2009
SEMESTER 1
EXAMINATION

Student Number

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

Question 1 (12 marks) Use the Question 1 writing booklet.

Marks

(a) Find the value of $\dfrac{3.6\times 7.4}{\sqrt{5.6+2.5}}$ correct to three significant figures. 1

(b) Find the integers a and b such that $\dfrac{\sqrt{5}}{2+\sqrt{5}}=a+b\sqrt{5}$. 2

(c) Solve $|2x+6|=3x-1$. 3

(d) Simplify $\dfrac{3x-x^2}{9-x^2}$. 2

(e) Express $0.\dot{1}\dot{8}$ as a fraction in simplest form. 2

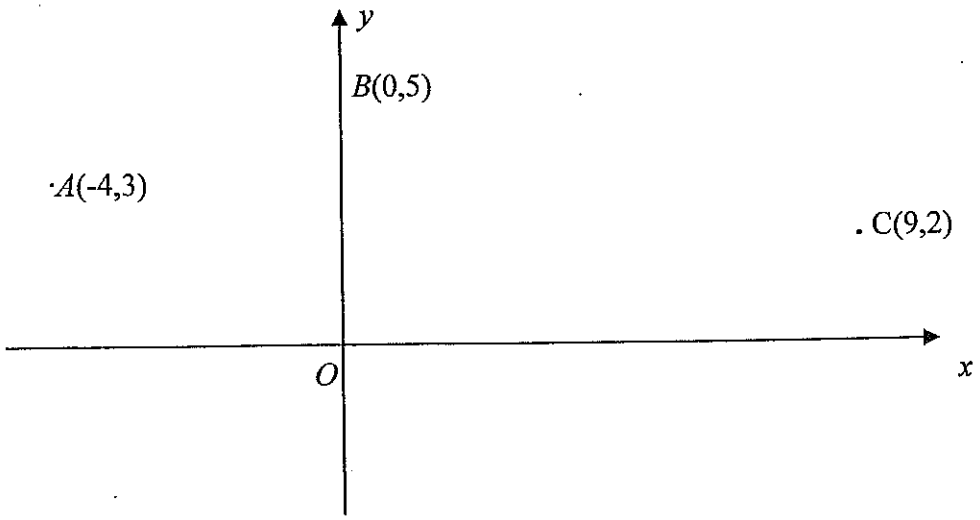
(f) The price of a skateboard has increased by 6.5% to \$89. Find the price before the increase. 2

End of Question 1

Question 2 (12 marks) Use the Question 2 writing booklet.

Marks

(a)



The diagram shows the origin O and the coordinates of the points $A(-4,3)$, $B(0,5)$ and $C(9,2)$.
Copy or trace this diagram into your writing booklet.

(i) Find the exact length of the interval BC. **2**

(ii) Show that the equation of the line k, drawn through A and parallel to BC is $x+3y-5=0$. ***Clearly indicate the line k on your diagram.*** **3**

(iii) Find the coordinates of D, the point where the line k meets the x axis. **1**

(iv) Prove that $ABCD$ is a parallelogram. **2**

(v) Find the perpendicular distance from the point B to the line k. **2**

(vi) Hence, or otherwise, find the area of $ABCD$. **2**

End of Question 2

Question 3 (12 marks) Use the Question 3 writing booklet.

Marks

(a) The quadratic equation $3x^2-5x+4=0$ has roots α and β.
Find:

(i) $\alpha+\beta$ — 1

(ii) $\alpha\beta$ — 1

(iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$ — 1

(b) Solve for real values of x: $4^x-4\left(2^x\right)-32=0$. — 3

(c) For the quadratic equation $x^2+(k+6)x-2k=0$, find:

(i) the discriminant in terms of k — 1

(ii) the values of k for which this equation has real roots. — 2

(d) Find the values of a, b and c given that — 3
$a\left(x^2-1\right)+b(x-1)^2+c(x+1)\equiv 2x^2-6$.

End of Question 3

Question 4 (12 marks) Use the Question 4 writing booklet.

Marks

(a) Differentiate the following with respect to x:

(i) $5x+\frac{1}{3x^2}$ — 2

(ii) $\left(4-3x^2\right)^6$ — 2

(b) Consider the curve $y=1+3x-x^3$ for $-2\le x\le 3$.

(i) Find the stationary points and determine their nature. — 3

(ii) Find any points of inflexion. — 2

(iii) Sketch the curve for $-2\le x\le 3$ labelling all critical points. — 2

(iv) Find the minimum value of y for $-2\le x\le 3$. — 1

End of Question 4

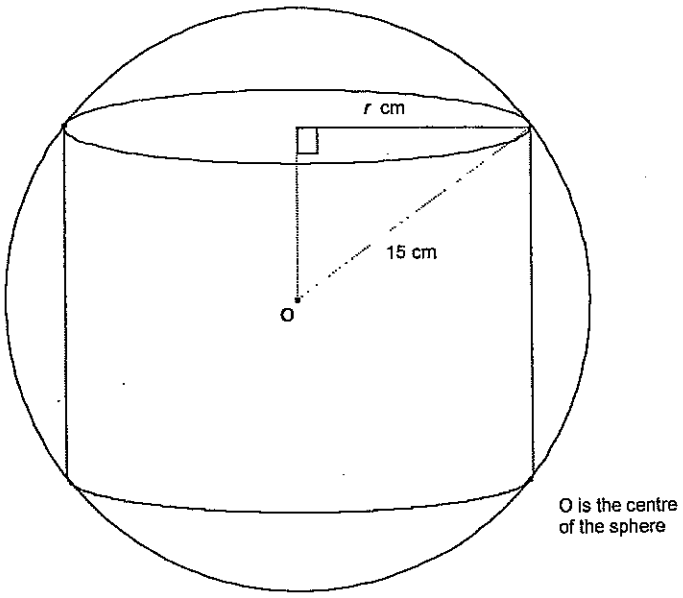
Question 5 (12 marks) Use the Question 5 writing booklet

Marks

(a) For all x in the domain $0 \le x \le 4$, a function $f(x)$ satisfies, $f(x) > 0$, $f'(x) > 0$ and $f''(x) < 0$. Sketch a possible graph of $y = f(x)$ in this domain. **3**

(b) Find the equation of the normal to the curve $y = 3x^2 - 4$ at the point where $x = 2$. **3**

(c) A cylindrical block of height h cm and base with radius r cm is cut from a solid sphere of radius 15 cm.



(i) Show that the volume of the cylinder is given by $V = 225\pi h - \frac{\pi}{4}h^3$. **2**

(ii) Find the height of the cylinder of maximum volume which can be cut from the sphere. **4**

End of Question 5

Question 6 (12 marks) Use the Question 6 writing booklet.

Marks

(a) Find:

(i) $\int(3x^5-4)\,dx$ 1

(ii) $\int x\sqrt{x}\,dx$ 2

(b) Evaluate $\int_0^2(2x-1)^3\,dx$. 3

(c) The curve of $y=f(x)$ has a gradient function $\frac{dy}{dx}=3x^2-2x+1$ and passes through the point (2,3). Find the equation of the curve. 2

(d) Show that the quadratic equation $mx^2+(m-4)x-4=0$ has real roots for all values of m. 2

(e) Find the value of k if $k>0$ and $\int_0^k(2x-1)\,dx=20$. 2

End of Question 6

Question 7 (12 marks) Use the Question 7 writing booklet

Marks

(a) Consider the function $f(x)=\dfrac{x}{x+1}$

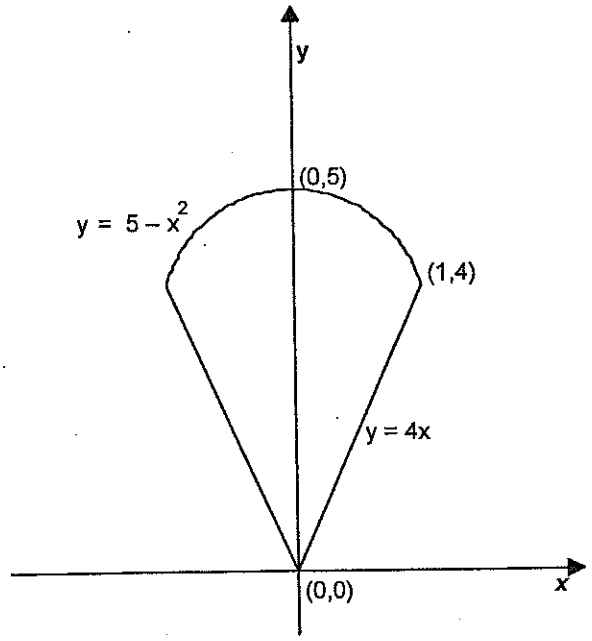
(i) Copy and complete the following table in your writing booklet. **1**

x	1	2	3	4	5
$f(x)$	$\frac{1}{2}$		$\frac{3}{4}$		

(ii) Using Simpson's Rule with five function values, evaluate: **2**

$$\int_1^5 f(x)\,dx.$$

(b) The diagram shows a cone and a paraboloid. It represents an ice cream cone which is completely full of ice cream and which has an additional scoop of ice cream on top. Units are in centimetres.



To calculate the volume of ice cream, the area bounded by the section of the line $y=4x$ between (0,0) and (1,4), the part of the parabola between (1,4) and (0,5) and the y-axis, was rotated about the y-axis.

(i) By integration, determine the total quantity of ice cream contained in the cone and the scoop on top. **3**

(ii) How many of these ice creams can be made from a one litre container of ice cream? (1 litre = 1000 cm^3) **1**

Question 7 continues on next page

(c) (i) By solving the equations simultaneously, show that the curves $y=\frac{1}{2}x^2$ and $y=2x$ intersect at the points (0,0) and (4,8). **2**

(ii) Sketch both curves on the same diagram and find the area of the region enclosed between them. **3**

End of Question 7

Question 8 (12 marks) — Use the Question 8 writing booklet.

Marks

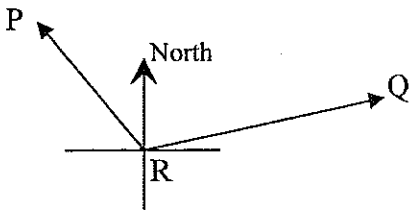
(a) Find the tenth term of the arithmetic series 2

$$4\frac{1}{2} + 3 + 1\frac{1}{2} + \ldots$$

(b) Show that: 2

$$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$$

(c) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330° at a speed of 180 km/h and Quentin flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.

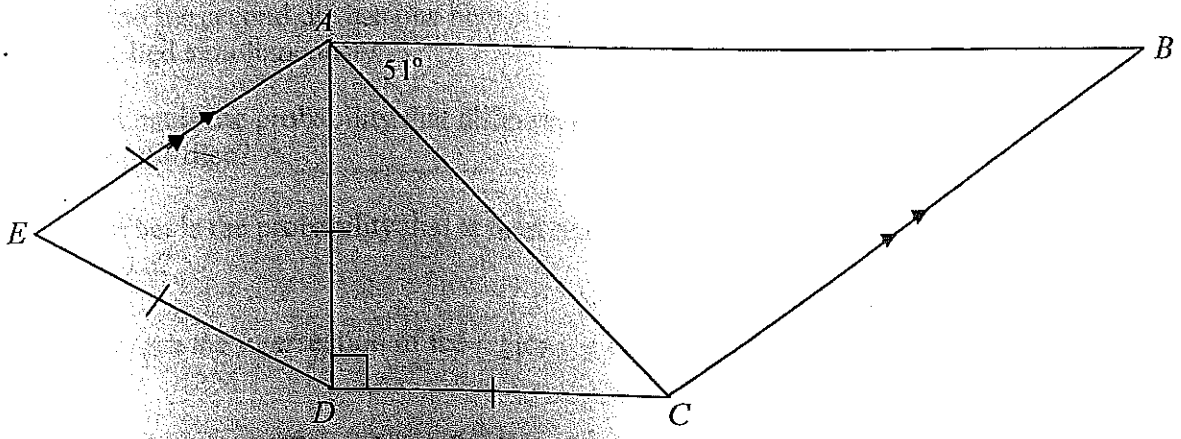


(i) Show that Peta and Quentin are 692 km (to the nearest km) apart after 2 hours? 2

(ii) What is the bearing of Quentin from Peta after 2 hours. (Answer to the nearest degree.) 2

(d) In the diagram below $AE = ED = AD = DC$, $\angle ADC = 90°$ and $AE \parallel BC$.

$\angle BAC = 51°$



(i) Find the size of $\angle EAB$. Give reasons for your answer. 3

(ii) Find the size of $\angle ABC$. Give reasons for your answer. 1

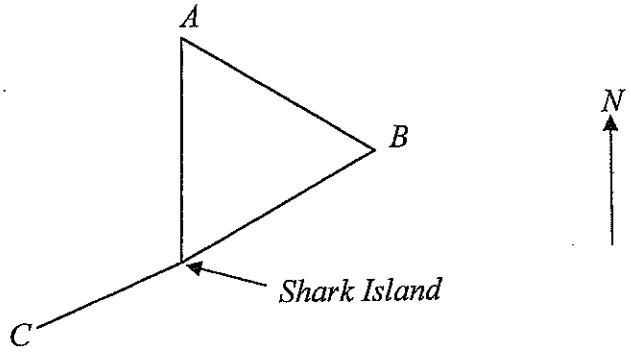
End of Question 8

Question 9 (12 marks) Use the Question 9 writing booklet.

Marks

(a) Three boats A, B and C are situated off Shark Island, as shown below.

DIAGRAM NOT TO SCALE



Boat A is due north of the island. Boat B is on a bearing of $060°$ from the island and Boat C is on a bearing of $240°$ from the island. Fish nets (of length 100 m) have been laid out between the island and each boat and also between boats A and B.

(i) Copy and complete the diagram showing all information. **1**

(ii) Calculate the triangular area that boats A and B have netted, to the nearest square metre. **2**

(iii) Calculate the length to the nearest metre of net needed between boats A and C. **2**

(b) Two dice are painted so that the first die has four blue and two red faces and the second die has one blue and five red faces. The two dice are rolled simultaneously.

(i) What is the probability that different colours show on the uppermost faces of the dice? **2**

(ii) What is the probability that at least one die shows blue on its uppermost face? **2**

(c) Graph the intersection of the regions $y < -x^2 + 3$ and $y \geq |x-3|$ **3**

End of Question 9

Question 10 (12 marks) Use the Question 10 writing booklet.

Marks

(a) Cans of fruit in a supermarket are stacked so that there are 3 cans in the top row, 5 in the next row, 7 in the next and so on.

If there are 20 rows in the display, find:

(i) the number of cans in the bottom row; **2**

(ii) the total number of cans in the display. **2**

(b) Evaluate $\sum_{n=5}^{14} 27-3n$. **2**

(c) Find the first and tenth terms of the geometric sequence where the second term is 12 and the fifth term is 324. **3**

(d) **(i)** For what values of x does the series $1+(x-2)+(x-2)^2+\ldots\ldots$ have a limiting sum? **2**

(ii) Find an expression for this sum? **1**

End of Question 10

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\qquad x>0$