STUDENT NUMBER: ________________________________

TEACHER'S NAME: ________________________________

BAULKHAM HILLS HIGH SCHOOL

Year 12

MATHEMATICS ADVANCED ASSESSMENT

HALF-YEARLY

March 2010

Time allowed – 3 hours + 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Start each question on a new page.
- Show all relevant working.
- Use black or blue pen.
- **NO** liquid paper is to be used.
- Approved Maths aids and calculators may be used.

QUESTION 1 [12 marks] **Marks**

(a) Evaluate, correct to three significant figures **2**

$$\frac{3.5^2}{1.8^2 - \sqrt{145}}$$

(b) Find integers a and $b$such **2**
$\left(2\sqrt{5} - 1\right)^2 = a\sqrt{5} + b.$

(c) Solve $\frac{2x-1}{3} - \frac{1-3x}{5} = 2$ **2**

(d) Find the primitive function of $3x + \sin 3x$ **3**

(e) Find the value of x for which
$|2x - 1| > 5$ **2**

(f) Factorise $9x^2 - 16$ **1**

QUESTION 2 [12 marks]

(a) Differentiate
(i) $x^2 . \cos x$ **2**

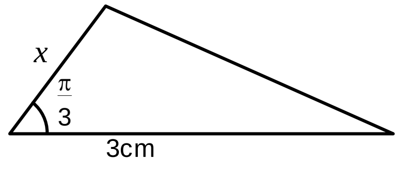
(ii) $\frac{4-x}{\sin x}$ **2**

(b) Solve $2\cos x + 1 = 0$ for $0 \leq x \leq 2\pi$ **2**

(c) Evaluate **2**

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3x \, dx$$

(d)



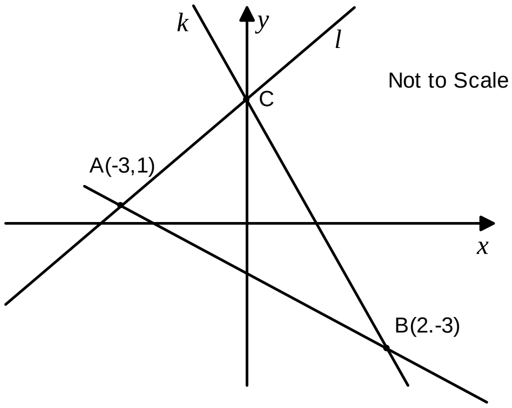
Not to Scale

If the area of this triangle is $6cm^2$, find the exact value of x. **3**

(e) Sketch the curve of $y = \tan x$ for $0 \leq x \leq \pi$ **1**

QUESTION 3 [12 marks] **Marks**

(a) Given points $A(-3,1)$ and $B(2,-3)$.
Point A lies on the line $l: 4x - 3y + 15 = 0$ and
Point B lies on the line k given by the equation $4x + y - 5 = 0$.



(i) Show that the point C, the point of intersection of the lines l and k, must lie on the $y - axis$. **2**

(ii) Find the gradient of the line AB **1**

(iii) Find the equation of AB **2**

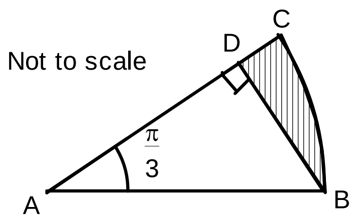
(iv) Find the perpendicular distance from point C to the line AB **2**

(v) Find the area of ΔABC. **2**

(b) Find the equation of the tangent to the curve $y = (x-1)(x+5)$ at the point where $x = 0$ **3**

QUESTION 4 [12 marks]

(a)



ABC is a sector with $\angle BAC = \frac{\pi}{3}$ and $AC = AB = 9cm$

(i) Calculate the area of sector ABC **1**

(ii) Calculate the area of the shaded region **3**

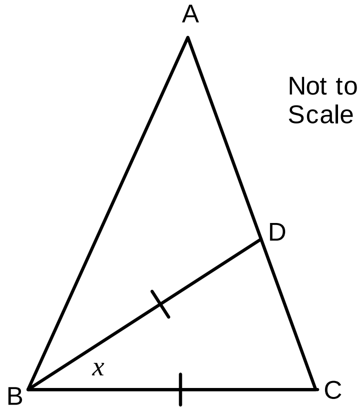
(b) Find the equation of the parabola with vertex $(1,4)$ and focus $(1,-2)$. **3**

(c) The tangent to the curve $y = \cos x$ at point P has a slope of $\frac{1}{2}$ for $0 \leq x \leq \frac{3\pi}{2}$
Find the coordinates of the point P. **3**

(d) Given $y = x^4 - x$. Is the function even or odd or neither? Justify your answer. **2**

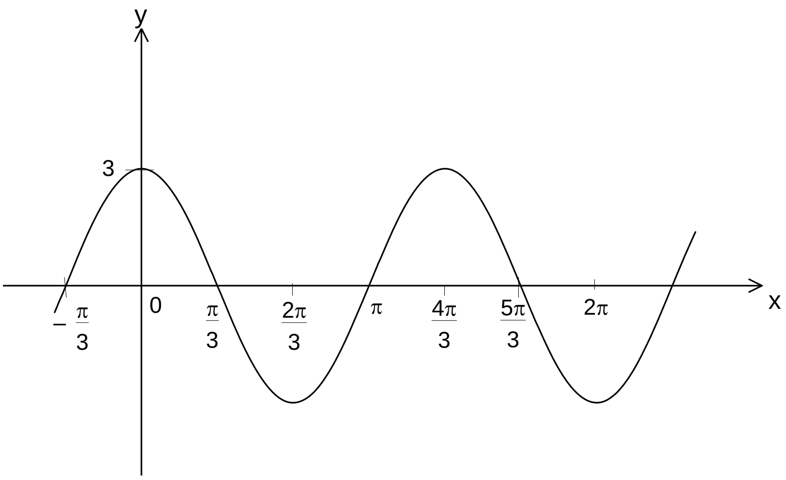
QUESTION 5 [12 marks] **Marks**

(a)

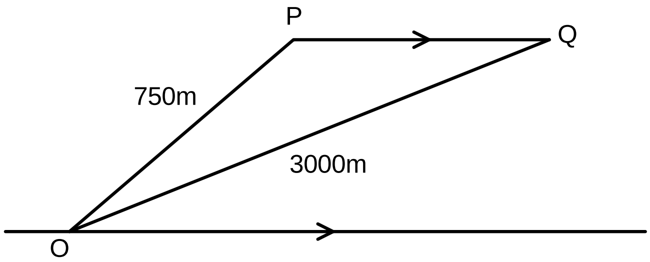


ΔABC is isosceles where $AB = AC$. **3**
D lies on AC such that $\angle ABD = 3\angle DBC$ and also $BD = BC$.
Find the value of x, giving reasons.

(b) The graph below can be represented by an equation in the form $y = a\cos nx$. **2**
Find the values of a and n.



(c)



An observer is standing at point O and sees a plane at P $750m$ from O. **4**
8 seconds later the plane is sighted at Q, $3000m$ from O.
The angles of elevation of P and Q from O are $73°$ and $7°$ respectively.
Find the speed of the plane.

(d) **(i)** Differentiate $y = (3x^2 - 7)^{10}$ **1**

(ii) Hence find $\int x(3x^2 - 7)^9 \, dx$ **2**

QUESTION 6 [12 marks] **Marks**

(a) **(i)** Graph **2**

$$y = \begin{cases} -\dfrac{3}{x-1} & for\ x < 0 \\ x^2 + 3 & for\ x \geq 0 \end{cases}$$

(ii) Find $f(-2) + f(2)$ **1**

(iii) Find $f(a^2)$ **1**

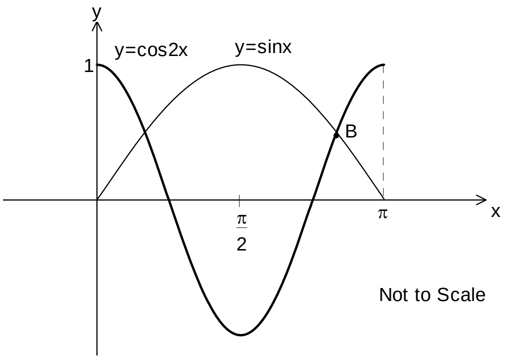
(b) Find the values of k for which the quadratic equation $2x^2 - kx + k = 0$ has real roots **3**

(c) Use Simpson's rule with five function values to approximate the volume generated by $y = \sin x$ rotated around $x - axis$ between $x = 0$ and $x = \frac{\pi}{2}$ **3**

(d) Find $\int 5 \cos x° \, dx$ **2**

QUESTION 7 [12 marks]

(a)



(i) Show that $y = \cos 2x$ and $y = \sin x$ meet at the point where $x = \frac{\pi}{6}$ **2**
Hence, write down the value of x at B.

(ii) Redraw the diagram in your booklet and shade the region bounded only by these two curves in between $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ **1**

(iii) Find the exact area of the shaded region shown in part **(ii)** **3**

(b) The point $P(x, y)$ moves so it's distance from the point $A(3,9)$ is always twice the distance from the point $B(6,6)$

(i) Find the locus of the point P **2**

(ii) Show that the locus in part (i) is a circle. **2**
State its centre and radius.

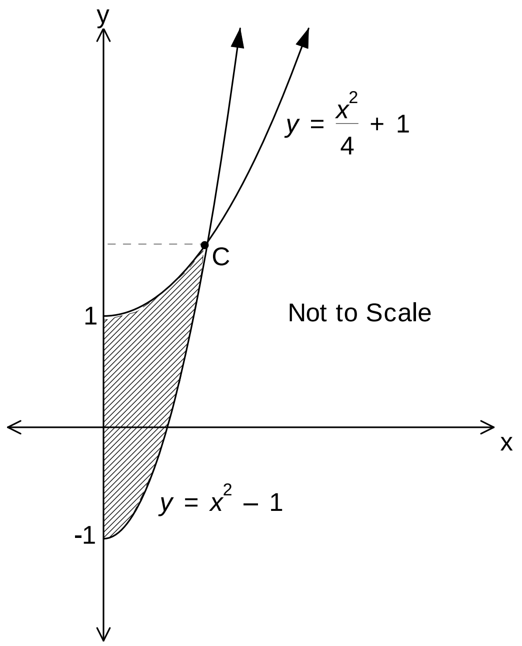
(c) Evaluate **2**

$$\sum_{x=2}^{10} (3x - 5)$$

QUESTION 8 [12 marks] **Marks**

(a) Find the area bounded by $x = y^2 - 2y - 3$ and the $y - axis$ **3**

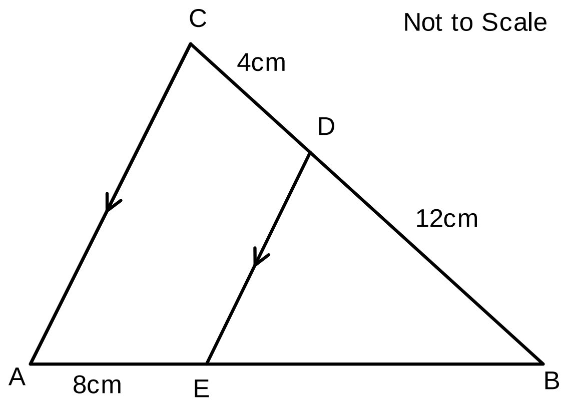
(b)



(i) Find the coordinates of C **2**

(ii) Find the exact volume when the shaded area is rotated around the $y - axis$. **4**

(c)



Given, $AC//ED$, $CD = 4cm, DB = 12cm$ and $AE = 8cm$

(i) Prove that $\Delta CBA///\Delta DBE$ **2**

(ii) Hence or otherwise, calculate the length of EB **1**

QUESTION 9 [12 marks] **Marks**

(a) **(i)** Prove that $\sec^2\theta - 2\tan\theta = (\tan\theta - 1)^2$ **2**

(ii) Hence or otherwise solve
$\sec^2\theta - 2\tan\theta = 0$ for $0 \le \theta \le 2\pi$ **2**

(b) Consider the geometric series

$$3 - 6x + 12x^2 - 24x^3 + \ldots$$

(i) For what values of x does this series have a limiting sum? **2**

(ii) If the limiting sum of the series is 2.5, find the value of x **2**

(c) The parabola is given by equation $2y = x^2 - 8x$
Find:

(i) The coordinates of the vertex **1**

(ii) The focal length **1**

(iii) The focus **1**

(iv) The equation of the directrix **1**

QUESTION 10 [12 marks] **Marks**

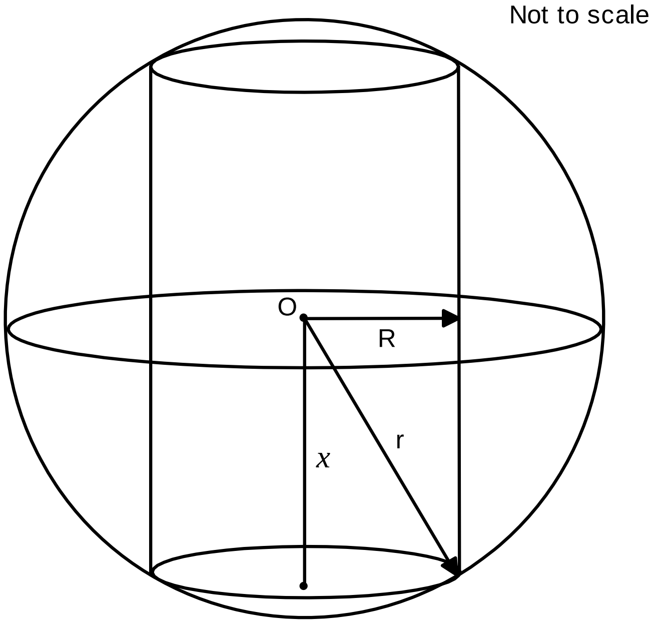
(a) Given that $f(x) = x^2\sqrt{10 - x}$

(i) Show that $f'(x) = \frac{5x(8-x)}{2\sqrt{10-x}}$ **2**

(ii) State the domain of the function **1**

(iii) Find all the stationary points on the curve $y = x^2\sqrt{10 - x}$ and determine their nature. **3**

(b) A cylinder is to be made to fit inside a sphere of radius $r\ cm$, as shown



(c)

Let $R =$ radius of cylinder and x be the distance of the base of the cylinder from the centre of the sphere as shown.

(i) Find an expression for the radius of the cylinder (R) in terms of r and x **1**

(ii) Show that the volume, V, of the cylinder is given by $V = 2\pi x(r^2 - x^2)$ **2**

(iii) Find in terms of r, the maximum volume of the cylinder. Leave your answer in exact form. **3**

Yr. 12 Half-yearly ass. task 2010 Total (120)

Question 1

a) $-1.39179\ldots = -1.39$ ✓

b) $(2\sqrt{5}-1)^2 = a\sqrt{5}+b$

$4\times5 - 2\times2\sqrt{5}+1 = a\sqrt{5}+b$

$20+1 - 4\sqrt{5} = b + a\sqrt{5}$

$b = 21$ ✓ $a = -4$ ✓

c) $\frac{2x-1}{3} - \frac{1-3x}{5} = 2$

$\frac{10x-5-3+9x}{15} = 2$ ✓

$19x = 30+8$

$\therefore x = 2$ ✓

d) $\int 3x + \sin 3x\,dx$

$= \frac{3x^2}{2} - \frac{1}{3}\cos 3x + C$ ✓ ✓ ✓

e) $|2x-1| > 5$

$2x-1>5$ or $2x-1<-5$

$x>3$ ✓ $x<-2$

$\therefore x<-2,\ x>3$ ✓

f) $(3x-4)(3x+4)$ ✓

Question 2 O.K.

a) (i) $\frac{d}{dx}(x^2\cdot\cos x)$

$= 2x\cdot\cos x + x^2\cdot(-\sin x)$ ✓ ✓

(ii) $\frac{d}{dx}\left(\frac{4-x}{\sin x}\right) = \frac{-1\cdot\sin x-(4-x)\cdot\cos x}{\sin^2 x}$

$= \frac{-\sin x - 4\cos x + x\cdot\cos x}{\sin^2 x}$ ✓ ✓

b) $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

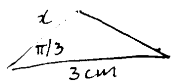
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ ✓ ✓

c) $\int_{\pi/12}^{\pi/9} \sec^2 3x\,dx = \left[\frac{1}{3}\tan 3x\right]_{\pi/12}^{\pi/9}$ ✓

$= \frac{1}{3}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{4}\right] =$

$= \frac{1}{3}[\sqrt{3}-1]$ ✓ or $0.244\ldots$

d)

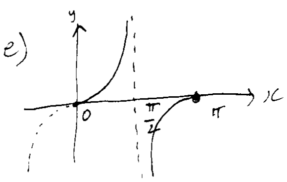


$A = 6 = \frac{1}{2}x\times3\times\sin\frac{\pi}{3}$ ✓

$12 = 3x\cdot\frac{\sqrt{3}}{2}$

$x = \frac{24}{3\sqrt{3}} = \frac{8}{\sqrt{3}}$ ✓

e) ✓



Question 3.

a) $A(-3,1)$ $B(2,-3)$

$\ell: 4x-3y+15=0$ $k: 4x+y-5=0$

i) $C\begin{cases}4x-3y+15=0\\4x+y-5=0\end{cases}$ $\therefore 4y-20=0$ ✓ $y=5$

$\therefore y=5$ $4x-3y+15=0$

$4x-15+15=0$ ✓

$\therefore (x=0)$

$\therefore C(0,5)$ $\therefore$ lies on y-axis

ii) $m_{AB} = \frac{1+3}{-3-2} = \frac{-4}{5}$ ✓

iii) AB: $y-1 = -\frac{4}{5}(x+3)$ ✓

$y = -\frac{4}{5}x - \frac{7}{5}$ ✓

or $4x+5y+7=0$

iv) $C(0,5)$ $AB: 4x+5y+7=0$

$\therefore d = \frac{|4\times0+5\times5+7|}{\sqrt{4^2+5^2}} = \frac{32}{\sqrt{41}}$ ✓ ✓

v) Area $= \frac{1}{2}\times AB\times h$ $AB=\sqrt{41}$

$= \frac{1}{2}\times\sqrt{41}\,\frac{32}{\sqrt{41}}$ ✓

$= 16$ (units²) ✓

b) $y = (x-1)(x+5) = x^2+4x-5$

$\therefore \frac{dy}{dx} = 2x+4$ $\therefore m = 2\times0+4 = 4$ ✓

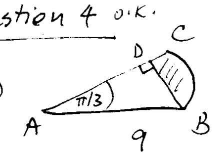
at $x=0$ $y=-5$ ✓

$\therefore$ tangent: $y+5 = 4(x-0)$ ✓

$y = 4x-5$

Question 4 O.K.

a)



i) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}\times$

$\therefore A = \frac{27\pi}{2}$ ✓

ii) $\Delta ADB = \frac{1}{2}AD\cdot AB\cdot$

$\cos\frac{\pi}{3} = \frac{AD}{9} = \frac{1}{2}$ $\therefore AD$

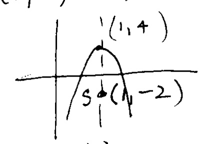
$\therefore$ Area $= \frac{1}{2}\times\frac{9}{2}\times9\cdot\frac{\sqrt{3}}{2}$

Area$_\Delta$ $= \frac{81\sqrt{3}}{8}$ ✓

$\therefore$ Area$_{\text{shaded}}$ $= \frac{27\pi}{2} - \frac{8\ldots}{\ldots}$

or $= \frac{108\pi - 81\sqrt{3}}{8} =$

b) $(1,4)$ $(1,-2)$



a

$(x-m)^2 = -4a(y\ldots$

$(x-1)^2 = -24(y\ldots$ ✓

(c) $y = \cos x$

$\frac{dy}{dx} = -\sin x$

$P(x\ldots$

$m = \ldots$

$\frac{1}{2} = -\sin x$ ✓

$\therefore x = \frac{7\pi}{6}$ ✓ (onl…

$\therefore P\left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{2}\right)$ ✓

Question 4d)

$y = x^4 - x = f(x)$

$f(-x) = (-x)^4 - (-x) = x^4 + x$

$\neq f(x)$ ∴ not even

$f(-x) \neq -f(x)$

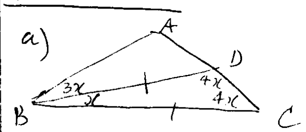
since $f(-x) = x^4 + x = -(x^4 - x)$

$\neq -f(x)$ ∴ not odd

∴ neither

Question 5 o.k.

a)



$\angle ABD = 3x$ ∴ $\angle ABC = 4x$ ✓

∴ $\angle ACB = 4x$ (base angles of an isosc. $\triangle ABC$ are =)

∴ $\angle BDC = 4x$ since $BD = BC$ ✓

∴ (opp. equal sides are equal ∠'s)

∴ $x + 4x + 4x = 180°$ (angle sum of $\triangle$)

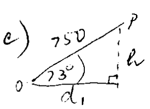
∴ $x = 20°$ ✓

b) $y = a\cos nx$

$a = 3$ (amplitude) ✓

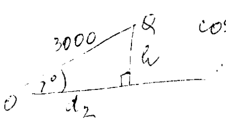
$T = \frac{4\pi}{3} = \frac{2\pi}{n}$ ∴ $n = \frac{3}{2}$ ✓

c)



$\cos 73° = \frac{d_1}{750}$ ✓

∴ $d_1 = 219.2787785$



$\cos 7° = \frac{d_2}{3000}$ ✓

∴ $d_2 = 2977.638455$

∴ $d_{TRAVELLED} = d_2 - d_1 = 2758.3597$ ✓

∴ speed $= \frac{2758.359676\,m}{8\,sec.}$ ✓

$= 344.7949596\ m/s$

d) i) $y = (3x^2 - 7)^{10}$

$\frac{dy}{dx} = 10(3x^2-7)^9 \times 6x$

$= 60x(3x^2-7)^9$ ✓

ii) ∴ $y = \int 60x(3x^2-7)^9 dx$ ✓

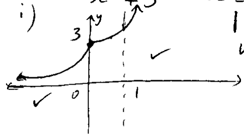
∴ $\frac{y}{60} = \int x(3x^2-7)^9 dx$

∴ $\int x(3x^2-7)^9 dx = \frac{1}{60}(3x^2-7)^{10} + c$ ✓

Question 6 o.k.

$y = \begin{cases} -\frac{3}{x-1} & x < 0 \\ x^2 + 3 & x \geq 0 \end{cases}$

i)



1 for each half with intercept ✓

ii) $f(-2) + f(2) = \frac{-3}{-2-1} + 2^2 + 3$

$= 8$ ✓

iii) $f(a^2) = (a^2)^2 + 3 = a^4 + 3$ ✓

Question 6

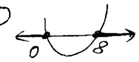
b) $2x^2 - kx + k = 0$

$\Delta = b^2 - 4ac \geq 0$

$(-k)^2 - 4 \times 2 \times k \geq 0$ ✓

$k^2 - 8k \geq 0$

$k(k-8) \geq 0$ ✓



∴ $k \leq 0$, $k \geq 8$ ✓

c) $y = \sin x$

$V = \pi \int_0^{\pi/2} \sin^2 x\, dx$

x	0	$\pi/8$	$\pi/4$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
$\sin^2 x$	0	0.1464	$\frac{1}{2}$		1

✓

$V \doteqdot \pi \times \frac{\frac{\pi}{8}}{3}\left(0 + 1 + 4\left(\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8}\right) + 2 \times \sin^2\frac{\pi}{4}\right) = \frac{\pi^2}{24} \times 6 = \frac{\pi^2}{4}$

1 mark off each error

1 mark for correct area answer → 1

or $V \doteqdot 2.4674$

d) $\int 5\cos x°\, dx = \int 5\cos\frac{x\pi}{180}\, dx$ ✓

$= 5 \times \frac{180}{\pi} \cdot \sin\frac{x\pi}{180} + C$ ✓

or $5 \times \frac{180}{\pi} \cdot \sin x° + C$

Question 7 o.k.

a) $y = \cos 2x$ and $y = \sin$

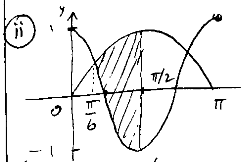
(i) $x = \frac{\pi}{6}$ ∴ $y = \cos\frac{\pi}{3} = \frac{1}{2}$ $y = \sin$

equal

∴ at $x = \frac{\pi}{6}$ pt. of intersection

∴ at B $x = \pi - \frac{\pi}{6} = \frac{5}{}$

(ii)



✓

(iii) $A = \int_{\frac{\pi}{6}}^{\pi/2} \sin x - \cos 2x\, d$

$= \left[-\cos x - \frac{1}{2}\sin 2x\right]_{\pi/6}^{\pi/}$

$= -\cos\frac{\pi}{2} - \frac{1}{2}\sin\pi + \cos\frac{\pi}{6}$

$= 0 - 0 + \frac{\sqrt{3}}{2} + \frac{1}{2} \times$

$= \frac{3}{4}\sqrt{3}$ ✓

b) $PA = 2PB$ $A(3,$ $B(6$

$\sqrt{(x-3)^2 + (y-9)^2} = 2\sqrt{(x-6)^2 + (y}$ ✓

$x^2 - 6x + 9 + y^2 - 18y + 81 = 4[x^2 - 12$

$9 + 81 - 4 \times 36 - 4 \times 36 = 3x^2 - 42x +$

$-\frac{198}{3} = x^2 - 14x + y^2 - 10$

$8 = (x-7)^2 + (y-5)$

ii) $8 = (x-7)^2 + (y-5)$

∴ centre $(7, 5)$

radius $r = \sqrt{8}$

Question 7 c)

$\sum_{x=2}^{10} 3x-5 = 1+4+7+\dots+25$ ✓

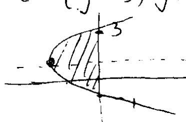
$= \frac{n}{2}(a+l)$ A.P $n=9$, $a=1$, $l=25$

$= \frac{9}{2}(1+25) = 117$ ✓

Q.8 O.K.

a) $x = y^2 - 2y - 3$ y-axis

$x = (y-3)(y+1)$



$\therefore A = \left| \int_{-1}^{3} y^2 - 2y - 3\,dy \right|$ ✓

$= \left| \left[\frac{y^3}{3} - y^2 - 3y \right]_{-1}^{3} \right| = \frac{32}{3}$ ✓ ✓

b) (i) $C \begin{cases} y = \frac{x^2}{4} + 1 \\ y = x^2 - 1 \end{cases}$ $\frac{x^2}{4} + 1 = x^2 - 1$

$\therefore 8 = 3x^2$

$x^2 = \frac{8}{3}$

$\therefore x = \pm\sqrt{\frac{8}{3}}$

$y = x^2 - 1 = \frac{8}{3} - 1$ $\therefore C\left(\sqrt{\frac{8}{3}}, \frac{5}{3}\right)$ ✓ ✓

(ii) $V_1 = \pi\int_{-1}^{5/3} y+1\,dy$ ✓ $y = x^2 - 1$, $y+1 = x^2$

$V_2 = \pi\int_{1}^{5/3} 4y-4\,dy$ $y = \frac{x^2}{4} + 1$, $4y - 4 = x^2$

$\therefore V = V_1 - V_2$

$\therefore V = \pi\int_{-1}^{5/3} y+1\,dy - \pi\int_{1}^{5/3} 4y-4\,dy$ ✓

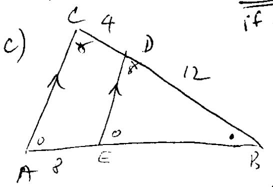
$= \pi\left[\frac{y^2}{2} + y\right]_{-1}^{5/3} - \pi\left[2y^2 - 4y\right]_{1}^{5/3}$ ✓

$= \pi\left[\frac{55}{18} + \frac{1}{2}\right] - \pi\left[-\frac{10}{9} - (-2)\right]$

$= \frac{32}{9}\pi - \frac{8}{9}\pi = \frac{8}{3}\pi$ ✓ ✓

if not splitted 3/4

c)



(i) $\angle CBA = \angle DBA$ (in common)

$\angle CA = \angle BDE$ ✓ corresp. ∠s

$\angle DEB = \angle CAB$ on parallel lines are equal

$\therefore \triangle CBA \,|||\, \triangle DBE$ ✓ (all matching angles are =)

(ii) $\frac{EB}{AB} = \frac{DB}{CB}$ all matching sides are in the same ratio

$\therefore \frac{EB}{8+EB} = \frac{12}{12+4}$

$16 \times EB = 12(8+EB)$

$4EB = 96$

$\therefore EB = 24$ ✓

Question 9 O.K.

a) i) $RHS = (\tan\theta - 1)^2$ ✓

$= \tan^2\theta - 2\tan\theta + 1$

$= \tan^2\theta + 1 - 2\tan\theta$ ✓

$= \sec^2\theta - 2\tan\theta = LHS$ proven

ii) $\sec^2\theta - 2\tan\theta = 0$ $0 \le \theta \le 2\pi$

$\therefore (\tan\theta - 1)^2 = 0$ ✓

$\tan\theta = 1$ ✓ $\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$ ✓

b) $3 - 6x + 12x^2 - 24x^3 + \dots$

i) $r = \frac{-6x}{3} = -2x$ ✓

lim. sum exists $|r| < 1$ $\therefore |-2x| < 1$

✓ $\begin{cases} |x| < \frac{1}{2} \\ -\frac{1}{2} < x < \frac{1}{2} \end{cases}$

ii) $S_\infty = \frac{a}{1-r} = \frac{3}{1-(-2x)} = 2.5$ ✓

$\therefore x = \frac{1}{10}$ ✓

c) $2y = x^2 - 8x$

i) $2y + 16 = x^2 - 8x + 4^2$

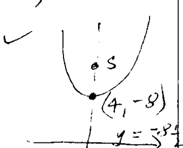
$2(y+8) = (x-4)^2$

$\therefore$ vertex $(4, -8)$ ✓

ii) $4a = 2$ $\therefore a = \frac{1}{2}$ ✓

iii) $S(4, -7\frac{1}{2})$ ✓

iv) $y = -8\frac{1}{2}$ ✓



Question 10 O.K.

a) $f(x) = x^2\sqrt{10-x} = x^2($

i) $f'(x) = 2x \cdot (10-x)^{\frac{1}{2}} + x$ ✓

$\therefore f'(x) = \frac{2x(10-x)^{\frac{1}{2}}}{1} - \frac{}{2(}$

$= \frac{2\cdot 2x(10-x) - x^2}{2\sqrt{10-x}}$ ✓

$\therefore f'(x) = \frac{5x(8-x)}{2\sqrt{10-x}}$

ii) $10 - x \ge 0$ $\therefore x \le 10$

iii) $f'(x) = 0 = \frac{5x(8-x)}{2\sqrt{10-x}}$

$\therefore 0 = 5x(8-x)$

$x = 0$ or $x = 8$

x	-1	0	1
$f'(x)$	$\frac{-45}{2\sqrt{11}}$	0	$\frac{35}{2\sqrt{9}}$
values	(−)		(+)

✓

x		1	
$f'(x)$		$\frac{35}{6}$	
		(+)	

$\therefore (0, 0)$ ✓ Min. turning

$(8, 64$ Max

Question 10 cont.

b) i) $R^2 = r^2 - x^2$

$\therefore R = + \sqrt{r^2 - x^2}$ ✓ (+ only)

ii) $V = \pi \times R^2 \times h$ ✓ $h = 2x$

$\therefore V = \pi \times (r^2 - x^2) \times 2x$ ✓

$\therefore V = 2\pi x (r^2 - x^2)$

iii) $\frac{dV}{dx} = 0$

$V = 2\pi r^2 x - 2\pi x^3$

$\therefore \frac{dV}{dx} = 2\pi r^2 - 6\pi x^2$ ✓

$0 = 2\pi r^2 - 6\pi x^2$

$x^2 = \frac{2\pi r^2}{6\pi} = \frac{r^2}{3} \therefore x = \frac{+r}{\sqrt{3}}$ ✓

$\therefore \frac{d^2V}{dx^2} = 0 - 12\pi x$

at $x = \frac{r}{\sqrt{3}}$ $\therefore \frac{d^2V}{dx^2} = -12\pi \times \frac{r}{\sqrt{3}} < 0$

$\therefore$ at $x = \frac{r}{\sqrt{3}}$ V is Max.

$\therefore V_{MAX} = 2\pi x (r^2 - x^2)$

$\therefore V_{MAX} = \frac{2\pi r}{\sqrt{3}} \left(r^2 - \frac{r^2}{3}\right)$ ✓

(or) $V_{MAX} = \frac{2\pi r}{\sqrt{3}} \left(\frac{2r^2}{3}\right)$

$= \frac{4\pi r^3}{3\sqrt{3}}$