}{2}e^6 - \frac{1}{2}$ ✓

Q2

a) $\int \frac{3x^2}{x^3+5}\,dx = \ln(x^3+5) + C$ ✓ ✓

b) (i) $y = x^2 \ln 3x$

$u = x^2 \quad v = \ln 3x$

$u' = 2x \quad v' = \frac{3}{3x} = \frac{1}{x}$ ✓

$y' = 2x\ln 3x + \frac{1}{x} \times x^2$ ✓

$= 2x\ln 3x + x$

(ii) $y = \frac{x}{3x-2} \quad \frac{u}{v}$

$u = x \quad v = 3x-2$

$u' = 1 \quad v' = 3$ ✓

$y' = \frac{1 \times (3x-2) - 3x}{(3x-2)^2}$

$= \frac{-2}{(3x-2)^2}$ ✓

c) i) $\log_a 10 = \log_a 2 \quad \log_a 5$ ✓

$= 0.36 + 0.83$

$=$ ✓

(ii) $\log_2 5 = \frac{\log_a 5}{\log_a 2} = \frac{0.83}{0.36}$ ✓ ✓

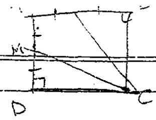
$=$

d) $\frac{1}{(3-\sqrt{2})} \frac{(3+\sqrt{2})}{(3+\sqrt{2})} + \frac{1}{(3+\sqrt{2})} \frac{(3-\sqrt{2})}{(3-\sqrt{2})} = \frac{6}{9-2}$ ✓

$= \frac{6}{7}$ ✓

Q3

a)



DM = AM = AN = NB (∴ M and N bisects equal sides of square)

In $\triangle CBN$ and $\triangle CDM$.

BC = DC (equal sides of square) ✓

NB = MD (Shown above) ✓

$\angle MDC = \angle NBC$ (∠s of a square is 90°)

$\therefore \triangle CBN \equiv \triangle CDM$ (SAS) ✓

ii) MC = NC (matching sides of congruent △s are =) ✓

b) (i) $S_1 = 1(2\times1-1)$
$= 1.$ ✓

$S_2 = 2(2\times2-1)$
$= 6.$ ✓

(ii) $T_n = S_n - S_{n-1}$

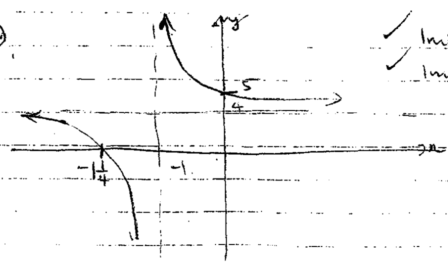
$S_{n-1} = (n-1)(2(n-1)-1)$
$= (n-1)(2n-3)$
$= 2n^2 - 5n + 3$ ✓

$T_n = (2n^2 - n) - (2n^2 - 5n + 3)$ ✓
$= 4n - 3$

c)(i) Domain {all real x, except $x \neq -1$} ✓

Range {all real y, except $y \neq 4$} ✓

(ii)



✓ 1mk intercepts

✓ 1mk curve

→ x int, y = 0

$-4 = \frac{1}{x+1}$

$x = -\frac{1}{4} - 1 = -\frac{5}{4}$

→ y int; x = 0

$y = \frac{1}{0+1} + 4 = 5$

Q4 a) (i) $\frac{AC}{\sin 73°} = \frac{6}{\sin 62°}$ ✓

$AC = 6.49...$ ✓

ii) $c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \frac{7^2 + 4^2 - AC^2}{2\times7\times4}$

$C = 66°$ ✓ ✓ (nearest degree)

b) (i) $m = \frac{8-0}{-2-3} = \frac{8}{-5}$ ✓

at (3,0)

$y - 0 = \frac{8}{-5}(x-3)$

$-5y = 8x - 24$

$8x + 5y - 24 = 0$ ✓

ii) sub (−1, 12).

① $3(-1) + (12) - 9 = 0$ ∴ lies on line

② $4(-1) - (12) + 16 = 0$ ∴ lies on line

∴ B(−1, 12) is the pt of intersection ✓

iii) $AC = \sqrt{(3+2)^2 + (8)^2}$ ✓
$= \sqrt{89}$ ✓

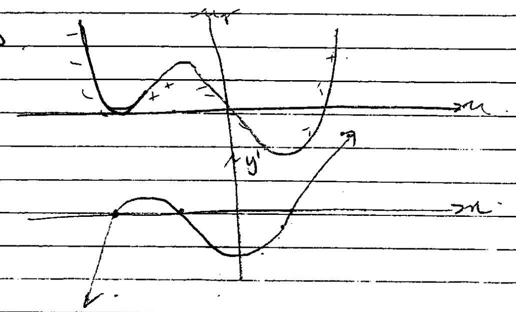
iv) $d = \left|\frac{8(-1) + 5(12) - 24}{\sqrt{8^2 + 5^2}}\right| = \frac{28}{\sqrt{89}}$ ✓ ✓

(v) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times \sqrt{89} \times \frac{28}{\sqrt{89}}$

$= 14 \text{ units}^2$

Q5 a)



b) $f(x) = 3x^4 - 4x^3$

(i) $f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1)$

ii) stat pt when $f'(x) = 0$

$0 = 12x^2(x-1)$

$x = 0$ or $x = 1$

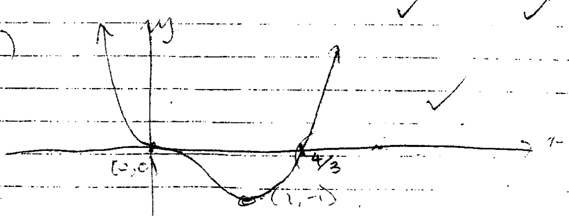
$f(0) = 0$ or $f(1) = -1$

$f''(x) = 36x^2 - 24x$

$f''(0) = 0$ ∴ $(0,0)$ is a horizontal inflection

$f''(1) = 12 > 0$ concave up ∴ $(1,-1)$ is a min pt.

iii)



Q5 iv) $f''(x) = 36x^2 - 24x$

$0 = 36x^2 - 24x$

$0 = x(3x-2)$

$x = 0$ $\quad x = \frac{2}{3}$

$f(0) = 0$ $\quad f(\frac{2}{3}) =$

pts of inflections are $(0,0)$ and $(\frac{2}{3}, -\frac{16}{27})$

v) concave down when $f''(x) < 0$

$x(3x-2) < 0$.

∴ concave down when $0 < x < \frac{2}{3}$

vi) $f(-1) = 3(-1)^4 - 4(-1)^3$

$= 7$

$f(2) = 3(2)^4 - 4(2)^3$

$= 16$

∴ max value is 16.

Q6 a) $6x^2 - x + 5 = 0$

(i) $\alpha + \beta = \frac{1}{6}$

(ii) $\alpha\beta = \frac{5}{6}$

(iii) $\alpha^2 + \beta^2 = \alpha^2 + \beta^2 + 2\alpha\beta - 2\alpha\beta$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= -\frac{59}{36}$

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\frac{1}{6})(-\frac{59}{36} - \frac{5}{6})$

$= -\frac{89}{216}$

Q6

b) $y = x^2 + 6x + 7$

$y = x^2 + 6x + 9 - 9 + 7$

$y = (x+3)^2 - 2$

$(x+3)^2 = (y+2)$

(i) focal length $= \frac{1}{4}$ ✓

ii) vertex $(-3, -2)$ ✓

iii) $y' = 2x + 6$

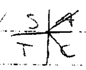
at $x = -1$ $m_T = 4$

$y = 1 - 6 + 7 = 2$

at $(-1, 2)$ $m_N = -\frac{1}{4}$ ✓

$y - 2 = -\frac{1}{4}(x+1)$

$x + 4y - 7 = 0$ ✓

c) $\cos\theta = \frac{1}{2}$ ✓ 

$\theta = 60$ or 300 ✓

Q7 a) RHS $= \frac{1}{x+3} + \frac{2}{x-3}$

$= \frac{x-3+2(x+3)}{(x+3)(x-3)}$

$= \frac{3x+3}{x^2-9}$ ✓

$=$ LHS

$\int \frac{3x+3}{x^2-9}\,dx = \int \frac{1}{x+3} + \frac{2}{x-3}\,dx$

$= \ln(x+3) + 2\ln(x-3)$ ✓ $+ C$

$= \ln(x+3)(x-3)^2 + C$

(b) $V = \pi \int_1^4 y^2\,dx$

$y = x - \frac{1}{x}$

$y^2 = (x - \frac{1}{x})^2$

$= x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2}$ ✓

$= x^2 - 2 + \frac{1}{x^2}$

$V = \pi \int_1^4 x^2 - 2 + \frac{1}{x^2}\,dx$ ✓

$= \pi \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_1^4$ ✓

$= \pi \left[\left(\frac{4^3}{3} - 2\times 4 - \frac{1}{4}\right) - \left(\frac{1^3}{3} - 2\times 1 - \frac{1}{1}\right)\right]$

$= \frac{63\pi}{4}$ ✓

c) $\frac{d}{dx}(x^2-3)^5 = 5(x^2-3)^4(2x)$

$= 10x(x^2-3)^4$ ✓

$\frac{1}{10}\int 10x(x^2-3)\,dx = \frac{1}{10}(x^2-3)^5 + C$ ✓

d) AP $a=1$ $d=3$

$T_n = a+(n-1)d < 10000$ ✓

$1+(n-1)3 < 10000$

$n < \frac{9999}{3}$

$n < 3334$

$\therefore n = 3333$

$\therefore$ largest term $= T_{3333} = 9997$ ✓

e) $x = \frac{-b \pm \sqrt{\Delta}}{2a}$

2 distinct root if $\Delta > 0$ ✓

$\Delta = b^2 - 4ac > 0$

$k^2 - 4\times1\times4 > 0$

$k^2 - 16 > 0$

$(k+4)(k-4) > 0$

Distinct root $\Rightarrow$ $k < -4$ or $k > 4$ ✓

Q8 a) $y = 4^x$

$y' = \ln 4 \times 4^x$ ✓

b) $y = \log_{10} x = \frac{\log x}{\log 10}$ ✓

$y' = \frac{1}{\log 10} \times \frac{1}{x}$

$= \frac{1}{x\log 10}$ ✓

b) $\ln x^2 = \ln(6-x)$ ✓

$x^2 = 6 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$ ✓

But $x \neq -3$ $\therefore x = 2$

c) $0.\dot{2}\dot{5} = 0.25 + 0.0025 + 0.000025 + \ldots$ ✓

GP $a = 0.25$ $r = 0.001$

$S_\infty = \frac{a}{1-r} = \frac{0.25}{1-0.001}$

$= \frac{0.25}{0.99}$

$= \frac{25}{99}$ ✓

Q8 (cont)

d) $f'(x) = (2x+1)^5$

$f(x) = \dfrac{(2x+1)^6}{6\times 2} + C$ ✓

at (1,5)

$5 = \dfrac{(2\times 1+1)^6}{12} + C$

$C = -55\frac{3}{4}$ ✓ $\quad -55\frac{3}{4}$

$f(x) = \dfrac{(2x+1)^6}{12} - 55\frac{3}{4}$

e) $x^2+5 \equiv A(x-1)^2 + B(x-1) + C$

$x^2+5 \equiv A(x^2-2x+1) + Bx - B + C$

$x^2+5 \equiv Ax^2 - 2xA + Bx + A - B + C$ ✓

$A = 1$

$-2A + B = 0$

$B = 2$

$A - B + C = 5$

$1 - 2 + C = 5$

$C = 6$

$\therefore A = 1,\ B = 2,\ C = 6$ ✓

Q4

a) $3^{2x} - 4(3^x) + 3 = 0$ ✓

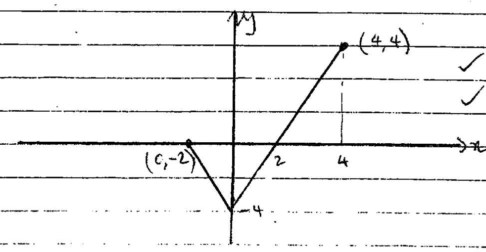
$(3^x - 3)(3^x - 1) = 0$

$3^x = 3$ or $3^x = 1$

$x = 1$ or $x = 0$ ✓

b) $y = |2x| - 4$

(i) y int; $x=0$	x int $y=0$	$x=4,\ y=4$
$y=-4$	$\lvert 2x\rvert = 4$	
	$x = \pm 2$	



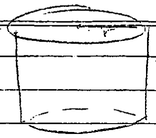
✓ 1mk intercepts
✓ 1mk domain

(ii) $\displaystyle\int_{-2}^{4} |2x| - 4\,dx = \int_{2}^{4} |2x| - 4\,dx + \int_{-2}^{2} |2x| - 4\,dx$

$= \frac{1}{2}\times 2\times 4 - \frac{1}{2}\times 4\times 4$ ✓

$= -4$ ✓ 1mk negative

Q9(c)



$V = 300$

$V = \pi r^2 h$

(i) $300 = \pi r^2 h$

$h = \dfrac{300}{\pi r^2}$ ✓

(ii) $A = 2\pi r^2 + 2\pi r h$ ✓

$A = 2\pi r^2 + 2\pi r\left(\dfrac{300}{\pi r^2}\right)$

$A = 2\pi r^2 + \dfrac{600}{r}$ ✓

$A = 2\pi r^2 + 600 r^{-1}$

(iii) $\dfrac{dA}{dr} = 4\pi r - 600 r^{-2}$

min when $\dfrac{dA}{dr} = 0$

$0 = 4\pi r - \dfrac{600}{r^2}$

$4\pi r^3 = 600$

$r = \sqrt[3]{\dfrac{600}{4\pi}} = 3.627\ldots$ ✓

r	3	3.62	4
$\frac{dA}{dr}$	−28	0	12.76
	\	—	/

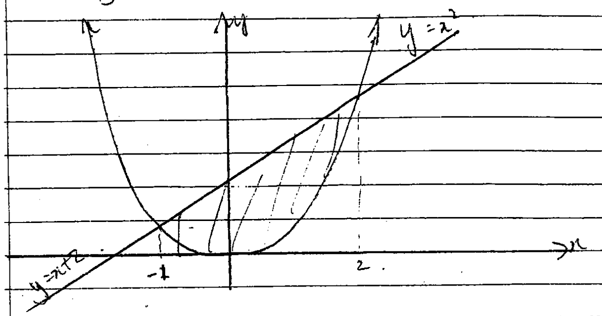
✓ min when $r = 3.627$

$A = 2\pi \times 3.62^2 + \dfrac{600}{3.627}$

$A = 218.85$ units^2 ✓

Q10.

a) $y = x^2$ — ① $y = x + 2$ — ②



① = ② $x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2,\ x = -1$ ✓

$\displaystyle\int_{-1}^{2} (x+2) - x^2 \, dx =$

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right]_{-1}^{2}$

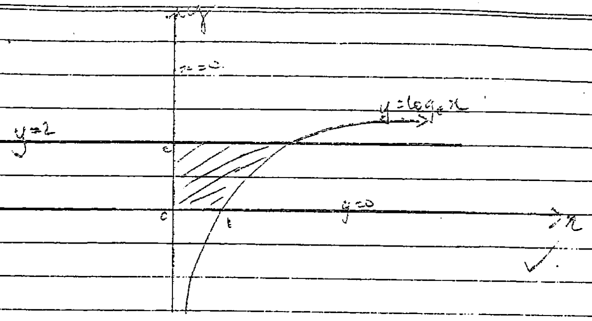
$= \left[\left(\dfrac{2^2}{2} + 2\times 2 - \dfrac{2^3}{3}\right) - \left(\dfrac{(-1)^2}{2} + 2(-1) - \dfrac{(-1)^3}{3}\right)\right]$

$= \dfrac{10}{3} - \left(-\dfrac{7}{6}\right)$

$= \dfrac{9}{2}$ units^2 ✓

Q10

b) (i)



b) ii) $V = \pi \int x^2 \, dy$

$y = \log_e x$

$x = e^y$

$x^2 = e^{2y}$ ✓

$V = \pi \int_0^2 e^{2y} \, dy$ ✓

$V = \pi \left[\frac{1}{2} e^{2y} \right]_0^2$

$V = \frac{\pi}{2} \left[e^4 - e^0 \right] = \frac{\pi}{2} (e^4 - 1)$

$V = \frac{\pi e^4 - \pi}{2}$ ✓

Q10c) $60 + 64 + 68 + \ldots \ldots 296$

(i) AP: $a = 60$ $d = 4$ $T_n = 296$ ✓

$= 296 = 60 + (n-1)4$

$n = 60$ ✓

∴ There are 60 lengths.

(ii) $S_n = \frac{n}{2}(a + l)$ ✓

$S_{60} = \frac{60}{2}(60 + 296)$

$S_{60} = 10680$ cm ✓

∴ 10680cm of timber needed.

(iii) 106.80 m of timber

0.1 m wide

$A = 10.68 m^2$ ✓✓

(if given $106800 cm^2$ 1mk only ?)