NAME: ____________________

CLASS: ____________________

DANEBANK
An Anglican School for Girls

2008
Year 12
Half Yearly Examination

Mathematics

Outcomes Examined: P2,P3,P4,P6,P7,P8,H2, H4, H5, H6,H7,H8,H9

Weighting of Task: 25%

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- All necessary working should be shown in every question otherwise full marks may not be awarded.
- Board-approved calculators may be used
- Start each new question in a new booklet

Total Marks – 84

Attempt All Questions 1 - 7

Questions are of equal value

This paper MUST NOT be removed from the examination room

Question	1	2	3	4	5	6	7
Mark							
Maximum	12	12	12	12	12	12	84

2008 2unit Mathematics Half Yearly

Total marks – 84

Attempt Questions 1- 7

All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate to 2 significant figures – $\dfrac{\sqrt{4.17^2 - 3.9}}{\pi}$ **2**

(b) .Find the primitive of $x^3 - 2x + 5$ **2**

(c) Factorise $8a^3 + 1$.

(d) Prove that $\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} = \tan\theta$ **2**

(e) A parabola satisfies all the following conditions. **2**

i. It is concave up;
ii. the focal length is one unit
iii. the directrix has equation y =1

Draw a possible diagram to show this information AND give a possible equation.

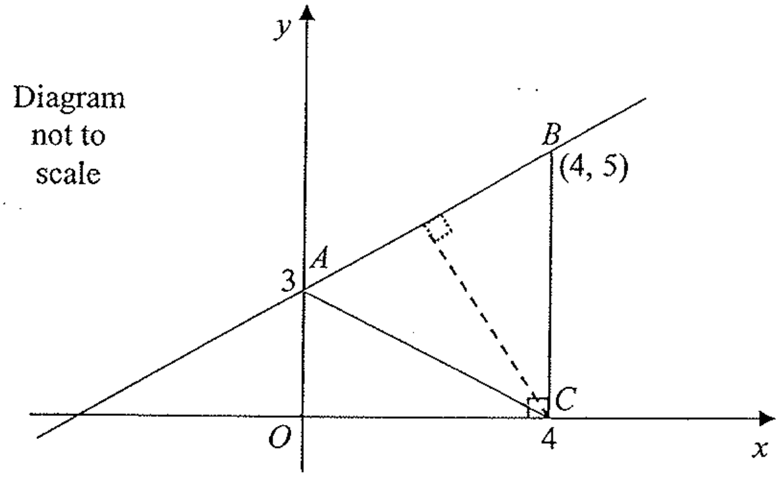
(f) Write 1 + 4 + 7 + ... + 19 as an expression in the form $\sum_{n=0}^{6.}$ **2**

Question 2 (12 marks) Use a SEPARATE booklet.

Marks

(a) In the diagram, the line AB cuts the y-axis at the point A (0, 3) and passes through the point B (4, 5). A perpendicular is dropped from B to meet the x-axis at C (4, 0).

Copy or trace the diagram into your working booklet.



(i) Calculate the length of the interval AB. 1

(ii) Find the gradient of the line AB. 1

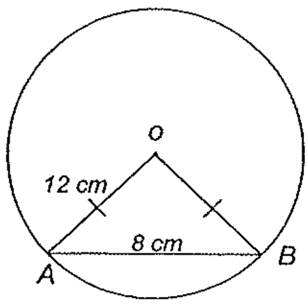
(iii) Show that the equation of the line AB is $x - 2y + 6 = 0$ 1

(iv) Find the equation of the line which is perpendicular to AB and which passes through C. 2

(v) Calculate the perpendicular distance from C to AB. 2

(vi) Find the area of the triangle ABC. 1

(b)



In the diagram $AB = 8\,cm$ and radius of the circle is $12cm$.

(i) Show that $\angle AOB = 39^0$ to the nearest degree. 2

(ii) Find the area of the triangle, correct to the nearest cm^2. 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) Differentiate with respect to x.

(i) $(x^3 - 1)^5$ 2

(ii) $\dfrac{4x^2}{x+3}$ 2

(b) (i) Find $\int \dfrac{2x + x^4}{x^3} dx$. 3

(ii) $\displaystyle\int_1^a (2x+3)dx = 0$, find value(s) of a 3

(c) Find the values of k for which the quadratic equation $kx^2 - 8k + k = 0$ has real roots. 2

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Explain (DO NOT FIND) how you can show that $y = 5x - 16$ is a tangent to the parabola $y = x^2 - 3x$. **1**

(b) Allia puts \$100 into a fund at the end of each month, which earns 7.2% interest per annum compounded monthly. **3**

Calculate the amount she will receive if she takes her investment out at the end of 5 years.

(c) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 6x - 9$. and the curve passes through the point (1,-2).

(i) Show the equation of the curve is $y = x^3 - 3x^2 - 9x + 9$. **2**

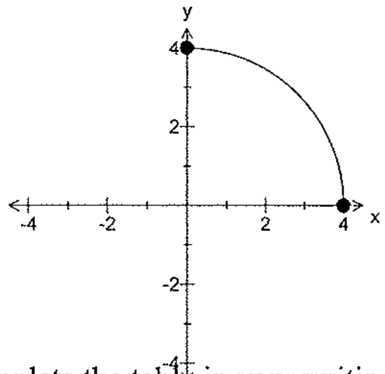
(ii) Find the co-ordinates of the stationary points and determine their nature. **2**

(iii) Find the co-ordinates of the point of inflexion **1**

(iv) Sketch the curve of $y = x^3 - 3x^2 - 9x + 9$. for the domain $-2 \leq x \leq 4$. **3**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The graph below represents $y = \sqrt{16 - x^2}$, $0 \le x \le 4$



(i) Copy and complete the table in your writing booklet. Leave answers to three decimal places. 1

x	0	1	2	3	4
y	4		3.464		

(ii) Use Simpson's rule with five function values to estimate $\int_0^4 \sqrt{16 - x^2}\, dx$. 2

(iii) Find the exact value of $\int_0^4 \sqrt{16 - x^2}\, dx$ in terms of π. 1

(iv) Hence, use the answers to parts (ii) and (iii) to approximate the value of π to 2 decimal places. 1

(b) (i) What is the condition for a geometric series to have a limiting sum? 1

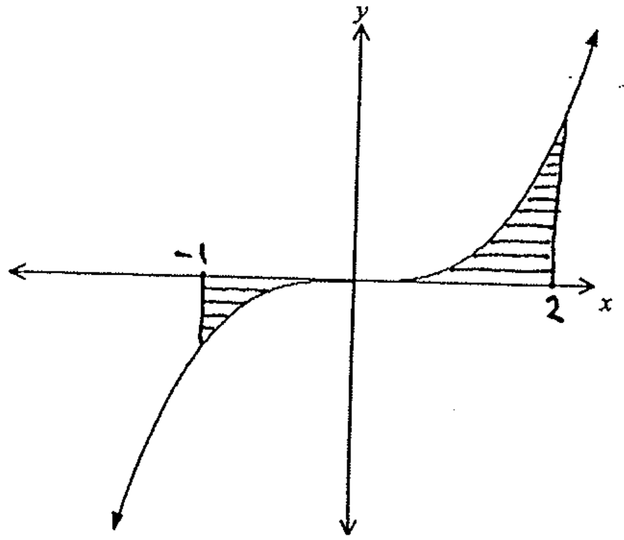
(ii) The limiting sum of a geometric series is 20. 2
What might the series be? (Give two possible answers).

Question 5 continued over the page

Marks

(c) Max was given this question to answer. 3

" Find the area bounded by the curve $y = x^3$ and the x-axis between $x = -1$ and $x = 2$.



This was his solution:

$$\int_{-1}^{2} x^3\,dx = \left[\frac{x^4}{4}\right]_{-1}^{2} \qquad \text{line 1}$$

$$= \left[\frac{2^4}{4}\right] - \left[\frac{(-1)^4}{4}\right] \qquad \text{line 2}$$

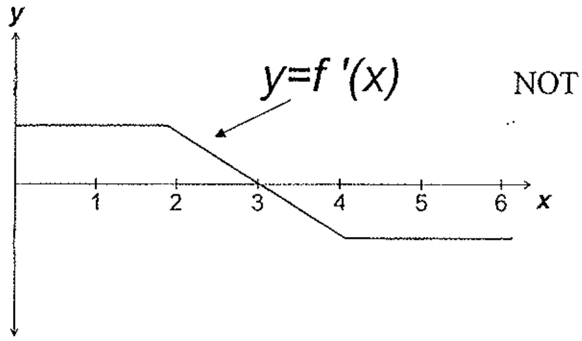
$$= 63\frac{3}{4}\,units^2 \qquad \text{line3}$$

Explain fully any errors made and find the actual area showing all working

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the volume of the solid formed when the area bounded by the parabola $y = 4 - x^2$ and the x-axis is rotated about the y-axis. 3

(b) 2



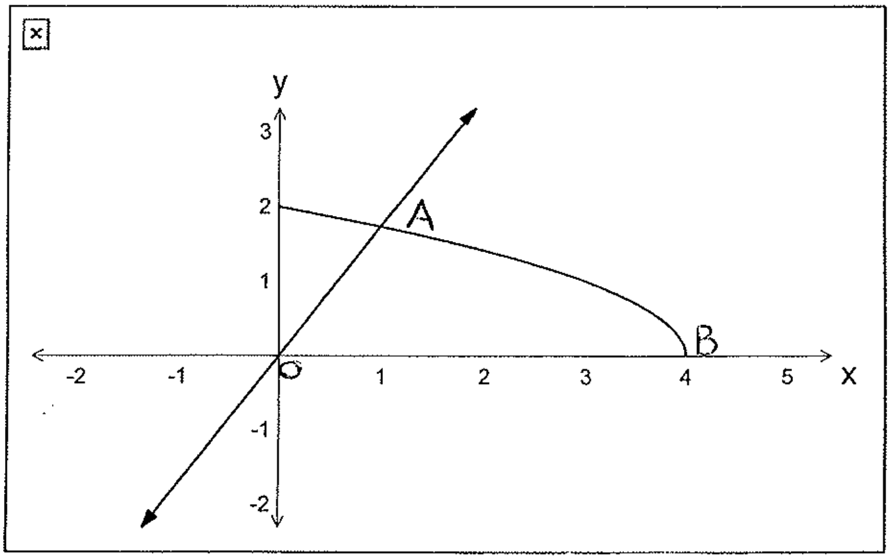
The above diagram shows a sketch of the **gradient** function of $y = f(x)$. In your writing booklet, draw a sketch of the function $y = f(x)$ given that $f(0) = 0$.

(c) The eight and fourteenth term of an arithmetic series. are -25 and -49.

(i) Find the 1st term and the common difference. 2

(ii) Find the sum of the first fourteen terms. 1

(d) The sketch below represents the curve $y = \sqrt{4-x}$ and the line $y = \sqrt{3}x$.



(i) Show that the point of intersection $A = \left(1, \sqrt{3}\right)$. 1

(ii) Hence, find the exact area bounded by OAB and the x $axis$. 3

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\sin^2 x = \frac{1}{4}$ *for* $0° \le x \le 360°$. 2

(b) Statistics have shown that the population of a particular town was decreasing in such a way that at the end of each year the population could be determined as follows.

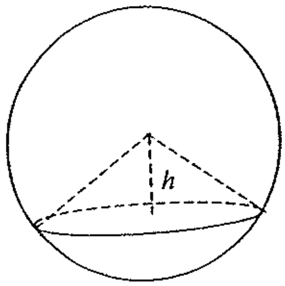
"10% of the population moved out at the end of the year and 500 people moved in during the year."

At the beginning of 2000, before the 10% moved out, the population of the town 10 000.

(i) Show that at the end of 2002 the population of the town was: $10\,000(0.9)^3 + 500(1 + 0.9 + 0.9^2)$. 2

(ii) This trend continued indefinitely. Find the population of the town at the end of the year 2019 (i.e. at the end of the 20th year). 2

(c) The diagram shows a cone in a sphere.
The vertex of the cone is at the centre of the sphere.
The radius of the sphere is 12cm.



(i) Show that the volume V of the cone is 2

$$V = \frac{\pi}{3}(144h - h^3)$$

(ii) Find the maximum volume of the cone in exact form. 4

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Half Yearly Solutions 2008 Mathematics

Marking Guidelines

1 (a) $\dfrac{\sqrt{4.17^2-3.9}}{\pi} = 1.16906429$

$= 1.2$ — 2 — 1 for calculation; 1 for significant fig

(b) $\int x^3-2x+5\,dx = \dfrac{x^4}{4} - x^2+5x+C$ — 2 — $\frac{1}{2}$ for each part

(c) $8a^3+1 = (2a+1)(4a^2-2a+1)$ — 2 — 1 for progress.

(d) To prove $\dfrac{\sin\theta(1-\sin^2\theta)}{\cos^3\theta} = \tan\theta$

$LHS = \dfrac{\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$

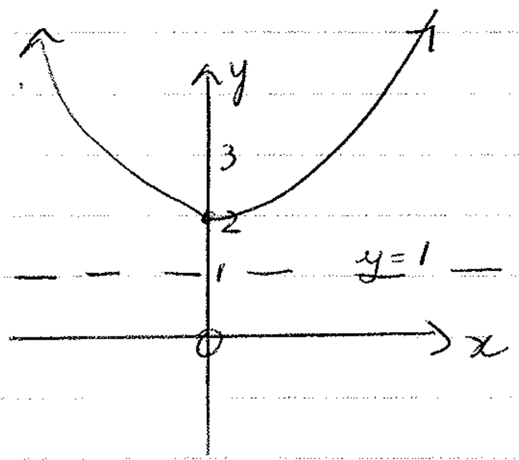
$= \dfrac{\sin\theta \times \cos^2\theta}{\cos^3\theta}$ — 1 mark

$= \dfrac{\sin\theta}{\cos\theta}$ — 2

$= \tan\theta$ — 1 mark

$= RHS$

(e)



2 — $\frac{1}{2}$ concave up; $\frac{1}{2}$ directrix; $\frac{1}{2}$ equation; $\frac{1}{2}$ vertex.

(f) $1+4+7+\cdots+19$

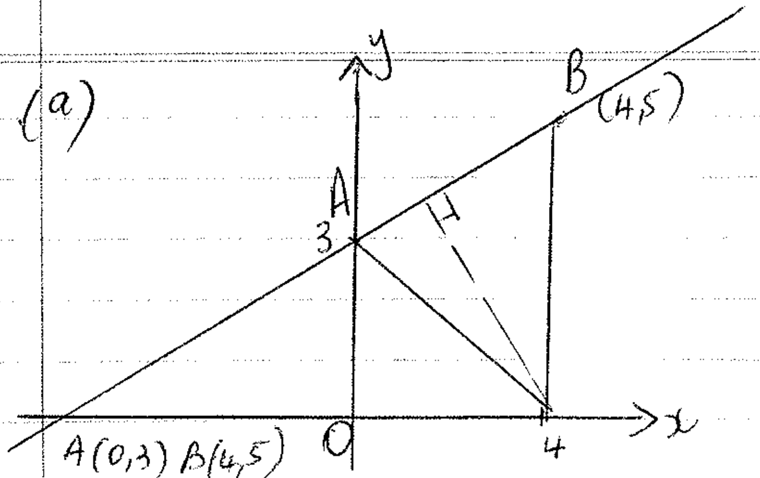
$a=1,\ d=3,\quad T_n = a+(n-1)d$ $\quad 1+6\times 3$

$\cancel{T_n}\ 3n+1 =$

$\sum_{n=0}^{6} 3n+1$ — 1 mark for making progress. 2 answer

Marking Gidel[illegible]

2 (a)



A(0,3) B(4,5)

(i) $d_{AB} = \sqrt{(5-3)^2 + (4-0)^2}$

$= \sqrt{20}$

$\underline{d = 2\sqrt{5} \text{ units}}$ | 1

(ii) $m\ AB = \dfrac{5-3}{4-0}$

$\underline{m = \tfrac{1}{2}}$ | 1

(iii) Eqn AB: $y - 5 = \tfrac{1}{2}(x-4)$ / $y - 3 = \tfrac{1}{2}(x-0)$

$2y - 10 = x - 4$ / $2y - 6 = x$

$x - 2y + 6 = 0$ / $x - 2y + 6 = 0$. | 1

(iv) Eqn l_2 $y - 0 = -2(x-4)$

$y = -2x + 8$ | 1 mark gradient

$2x + y - 8 = 0$. | 2 correct eqn

(v) $C(4,0)$ $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ | 1 mark for wrong line / wrong point

$= \left|\dfrac{1\times 4 - 2\times 0 + 6}{\sqrt{1^2 + (-2)^2}}\right|$

$= \left|\dfrac{4+6}{\sqrt{5}}\right|$

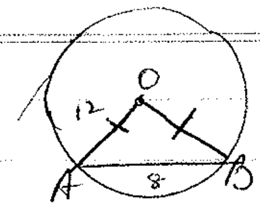
$= \dfrac{10}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= 2\sqrt{5}$ units. or 4.47 (2dp) | 2 answer

$\tfrac{1}{2} \times$ (i) $\times$ (v)

(vi) Area $= \tfrac{1}{2} \times 2\sqrt{5} \times 2\sqrt{5}$ | $\tfrac{1}{2}$ this stage

$= 10 \text{ units}^2$. | 1 correct answer

(b) 

Marking Guidelines

(i) $\cos \angle AOB = \dfrac{12^2 + 12^2 - 8^2}{2 \times 12 \times 12}$ — $\frac{1}{2}$

$= \dfrac{224}{288}$ — 2 marks

$\angle AOB = \cos^{-1}\left(\dfrac{224}{288}\right)$

$= 38^\circ$ 24

$= 39^\circ$ to nearest degree. — 2 marks

(ii) Area $= \frac{1}{2} ab\sin C$ — -1 if cos

$= \frac{1}{2} \times 12 \times 12 \times \sin 39^\circ$ — 1

$= 45\ cm^2$. (nearest) — 2 — 1

3 (a) Marking Guidel[ines]

(i) $\frac{d}{dx}(x^3-1)^5 = 5(x^3-1)^4 \times 3x^2$ 1 mark each part

$= 15x^2(x^3-1)^4$

(ii) $\frac{d}{dx}\frac{4x^2}{x+3} = \frac{(x+3)8x - (4x^2)1}{(x+3)^2}$ $u = 4x^2$ $u' = 8x$ $v = x+3$ $v' = 1$.

1 mark for the 1½ of rule correctly

2 simplify

$= \frac{8x^2+24x-4x^2}{(x+3)^2}$

$= \frac{4x^2+24x}{(x+3)^2}$.

(b) (i) $\int \frac{2x+x^4}{x^3}\,dx = \int 2x^{-2}+x\,dx$

$= \frac{-2}{x} + \frac{x^2}{2} + c$. 3

1 mark rewrite
1 correct answer
1 if + c only

(ii) $\int_1^a (2x+3)\,dx = 0$

$\therefore [x^2+3x]_1^a = 0$ 3

1 mark integrate
1 mark substitute
1 mark solution

$(a^2+3a)-(1^2+3\times1) = 0$

$a^2+3a-4 = 0$

$(a+4)(a-1) = 0$

$\therefore a = -4, 1$.

(c) $kx^2-8x+k=0$

$\Delta = b^2-4ac$

$= (-8)^2-4(k)(k)$

$= 64-4k^2$

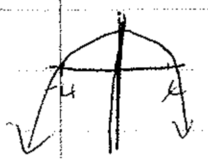
For real roots $\Delta \geq 0$

$\therefore 64-4k^2 \geq 0$

$16-k^2 \geq 0$

$(4-k)(4+k) \geq 0$

$\therefore -4 \leq k \leq 4$



$\frac{1}{2}$ $\Delta \geq 0$

$\frac{1}{2}$ $\Delta = 64-4k^2$

$-\frac{1}{2}$ if only > no[t ≥]

$-\frac{1}{2}$ final inequality incorrect.

Marking Guideline

4(a) If $y = x^2 - 3x$ and $y = 5x - 16$ have one point of intersection by solving them simultaneously, then the line $y = 5x - 16$ is a tangent to $y = x^2 - 3x$. — 1 for reasonable explanation

(b) $P = \$100$ $r = 7.2\%$ p.a $= 0.6\%$ pm $= 0.006$ pm. $n = 5 \times 12$ $n = 60$. — -1 if wrong interest rate and n

A_1 = first \$100 invested — -1 if in wrong order

$= \$100 \times 1.006^{60}$

$A_2 = \$100 \times 1.006^{59}$

$A_3 = \$100 \times 1.006^{58}$

$\vdots$

$A_{60} = \$100 \times 1.006^{1}$

$\therefore$ Total $= \$100 \times 1.006^{60} + \$100 \times 1.006^{59} + \dots \$100 \times 1.006$

$= \$100 \times 1.006 \left(1.006^{59} + 1.006^{58} + 1.006^{57} + \dots + 1\right)$ — 1 mark

$= \$100 \times 1.006$ (sum of geometric series with $a = 1$, $r = 1.006$ $n = 60$

$= \$100 \times 1.006 \left[\frac{a(r^n - 1)}{r - 1}\right]$

$= \$100 \times 1.006 \left[1\left(\frac{1.006^{60} - 1}{1.006 - 1}\right)\right]$ — 1 mark

$= \$7239.65$.

Marking Guide

(i) $\frac{dy}{dx} = 3x^2 - 6x - 9$

(i) $y = \frac{3x^3}{3} - \frac{6x^2}{2} - 9x + C$ — 2 { 1 for integration, 1 for constant

$y = x^3 - 3x^2 - 9x + C$

$\therefore -2 = 1^3 - 3(1)^2 - 9(1) + C$

$\therefore C = 9$

$\therefore y = x^3 - 3x^2 - 9x + 9$

(ii) $\frac{dy}{dx} = 3x^2 - 6x - 9$

$\frac{d^2y}{dx^2} = 6x - 6$

Stationary points when $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

When $x = 3$, $y = 3^3 - 3(3)^2 - 9(3) + 9$

$= -18$

and $\frac{d^2y}{dx^2} = 6(3) - 6$

$= 12$

> 0

$\therefore$ Minimum turning point at $(3, -18)$ — 2 { 1 points, 1 nature

When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) + 9$

$= 14$

and $\frac{d^2y}{dx^2} = 6(-1) - 6$

$= -12$

< 0

$\therefore$ Maximum turning point at $(-1, 14)$.

Marking Guide

(iii) Pt of inflexion when $\frac{d^2y}{dx^2} = 0$

$\therefore 6x - 6 = 0$

$x = 1, \quad y = 1^3 - 3(1)^2 - 9(1) + 9$
$\qquad = -2$ — ½ point

Testing concavity — 1 — ½ test

x	0	1	2
$\frac{d^2y}{dx^2}$	<0	0	>0

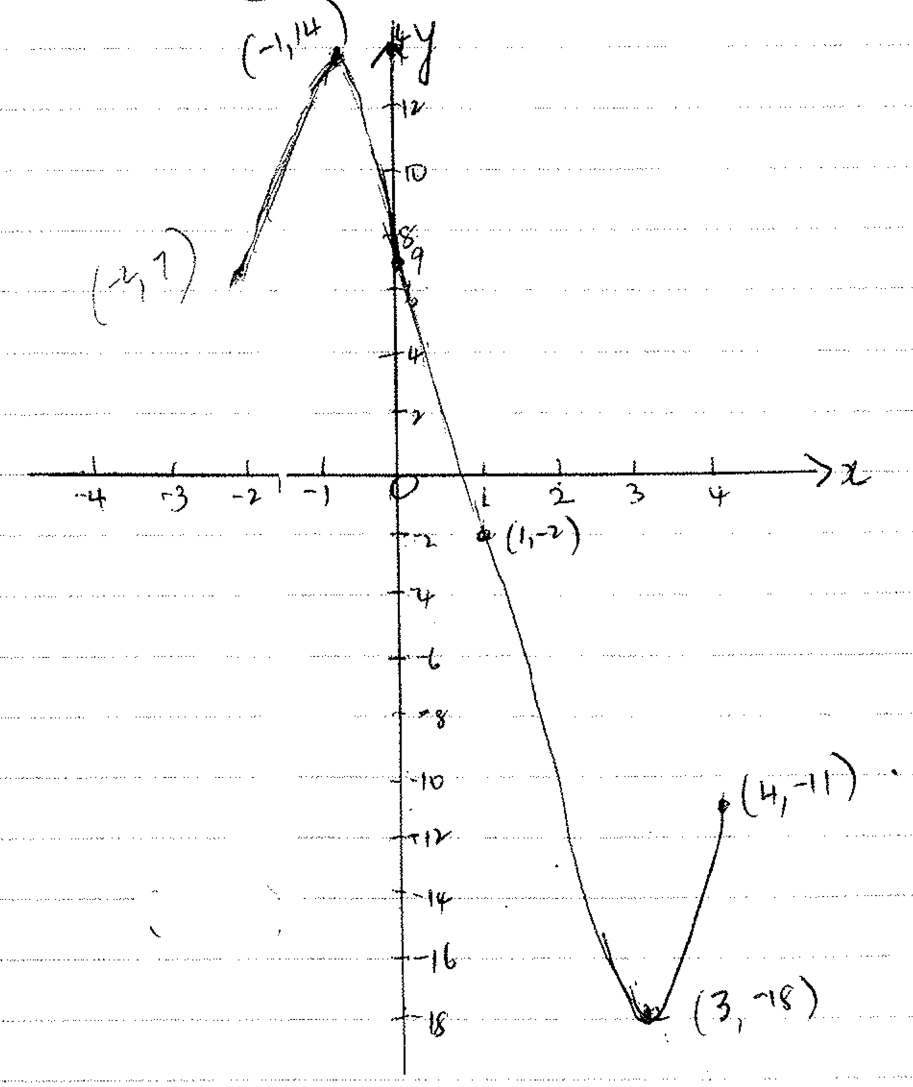
$\therefore$ change in concavity

$\therefore (1, -2)$ is a pt of inflexion

(iv) End points: When $x = -2$, $y = (-2)^3 - 3(-2)^2 - 9(-2) + 9$
$\qquad = 7 \quad \therefore (-2, 7)$

When $x = 4$, $y = 4^3 - 3(4)^2 - 9(4) + 9$
$\qquad = -11 \quad \therefore (4, -11)$

Y intercept $(0, 9)$.



½ for each end point, turning point, inflexion point & intercept

Marking Guide

5 (a)(i) $y = \sqrt{16-x^2}$

x	0	1	2	3	4
y	4	3.873	3.464	2.646	0

$-\frac{1}{2}$ for any error

1 correct answer

(ii) $\int_0^4 \sqrt{16-x^2}\,dx \doteqdot \frac{1}{3}\left[4+0+4(3.873+2.646) + 2(3.464)\right]$

$= \frac{1}{3}(4+26.076+6.928) = 37.004$ — 1 mark

$= 12.3346$

$= 12.335$ (3dp). — 2 answer

(iii) $\int_0^4 \sqrt{16-x^2}\,dx = \frac{1}{4}\pi(4)^2$

$= 4\pi$ — 1

(iv) $\therefore 12.337 = 4\pi$

$\therefore \pi = \frac{12.335}{4}$

$\doteqdot 3.08375$

$\doteqdot 3.08$ (to 2dp) — 1

(b) (i) $-1 < r < 1$ — 1

(ii) $S_\infty = \frac{a}{1-r}$

$20 = \frac{a}{1-r}$

$20(1-r) = a$ — 1 mark

If $r = 1/2$, $20(1-\frac{1}{2}) = a$

$\therefore a = 10$.

$\therefore$ Series could be $10, 5, 2\frac{1}{2} \ldots$

If $r = -\frac{1}{2}$, $20(1+\frac{1}{2}) = a$

$a = 30$

$\therefore$ Series could be $30, -15, 7\frac{1}{2}, \ldots$

1 each series — 1 mark

(c) Error, is in line 3 with the calculation $3\frac{3}{4}$ units.

Marking Guidelines

Correct working Area $= \left|\int_{-1}^{0} x^3\,dx\right| + \int_{0}^{2} x^3\,dx$

$= \left|\left[\frac{x^4}{4}\right]_{-1}^{0}\right| + \left[\frac{x^4}{4}\right]_{0}^{2}$

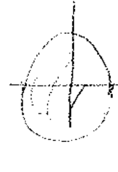
$= \left|\frac{0}{4} - \left(\frac{-1}{4}\right)^4\right| + \left(\frac{2^4}{4} - 0\right)$ /

$= 4\frac{1}{4}$ units^2. /

- You must take absolute value of area under the curve. /

Marking Guideline

6. (a) $V = \pi\int_0^4 x^2\,dy$ $y = 4 - x^2$, $x^2 = 4 - y$



$= \pi\int_0^4 4 - y\,dy$

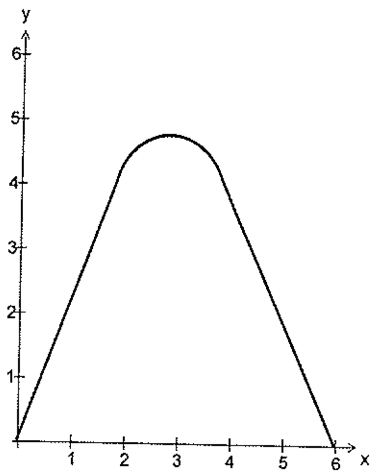
$= \pi\left[4y - \frac{y^2}{2}\right]_0^4$

$= \pi\left[(16 - 8) - (0 - 0)\right]$

$= 8\pi u^3$

3 { 1 for $\int_0^4$
1 for $x^2 = 4 - y$
$\frac{1}{2}$ correct integration
$\frac{1}{2}$ correct answer }

(b)



2 { $\frac{1}{2}$ for identifying 2 straight lines with correct gradients.
$\frac{1}{2}$ identifying parabola
$\frac{1}{2}$ with max turning point }

(c)

(i) $T_8: \; a + 7d = -25$ ①

$T_{14}: \; a + 13d = -49$ ②

② − ① $6d = -24$

$d = -4$

$\therefore a + 7 \times -4 = -25$

$a = 3$

2 { 1 for 2 correct equations
1 mark correct answer }

(ii) $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{14}{2}\left[2 \times 3 + 13 \times -4\right]$

$= -322$

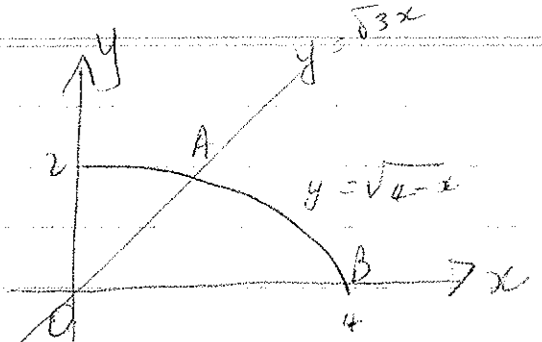
$a = 3$, $d = -4$, $n = 14$

1 for a, d, n

2 answer

Marking Guide

6 (d)



(i) $y = \sqrt{3}x$ & $y = \sqrt{4-x}$ solve simultaneously

$\therefore \sqrt{4-x} = \sqrt{3}x$ ← ½ — 1 mark for correct working

Squaring $4 - x = 3x^2$

$3x + x - 4 = 0$

$(3x+4)(x-1) = 0$

$\therefore x = -4/3,\ 1$

When $x = 1$, $y = \sqrt{3}(1)$

$\therefore A$ is $(1, \sqrt{3})$.

(ii) Area $= \int_0^1 \sqrt{3}x\,dx + \int_1^4 \sqrt{4-x}\,dx \quad ((4-x)^{\frac{1}{2}}$ — 1 mark

$= \left[\frac{\sqrt{3}x^2}{2}\right]_0^1 + \left[\frac{(4-x)^{1\frac{1}{2}}}{-1\frac{1}{2}}\right]_1^4$ — 1 correct integration

$= \left[\frac{\sqrt{3}}{2}(1) - 0\right] - \frac{2}{3}\left[(4-4)^{1\frac{1}{2}} - (4-1)^{1\frac{1}{2}}\right]_1^4$

$= \frac{\sqrt{3}}{2} - \frac{2}{3}\left(0 - 3^{3/2}\right)$ — 3

$= \frac{\sqrt{3}}{2} + \frac{2}{3}\,3\sqrt{3}$

$= \frac{\sqrt{3}}{2} + 2\sqrt{3}$ units^2 — 1 answer

$= \frac{5\sqrt{3}}{2} = 4.3$ units^2 — 4.3 (2½ only)

7(a) $\sin^2 x = \frac{1}{4}$ | Marking Guide

$\sin x = \pm\frac{1}{2}$ | 1 solutions &

$x = 30°, 150°, 210°, 330°$ 2 | 1 answers

(−½ for ommissions)

(b) (i) start 2000 → population of 10 000

i.e. end 2000 after 1 year population is

end 2000 $A_1 = 10\,000 \times 0{\cdot}9 + 500$

end 2001 $A_2 = A_1 \times 0{\cdot}9 + 500$

$= (10\,000 \times 0{\cdot}9 + 500) \times 0{\cdot}9 + 500$ | 1 establishing

$= 10\,000 \times 0{\cdot}9^2 + 500(1 + 0{\cdot}9)$ | pattern

end 2002 $A_3 = A_2 \times 0{\cdot}9 + 500$

$= [10\,000 \times 0{\cdot}9^2 + 500(1+0{\cdot}9)] \times 0{\cdot}9 + 500$

$= 10\,000 \times 0{\cdot}9^3 + 500(1 + 0{\cdot}9 + 0{\cdot}9^2)$ | 2 showing answer

⋮

$A_{20} = 10\,000 \times 0{\cdot}9^{20} + 500(1 + 0{\cdot}9 + 0{\cdot}9^2 + \dots + 0{\cdot}9^{19})$ | ½

$= 10\,000 \times 0{\cdot}9^{20} + 500(\text{sum of geometric series})$

$= 10\,000 \times 0{\cdot}9^{20} + 500\left[\frac{a(1-r^n)}{1-r}\right]$ $a=1$ $n=20$ $r=0{\cdot}9$ | 1 geometric series

$= 10\,000 \times 0{\cdot}9^{20} + 500\left[\frac{1(1-0{\cdot}9^{20})}{1-0{\cdot}9}\right]$

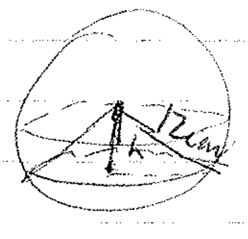
$= 1215{\cdot}766\ldots + 500 \times 8{\cdot}7842$ | 1½

$= 5607\ 853273$

end of 2019 = 5608 people | 2 correct answer

(c) (i) $r = \sqrt{144 - h^2}$ | 1

$V = \frac{1}{3}\pi r^2 h$



$= \frac{1}{3}\pi\left(\sqrt{144-h^2}\right)^2 h$ | 2

$= \frac{\pi}{3}(144h - h^3)$ | 1

(ii) $\frac{dV}{dh} = \frac{\pi}{3}(144 - 3h^2)$ | ½

$\frac{d^2V}{dh^2} = \frac{\pi}{3}(-6h)$ | ½

$= -3\pi h$ | 2 | ½

< 0

∴ Max for all h since $h > 0$ | ½

Marking Guide

Maximum volume when $\frac{dV}{dh} = 0$

$\therefore \frac{\pi}{3}(144 - 3h^2) = 0$

$3h^2 = 144$

$h^2 = 48$

$h = \pm\sqrt{48}$ 1

Since $h > 0$ for length

$h = 4\sqrt{3}$

$\therefore$ Volume $= \frac{\pi}{3}\left(144 \times 4\sqrt{3} - (4\sqrt{3})^3\right)$

$= \frac{\pi}{3}\left(576\sqrt{3} - 192\sqrt{3}\right)$

$= 128\sqrt{3}\,\pi$ cm^3 1