Student Number

Year 12 2011

Semester 1 EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt questions 1-10
- All questions are of equal value

Question 1 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Express $\sqrt[5]{47}$ as a decimal correct to:

(i) Two decimal places **1**

(ii) Four significant figures **1**

(b) Simplify $\frac{x}{2} - \frac{x-2}{3}$ **2**

(c) Solve $x^2 + 6x - 16 \leq 0$ **2**

(d) Simplify $\sqrt{75} - \sqrt{12}$ **2**

(e) Factorise fully $x^2(x-3) - 4(x-3)$ **2**

(f) For two years Lewis and Mataya continued to debate who was the fittest and so decided to have another 1500m race. **2**

Lewis' previous best time was 5min 48secs, but with much practise and training he had reduced this time by 25%.

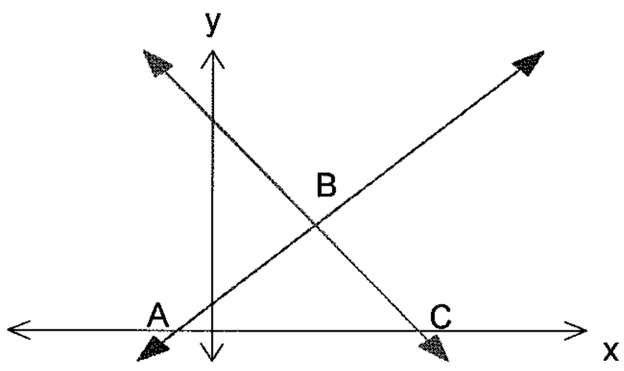
Mataya's previous best time was 4min 34secs, but with little effort at all she was able to reduce this time by 5%.

Who was the fastest and by how much?

End of Question 1

Question 2 (12 marks) Start a new sheet of writing paper. **Marks**

(a)



The line AB cuts the x-axis at A(-1,0) and the y-axis at 2.

(i) Copy the diagram onto your paper and mark in the given information. **1**

(ii) Find the equation of the line AB. **2**

(iii) The line BC (as shown in the diagram) has the equation $8x + 3y = 48$. **2**

Show that B has the co-ordinates (3,8)

(iv) M is the mid-point of AB. Find the coordinates of M. **1**

(v) Find the gradient of line BC. **1**

(vi) P is the mid-point of AC. A line is drawn from M to P. Show that the gradient of MP is equal to the gradient of BC. **3**

(vii) Find the coordinates of R such that MBRP is a parallelogram. **2**

End of Question 2

Question 3 (12 marks) Start a new sheet of writing paper. **Marks**

(a) Differentiate

(i) $\sqrt{x^3}$ **2**

(ii) $\dfrac{x^3 + 2x - 1}{x + 3}$ **2**

(b)

(i) $\int_1^4 (3x - 2)dx$ **2**

(ii) $\int \dfrac{1}{(4x + 5)^3} dx$ **2**

(c) Find the domain and range of $f(x) = \sqrt{2x - 6}$ **2**

(d) Tom sets a pendulum swinging and notices that each swing is 80% as long as the preceding swing. The first swing is 20cm, the second swing is 16cm, and it continues to swing until coming to rest. **2**

What is the total distance the pendulum swings?

End of Question 3

Question 4 (12 marks) Start a new sheet of writing paper. **Marks**

(a) Consider the function $y = x^3 - 3x^2 - 9x + 30$.

(i) Find the stationary points and determine their nature **4**

(ii) Find any points of inflection. **2**

(iii) Sketch the curve. **2**

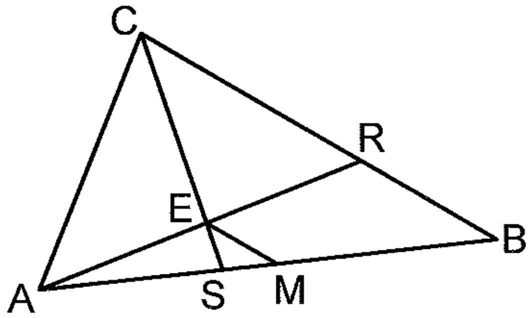
(iv) Find the coordinates of the point(s) where the line $y = 30 - 9x$ intersects the curve. **2**

(v) Find the area enclosed by the line and the curve. **2**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a)



In the diagram, CS bisects ∠ACB, AE is perpendicular to CS and M is the mid-point of AB. AE produced cuts CB at R.

Copy the diagram onto your paper.

(i) Prove that $\Delta ACE \equiv \Delta RCE$. **3**

(ii) Explain why $AE = ER$ **1**

(iii) Explain why EM is parallel to RB **3**

(b) **(i)** Sketch the area represented by **1**

$$\int_0^1 \sqrt{1-x^2}\,dx$$

(ii) Hence find the value of the integral in part (i) **1**

(c) On the same set of axes, graph $y = |2x - 1|$ and $y = -x$. **3**

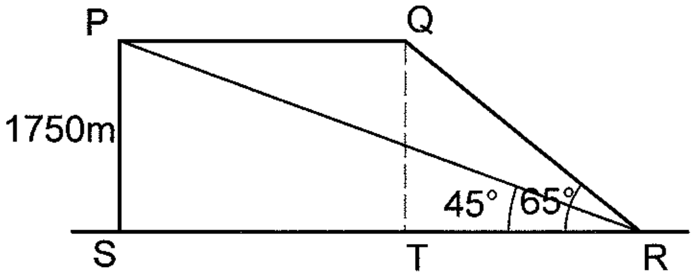
Use your graph, or otherwise, to explain why $|2x - 1| + x = 0$ has no solutions.

End of Question 5

Question 6 (12 marks) Start a new sheet of writing paper. **Marks**

(a) By evaluating the discriminant of the quadratic equation $2x^2 - 5x + 7 = 0$, determine whether the parabola $y = 2x^2 - 5x + 7$ is positive definite, negative definite or indefinite? **2**

(b) From a point R on level ground, Luke observes a plane flying at a constant altitude of 1750 metres from point P to point Q. At point P, the plane's angle of elevation is 45˚. A few minutes later, it reaches point Q where its angle of elevation measures 65˚.



Copy the diagram.

(i) Show that the exact length of PR is $1750\sqrt{2}$. **1**

(ii) Using triangle QRT find the length of QR, correct to three significant figures. **2**

(iii) Hence in triangle PQR find the distance between P and Q correct to three significant figures. **3**

(c) If α and β are the roots of $x^2 - 5x + 2 = 0$ find

(i) $\alpha + \beta$ **1**

(ii) αβ **1**

(iii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ **2**

End of Question 6

Question 7 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Find the focus and directrix of the parabola $x^2 - 8x - 16y + 48 = 0$ **3**

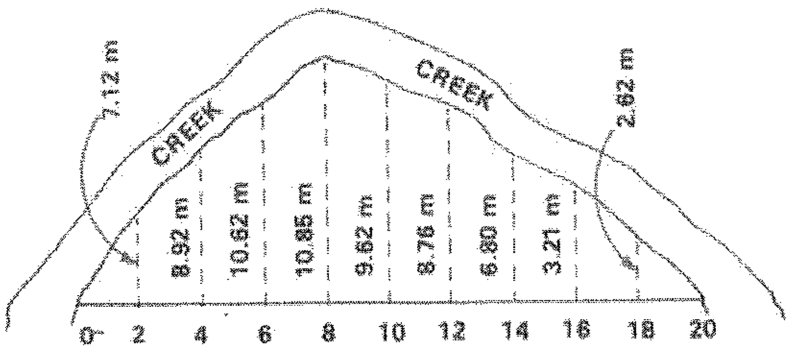
(b) Evaluate the following using series. **3**

$$\sum_{n=1}^{25}(100 - 3n)$$

(c) The sum of the first three terms of an arithmetic progression is 24, and the sum of the next three terms is 51. **3**

Find the first term and the common difference of this progression.

(d) Jacob gets a job as a surveyor and wants to calculate the land area next to a creek. He makes a straight line traverse the bend of a creek and measures perpendicular offsets every two metres as shown. By application of Simpson's Rule, help Jacob find the approximate area bounded by the traverse line and the creek. **3**



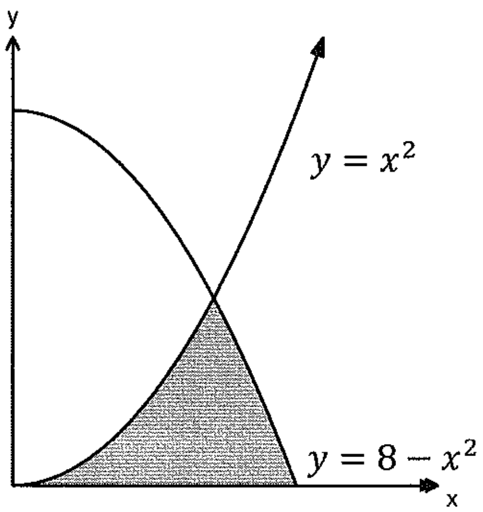
End of Question 7

Question 8 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Is $4x + 3y = 7$ a tangent to the circle $(x-2)^2 + (y-2)^2 = 25$? **3**

Justify your answer.

(b) Calculate the area between $= x^2$, $y = 8 - x^2$ and the x-axis. (The shaded area on the diagram.) **6**



(c) Find the co-ordinates of the point on the parabola $y = 3x^2 + 6x - 4$ where the tangent is perpendicular to the line $x - 6y - 3 = 0$. **3**

Question 9 (12 marks) **Start a new sheet of writing paper.** **Marks**

(a) Jarrod drops a basketball off the top of a tall building. The ball falls through a distance of 16m in the first second, 24m in the second second, 32m in the third second, and so on.

(i) How far does the ball fall in the eighth second? **1**

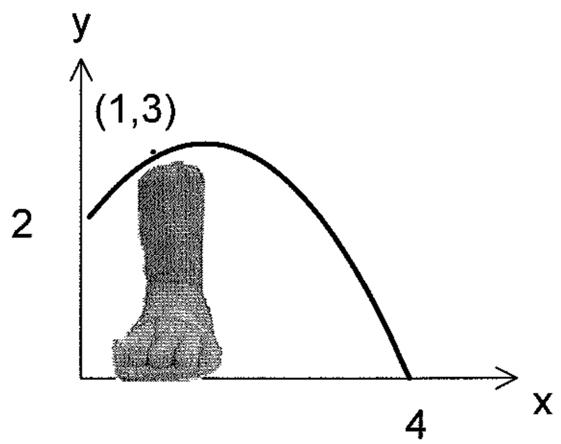
(ii) What is the total height through which the ball falls in 8 seconds? **2**

(ii) If the basketball were dropped from the top of a building 280m above the ground, how long would it take to hit the ground? **2**

(b) In Visual Arts Skye creates a piece of work involving an ornamental jet of water on following a parabolic shape with an equation of the form $y = ax^2 + bx + c$.

It starts from a point 2 metres above the ground and just passes over the top of a piece of sculpture 3 metres high and at a distance of 1 metre horizontally from the starting point.

The jet of water strikes the surface of a small pool, at ground level, at a distance of 4 metres horizontally from the starting point.



(i) Find the equation of the parabolic jet of water. **3**

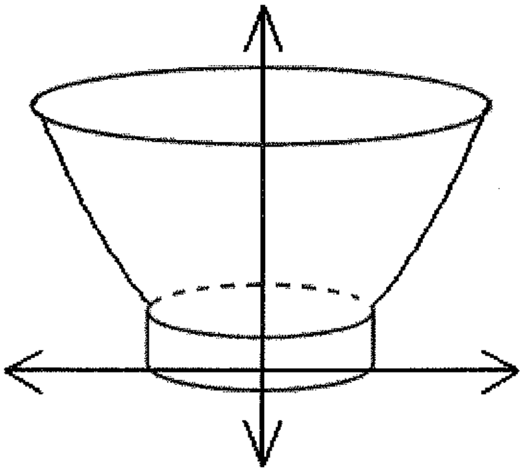
(ii) Find how far the jet of water rises above the ground. **2**

(iii) Find, correct to the nearest degree, the angle to the horizontal at which the jet emerges from its starting point **2**

End of Question 9

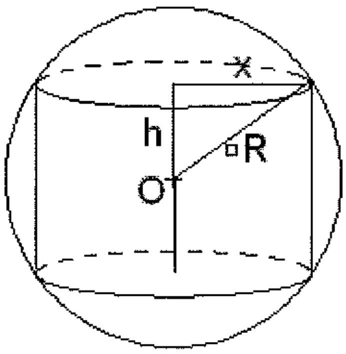
Question 10 (12 marks) Start a new sheet of writing paper. **Marks**

(a) Laura takes up pottery and fashions a bowl with the following shape. The portion of the curve $y = x^2$ from $x = \frac{1}{2}$ to $x = 2$ and the interval $x = \frac{1}{2}$ from $y = 0$ to $y = \frac{1}{4}$ are rotated about the y-axis to from a bowl with a base as shown.



Calculate the volume of liquid it can hold if the base is hollow. **6**

(b) A cylinder, radius x units and height 2h units, is inscribed in a sphere of radius R units where R is a constant.



(i) Show that the volume of the cylinder is given by: **2**

$$V = 2\pi h(R^2 - h^2)$$

(ii) Show that the cylinder has a maximum volume when $h = \frac{R}{\sqrt{3}}$ units. **4**

End of Question 10

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$