2012
SEMESTER 1
HIGHER SCHOOL CERTIFICATE EXAMINATION

Student Number

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided as a separate sheet
- A multiple choice answer sheet is provided
- Answer Section 1 on multiple choice answer sheet provided
- All necessary working should be shown in Section 2

Total Marks – 100

Section 1 - 10 Marks

- Attempt questions 1-10
- 1 mark objective-response questions

Section 2 – 90 Marks

- Attempt questions 11-16
- All questions are 15 marks

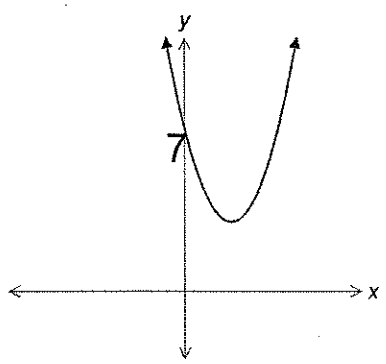
SECTION 1
10 marks
Attempt Questions 1 to 10.
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

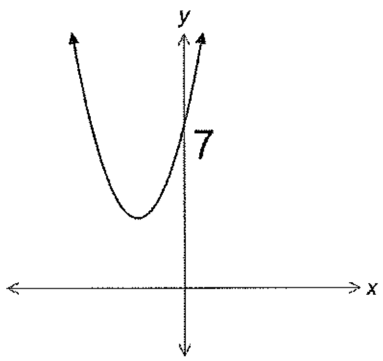
1. Which is a graph of the following function

$$f(x) = -(x+2)^2 - 3$$

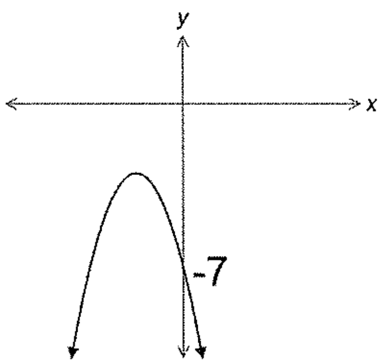
(A)



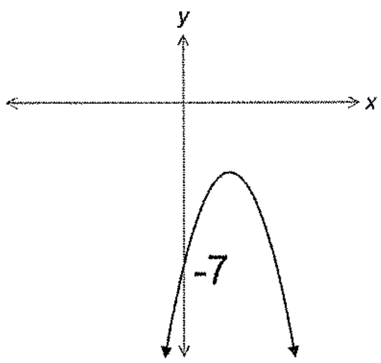
(B)



(C)



(D)



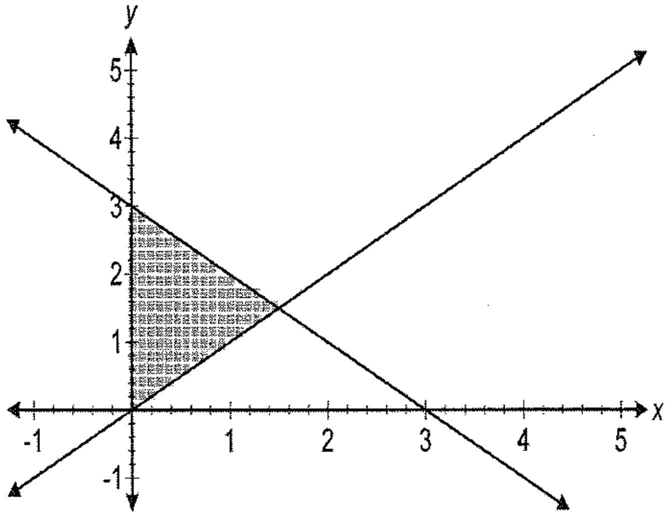
2. From point P the bearing of point R is $065°$ and from point Q the bearing of R is $315°$. If point Q is 8.8 kilometres from P on a bearing of $090°$ then the distance from R to Q is closest to:

(A) 3.78 km **(B)** 3.94 km **(C)** 3.96km **(D)** 5.37 km

3. Solve for x: $log_4 65 = x$

(A) 1.21 **(B)** 1.81 **(C)** 3.06 **(D)** 3.01

4. The equations of the two axes below are $y = 3 - x$ and $y = x$.



The shaded region could be described by which of the following sets of inequations?

(A) $x \geq 0, \quad y \geq 0, \quad y \leq x, \quad x + y \leq 3$

(B) $x \geq 0, \quad y \geq 0, \quad x \leq y, \quad x + y \leq 3$

(C) $x \geq 0, \quad y \geq 0, \quad y \leq x, \quad x - y \leq 3$

(D) $x \geq 0, \quad y \geq 0, \quad x \leq y, \quad x - y \leq 3$

5. The quadratic equation whose roots are at x = 3 and x = 5 is given by

(A) $(x - 3)(x - 5) = 1$

(B) $(x + 3)(x + 5) - 9 = (x + 3)(x + 5) - 25$

(C) $(x + 3)(x + 5) = 0$

(D) $x^2 - 8x = -15$

6. If $tan\theta = \frac{5}{12}$ and $180° \leq \theta \leq 360°$, then $sec\theta =$

(A) $\frac{12}{13}$ **(B)** $-\frac{12}{13}$ **(C)** $\frac{13}{12}$ **(D)** $-\frac{13}{12}$

7. If $x > 1$ and $\frac{\sqrt{x}}{x^3} = x^m$ what is the value of m?

(A) $-\frac{7}{2}$ **(B)** -3 **(C)** $-\frac{5}{2}$ **(D)** $\frac{1}{6}$

FOR THE FOLLOWING THREE QUESTIONS THERE MAY BE MORE THAN ONE ANSWER

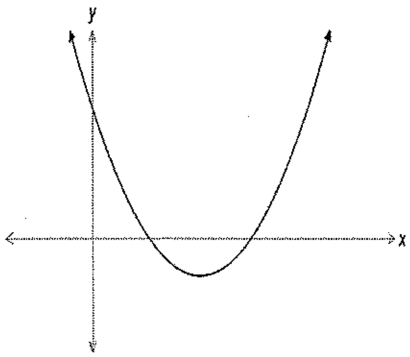
8. What are the asymptotes of the graph of

$$y = \frac{1}{x-3}$$

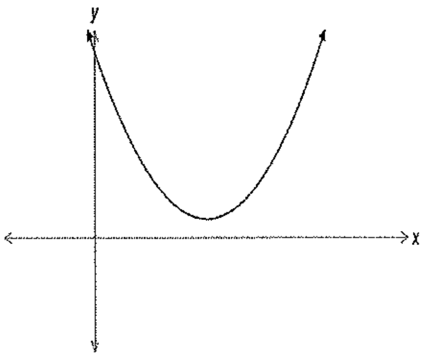
(A) $x = 3$ **(B)** $y = 3$ **(C)** $y = 0$ **(D)** $x = 0$

9. For which of these graphs is $\Delta > 0$

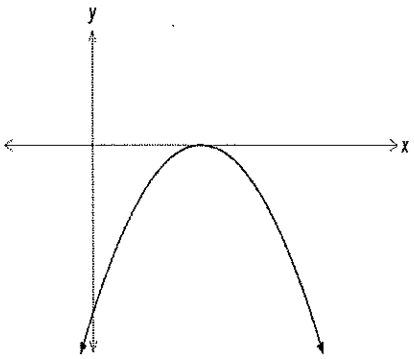
(A)



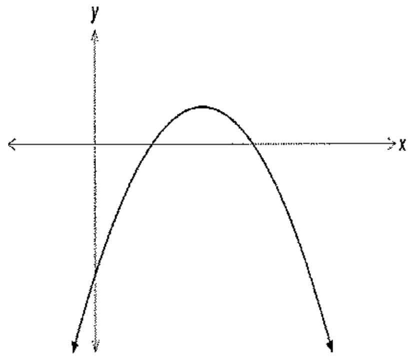
(B)



(C)



(D)



10. The x coordinate of the function $f(x) = \frac{x^3}{3} - 5x^2 + 2x + 10$ where its gradient is -14 is:

(A) -10 **(B)** -2 **(C)** 2 **(D)** 8

End of Section I

SECTION 2

Question 11 (15 marks) **Start a new sheet of writing paper.**

Marks

(a) If $f(x) = x^2 - 4$, calculate $f(-2)$ **1**

(b) Find integers a and b such that **2**

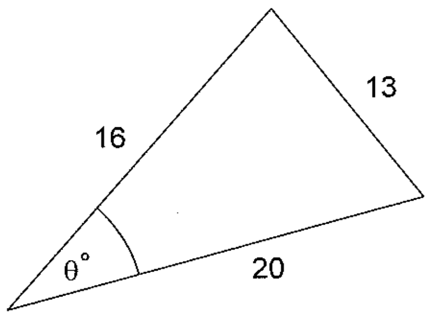
$$\frac{2}{2-\sqrt{3}} = a + \sqrt{b}$$

(c) Express $2.1\dot{7}$ as a fraction **1**

(d) Simplify **2**

$$\frac{16a^4b^8 - 36a^8b^4}{8a^4b^6 + 12a^6b^4}$$

(e) Find θ (to the nearest minute) **2**



(f) Find the distance between $(\sqrt{5}, -\sqrt{5})$ and $(-\sqrt{5}, 3\sqrt{5})$ **2**

(g) Find $\lim_{x\to 1}(5x)(2+3x)^2$ **1**

(h) If $f(x) = 5x^{-6}$, find $f'(x)$ **1**

(i) Evaluate **2**

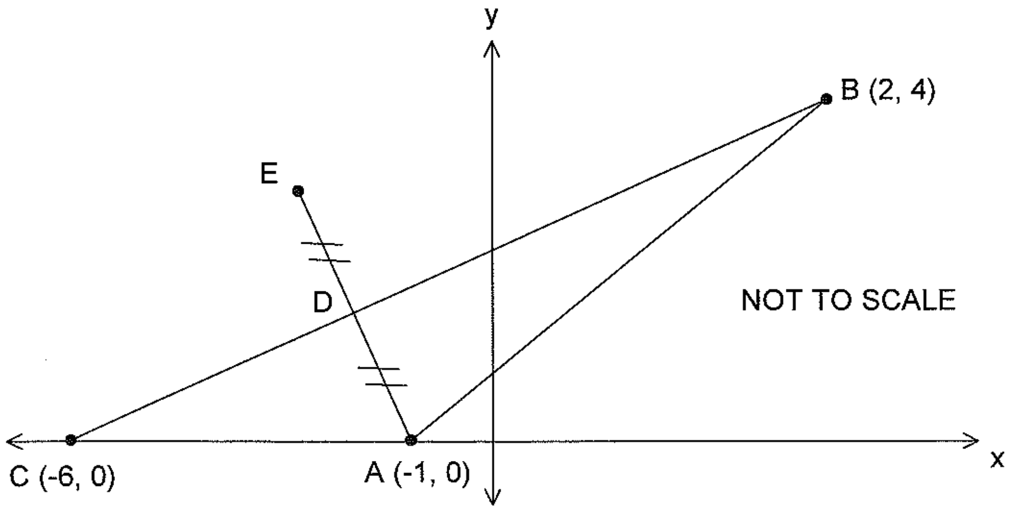
$$\int_0^3 2dx$$

(j) Find the primitive function for $x^4 + 2x + 3$ **1**

End of Question 11

Question 12 (15 marks)   Start a new sheet of writing paper.   Marks

(a)



In the diagram A, B and C are the points $(-1,0)$, $(2,4)$ and $(-6,0)$ respectively. D has coordinates $(-2,2)$ and is the midpoint of AE.

(i) Find the length of the interval AB. **1**

(ii) Find the equation of the circle with centre at B which passes through the point A. **2**

(iii) Find the size of $\angle CAB$ (to the nearest degree) **2**

(iv) Find the midpoint of BC. **1**

(V) Show that the equation of the line BC is $x - 2y + 6 = 0$. **2**

(vi) Find the perpendicular distance of A from the line BC in simplest exact form. **2**

(vii) What type of quadrilateral is ABEC? Give reasons for your answer. **2**

(b) (i) Sketch the locus of a point $P(x, y)$ which is equidistance from the point $(0,3)$ and the line $y = -3$. **1**

(ii) Determine the equation of the locus **2**

End of Question 12

Question 13 (15 marks) Start a new sheet of writing paper. Marks

(a) Consider the curve given by $y = 2x^3 - 9x^2 + 12x$.

(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Show that a point of inflexion occurs at $x = \frac{3}{2}$ **1**

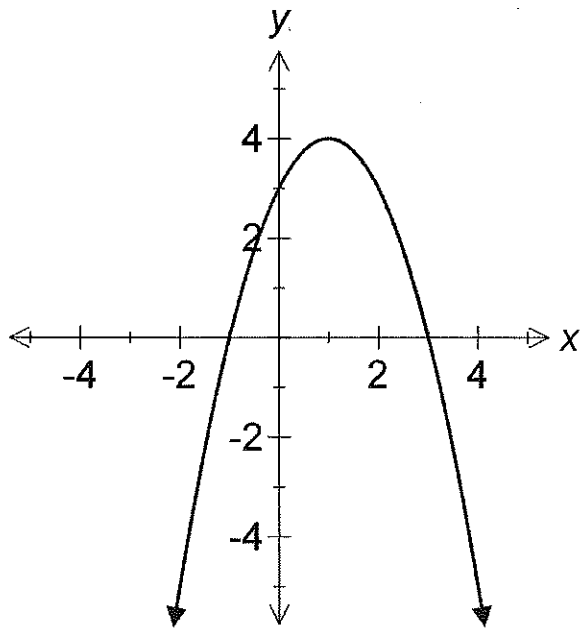
(iii) Sketch the graph $y = f(x)$, indicating clearly any important features. **2**
(Make the sketch approximately $\frac{1}{3}$ of a page)

(iv) For what values of x is the curve concave up? **1**

(b) The height in metres of a projectile for a horizontal displacement of s metres is given by $h = 108 + 40s - s^2$. Find the maximum height **2**

(c) Find the value of k if the sum of the roots of the equation, **2**
$x^2 - (4 - k)x + (k - 2) = 0$, is equal to the reciprocal of the products of the roots.

(d)



The parabola above is drawn to scale.

(i) State the coordinates of the points where the parabola crosses the *x* axis and the *y* axis. **2**

(ii) Hence, or otherwise, find the equation of the parabola drawn above in the form of $y = ax^2 + bx + c$ **2**

End of Question 13

Question 14 (15 marks) **Start a new sheet of writing paper.** **Marks**

(a) Calculate the angle between the line $3y - 5x = 17$ and the positive direction of the x-axis, correct to the nearest degree. **2**

(b) Find all the values of x in the interval $0° \leq x \leq 360°$ for which $2cosx = -\sqrt{3}$ **3**

(c) Differentiate:

(i) $y = x^2 e^x$ **2**

(ii) $f(x) = \dfrac{e^x + 1}{2x}$ **2**

(d) Find the equation of the normal to the curve $y = x^4 + x - 1$ at the point where $x = 1$. **3**

(e) Consider the function given by **3**

$$y = \frac{8}{2 + x^2}$$

Using Simpson's rule with 5 function values, estimate the area under the curve from 0 to 4.

End of Question 14

Question 15 (15 marks) **Start a new sheet of writing paper.** **Marks**

(a) Differentiate:

(i) $y = \frac{2}{x} + 3x^2 - 1$ **2**

(ii) $f(x) = \ln(e^x + 3)$ **2**

(b) Evaluate

(i) $\int (6x - 4)dx$ **1**

(ii) $\int (3x - 4)^{10} dx$ **1**

(iii) $\int \left(\frac{x^2 + 2x - 4}{x}\right) dx$ **2**

(iv) $\int_1^3 (3x^2 - 2x + 5)dx$ **2**

(c) Given that $f'(x) = 2x + 2$, and $f(2) = 13$. Find $f(x)$. **2**

(d) Consider **3**

$$\int_1^m \frac{dx}{\sqrt{x}} = 6$$

Determine the value of m

End of Question 15

Question 16 (15 marks) **Start a new sheet of writing paper.** **Marks**

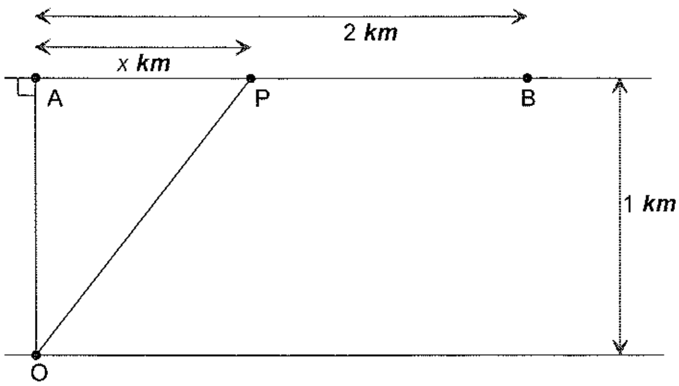
(a) Find the equation of the normal to the curve $y - 2e^{2x} - e^{x}$ at the point where $x = 0$ **3**

(b) Express $5x^2 + 2x - 3$ in the form of $A(x+1)^2 + B(x+1) + C$ **3**

(c) Solve the following equation giving all real roots in exact form **3**

$$(x-3)^4 - 18(x-3)^2 + 32 = 0$$

(d)



The diagram shows a straight section of a river with parallel riverbanks 1 km wide.

Ben is at point *O* on the bank. He needs to reach point B on the opposite bank. The point *A* is directly opposite him on the other side of the river and the distance between *A* and *B* is 2 kilometres.

Ben can swim at 6km/h and jog at 10km/h. He wants to swim in a straight line to the other side of the river, to a point *P* (between *A* and *B*), and then jog the rest of the way to *B*. Let the distance from *A* to *P* be *x*.

(i) Show that the time *T*, in hours that Ben takes to reach *B* is given by: **2**

$$T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$$

(ii) Show that if Ben wishes to minimize the time taken to complete the journey from *O* to *B*, then he should swim to a point *P*, 0.75km from *A*. **3**

(iii) Find the minimum time it takes Ben to complete his journey, to the nearest minute **1**

End of Question 16

MATHEMATICS
SEMESTER 1 EXAMINATION

MULTIPLE CHOICE ANSWER SHEET

STUDENT NUMBER:_______________

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$