2010

Semester 1 Exam

Student Number

HSC Mathematics

PER · ARDUA · AD · ALTIORA

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

Question 1 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Evaluate $\sqrt{\dfrac{5^2 - 11^2}{197 - 11^2}}$, correct to 3 significant figures. **2**

(b) Factorise $3x^2 - 48$ completely. **2**

(c) Solve $\dfrac{x+6}{4} - \dfrac{x-3}{5} = 3$. **2**

(d) Annabelle purchased a silver bracelet in Copenhagen marked at A $300, including 25% GST. If the GST was deducted from the price, how much did she pay for the bracelet? **2**

(e) Find a primitive of $4x^3 - 3$. **2**

(f) Solve this pair of simultaneous equations: **2**
$2x - y = 7$
$x + 2y = 1$

End of Question 1

Question 2 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) A(1,5), B(4,2), C(10,-4) and D(-2,-4) are four points on the number plane.

(i) Find the gradient of BC. **2**

(ii) Calculate the length of AC. **2**

(iii) Determine the equation of the line BC. **2**

(iv) Prove that A, B and C are collinear. **2**

(v) Calculate the perpendicular distance of D from the line AC. **2**

(iv) Calculate the area of ΔACD. **2**

End of Question 2

Question 3 (12 marks) Start a new sheet of writing paper. Marks

(a) James collects football cards. In his top drawer he has 5 Newcastle Knights, 5 St George-Illawarra and 2 Brisbane Broncos cards to swap. James pulled from his drawer 2 cards in succession while the lights were out.

Find the probability that:

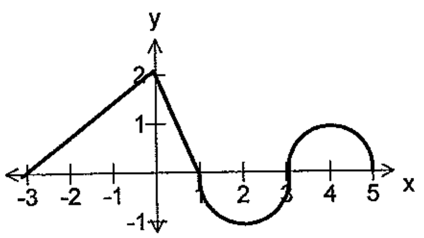
(i) The first card chosen was a Knight **1**

(ii) Both cards were Knights. **1**

(iii) Both cards were from the same club. **2**

(iv) At least one card was a Bronco. **2**

(b)



The figure illustrated represents the function y = h(x) and consists of two straight lines and two semi-circles of radius 1.

(i) Calculate the value of $\int_{-3}^{5} h(x)\,dx$ **2**

(ii) Find the area contained between $y = h(x)\,dx$ and the x axis. **2**

(c) Find the exact value of $\sin(-405^\circ)$ **2**

End of Question 3

Question 4 (12 marks) Start a new sheet of writing paper. Marks

(a) For the curve $x^3 - 6x^2 + 9x$:

(i) Find the coordinates of the points at which the curve intersects with the x-axis. **2**

(ii) Find the turning points, and determine their nature. **2**

(iii) Sketch a diagram of the graph **2**

(iv) Find the area of the region enclosed between the curve and the x axis **2**

(b) The roots of $2x^2 - 3x + 4 = 0$ are α and β. Find the value of

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(iv) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ **1**

End of Question 4

Question 5 (12 marks) Start a new sheet of writing paper. Marks

(a) For what value of k does $x^2 - kx + k + 3 = 0$ have equal roots? **2**

(b) Differentiate

(i) $(x^2 + 5x - 1)^8$ **2**

(ii) $\dfrac{x^2 - 3x + 1}{3x + 4}$ **2**

(c) Use Simpson's rule to approximate $\int_2^4 f(x)dx$ where the values of f(t) are given in the table (correct to 3 decimal places). **3**

t	2	2.5	3	3.5	4
f(t)	3.7	1.2	9.8	4.1	2.7

(d) A triangular field *ABC* has sides *AB* = 85m and *AC* = 50 m. *B* is on a bearing of 065° from *A* and *C* is on a bearing of 166° from *A*.

(i) Draw a diagram representing this information. **1**

(ii) Find the length of *BC*, correct to the nearest metre. **2**

End of Question 5

Question 6 (12 marks) Start a new sheet of writing paper. **Marks**

(a) The sum of 12 terms of an arithmetic series is 186 and the 20th term is 83. Find the sum of 40 terms. **2**

(b) Write an expression for the nth term of the following series: **2**

$$\frac{1}{4} + 1 + 4 + \ldots$$

(c) A curve has $\frac{dy}{dx} = 6x^2 + 12x - 5$. If the curve passes through the point (2,-3), find the equation of the curve. **2**

(d) Find all the values of x for which the curve $y = (x-3)^3$ is concave downwards. **2**

(e) Find the indefinite integral of

(i) $\int \frac{3}{x^2}\, dx$ **2**

(ii) $\int \sqrt{(5x+2)^5}\, dx$ **2**

End of Question 6

Question 7 (12 marks) Start a new sheet of writing paper. **Marks**

(a) Students studying at least one of the languages, French and Japanese, attend a meeting. Of the 35 students present 24 study French and 29 study Japanese.

(i) What is the probability that a randomly chosen student studies French? **1**

(ii) What is the probability that two randomly chosen students both only study French? **2**

(iii) What is the probability that a randomly chosen student studies both languages? **1**

(b) Find the sum to infinity of the geometric progression **2**

$$1 + (\sqrt{2} - 1) + (\sqrt{2} + 1)^2 + \ldots$$

Leaving your answer as a surd in rational form.

(c) A brand of rechargeable batteries provides power for 20 hours when first purchased fully charged. After its first recharge, it only provides power for a further 18 hours. After its second recharge, it only supplies power for 16.2 hours. Each subsequent recharging results in the battery having 90% of its previous power available.

(i) What is the power available after the third recharge? **2**

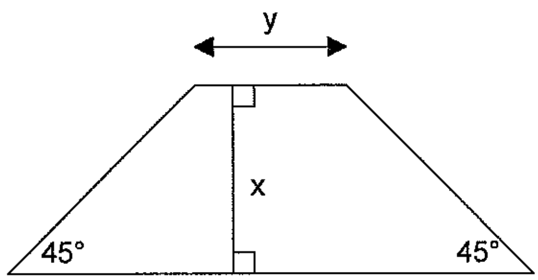
(ii) How many hours could you expect to get out of the battery? **2**

(iii) If the battery is thrown away when its charge level after recharging is less than one hour, how many times would it be recharged? **2**

End of Question 7

Question 8 (12 marks) Start a new sheet of writing paper. Marks

(a) An Isosceles trapezium has base angles of 45° and a perimeter of 24 meters.

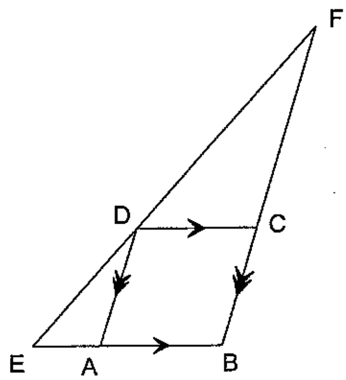


(i) Show that $y = 12 - x - \sqrt{2}\,x$ **2**

(ii) Obtain an expression for the area of the trapezium in terms of x. **2**

(iii) Find the dimensions of the trapezium of maximum area. **3**

(b) In the figure, ABCD is a parallelogram with BA produced to E and BC produced to F. EF passes through D. Also, AB=4cm, EA=4cm, BC=3cm and CF=5cm.



(i) Show that $\angle EAD = \angle DCF$ **2**

(ii) Prove that $\Delta DEA \mid\mid\mid \Delta CFD$ **2**

(iii) If DE = 4cm, find the length of DF. **1**

End of Question 8

Question 9 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) The number of unemployed people u at time t was studied over a period of time. At the start of this period, the number of unemployed was 800 000.

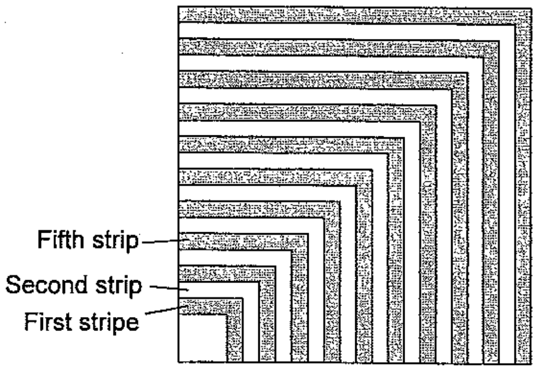
(i) Throughout the period, $\dfrac{du}{dt} < 0$

What does this say about the number of unemployed during the period? **2**

(ii) It is also observed that, throughout the period, $\dfrac{d^2u}{dt^2} > 0$ **2**

Sketch the graph of u against t.

(b)



A logo is made up of 20 squares with a common corner, as shown in the diagram. The odd-numbered 'stripes' between successive squares are shaded in the diagram. The shaded stripes are painted in gold paint, which costs \$9 per square centimetre.

The side length of the nth square is (2n+4) cm. The nth stripe lies between the nth stripe and the (n+1)th square.

(i) Show that the area of the nth stripe is $(8n+20)$ cm^2. **2**

(ii) Hence find the areas of the first and last stripes. **2**

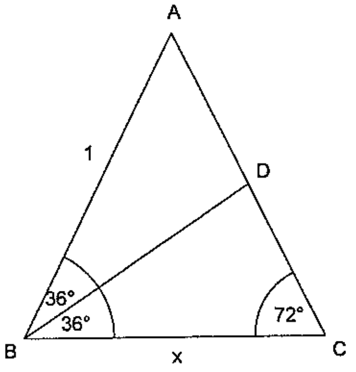
(iii) Hence find the total cost of the gold paint for the logo. **2**

(c) In the game of Yahtzee 5 dice are rolled. Find the probability of rolling all the same number. **2**

End of Question 9

Question 10 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a)



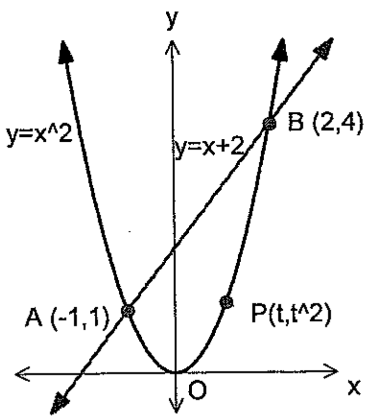
In the diagram ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$ and AB = AC =1. Angle ABC is bisected by BD, and BC=x.

Copy the diagram into your workbook.

(i) Show that triangles ABC and BCD are similar **2**

(ii) By using (i) find the exact value of x. **2**

(b)



In the diagram A(-1,1) and B(2,4) are the points of intersection of the parabola $y = x^2$ with the line $y = x + 2$. The point $P(t, t^2)$ is a variable point on the parabola below the line.

(i) Find the area of the parabolic segment APB, i.e. the area below the line and above the parabola. **3**

(ii) Show that the perpendicular distance from the point P to the line $y = x + 2$ is given by: **2**

$$d = \left| \frac{t - t^2 + 2}{\sqrt{2}} \right|$$

(iii) Using the above result show that the maximum area of triangle APB is three quarters of the area of the parabolic segment APB. **3**

End of Question 10

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$