Girraween High.
Yr 12 HALF Yearly. 1998. 2/3 unit (Common)

QUESTION 1 Use a *separate* piece of paper

Marks

a) Find the value of $\dfrac{6}{5\times 7-4}$ correct to 3 significant figures. 2

b) By rationalising the denominator, express $\dfrac{6}{\sqrt{7}+2}$ in the form $a\sqrt{7}-b$. 2

c) Evaluate $\displaystyle\lim_{x\to -2}\frac{x^2+x-2}{x+2}$ 2

d) Differentiate the following with respect to x : 9

(i) e^{7x}

(ii) $\dfrac{5}{x^2}$

(iii) $x^2\log x$

(iv) $\dfrac{e^x}{7x-3}$

e) Find: 9

(i) $\int (3x^2-2x+1)dx$

(ii) $\int \sqrt{6x+5}\,dx$

(iii) $\int_0^1 e^{2x}dx$

QUESTION 2 Use a *separate* piece of paper 13

A, B and C are the points $(5,0)$, $(8,4)$ and $(0,10)$ respectively.

(i) On a number plane draw ΔABC

(ii) Find the slope of the line BC.

(iii) Prove that $AB \perp BC$.

(iv) Calculate the length of AB.

(v) Prove $\Delta ACO \equiv \Delta ACB$, giving reasons for your answer.

(vi) Calculate the area of quadrilateral $ABCO$, where O is the origin.

QUESTION 3 Use a *separate* piece of paper

Marks

a) The probability that Nathan will be present in class on any day is $\frac{3}{4}$ and the probability that Matthew is present is $\frac{19}{20}$. 4

What is the probability that :

(i) they will both be present on any particular day ?

(ii) at least one of them will be absent from class on any particular day ?

b) Use Simpson's Rule to find an approximation for $\int_1^3 f(x)\,dx$ using the values in the table below : 3

x	1	1.5	2	2.5	3
$f(x)$	8.6	11.9	23.7	39.8	56.7

c) A parabola has the equation $y+11=(x+3)^2$. Find : 4

(i) the coordinates of the vertex.

(ii) the focal length.

(iii) coordinates of the focus.

QUESTION 4 Use a *separate* piece of paper

a) The 6th term of an arithmetic sequence is 58 and the 9th term is 112. Find : 7

(i) the common difference.

(ii) the first term.

(iii) the sum of the first 100 terms of the sequence.

b) Find the sum of the infinite geometric series $6+2+\frac{2}{3}+\ldots\ldots\ldots$ 3

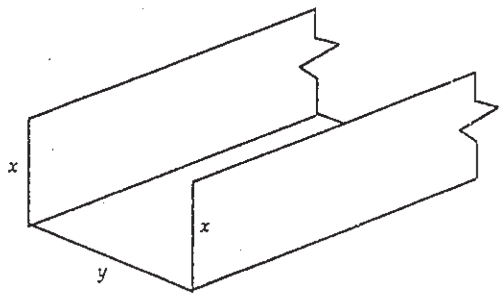
c) Find the equation of the tangent to the curve $y=e^{2x}+x$ at the point $(0,1)$. 4

QUESTION 5 Use a *separate* piece of paper

Marks

a) Find the volume of the solid formed when the curve $y = x^2 - 1$ is rotated about the y axis from $y = -1$ to $y = 3$. 4

b) A home guttering company makes metal gutters from material which is 36 cm wide. The gutter is open at the top and it has a rectangular cross section. 8



(i) Show that $y = 36 - 2x$

(ii) Show that the area, A cm^2, of the rectangular cross section is given by $A = 36x - 2x^2$

(iii) Find the value of x for which the area will be a maximum

QUESTION 6 Use a *separate* piece of paper

Marks

a) Solve $(x+3)^2 + 5(x+3) + 6 = 0$ 4

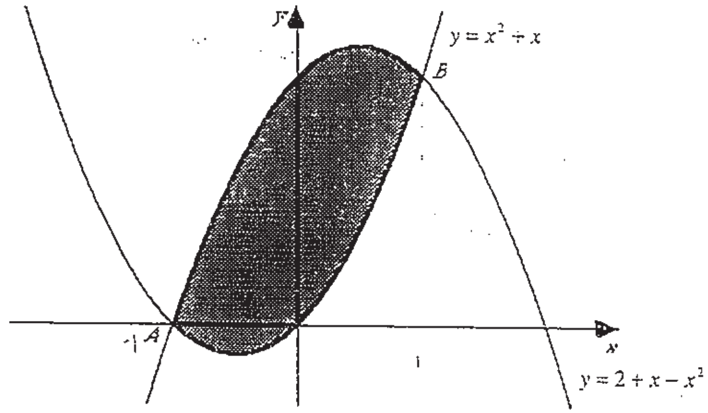
b) If α and β are roots of the equation $3x^2 - 2x - 1 = 0$, find the value of: 4

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2 + \beta^2$

c) 7



The diagram shows the curves $y = x^2 + x$ and $y = 2 + x - x^2$.

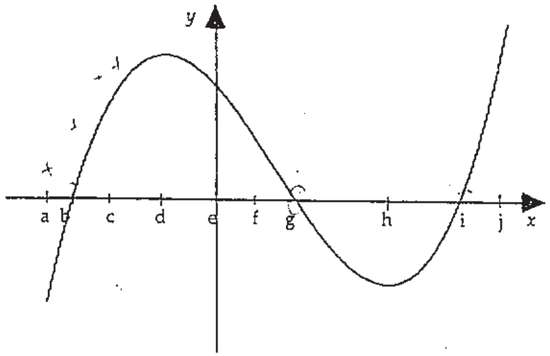
(i) Find the coordinates of A and B.

(ii) Calculate the area of the shaded region.

QUESTION 7 Use a *separate* piece of paper

Marks

a) The diagram shows the graph of $y = f(x)$ — 12



(i) At what values of x on the function is :

(α) $f(x) = 0$?

(β) $f'(x) = 0$?

(γ) $f''(x) = 0$?

(ii) Between which values of x on the function is :

(α) the first derivative positive ?

(β) the second derivative negative ?

(γ) the function decreasing ?

(iii) Sketch the graph of $y = f'(x)$.

b) Given that $\frac{d}{dx}\left\{(\log x)^2\right\} = \frac{2\log x}{x}$, evaluate $\int_1^e \frac{\log x}{x}\,dx$ — 3

Question 1

a) $\frac{\cdot 6}{5\times 7-4} = 0.193548387\ldots$ ②

$= 0.194$ (to 3 sig figs)

b) $\frac{6}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}$

$= \frac{6\sqrt{7}-12}{7-4}$

$= \frac{6\sqrt{7}-12}{3}$

$= 2\sqrt{7}-4$ ②

c) $\lim_{x\to -2}\frac{x^2+x-2}{x+2}$

$\lim_{x\to -2}\frac{(x+2)(x-1)}{(x+2)}$

$= \lim_{x\to -2}(x-1)$

$= -2-1$

$= -3$ ②

d)(i) $f(x) = e^{7x}$

$f'(x) = 7e^{7x}$ ①

(ii) $f(x) = \frac{5}{x^2}$

$f'(x) = \frac{-5(2x)}{x^4}$

$= \frac{-10}{x^3}$ ②

(iii) $f(x) = x^2\log x$

$f'(x) = (x^2)(\frac{1}{x}) + (\log x)(2x)$

$= x + 2x\log x$ ③

(iv) $f(x) = \frac{e^x}{(7x-3)}$

$f'(x) = \frac{(7x-3)(e^x)-(e^x)(7)}{(7x-3)^2}$

$= \frac{7xe^x-3e^x-7e^x}{(7x-3)^2}$

$= \frac{7xe^x-10e^x}{(7x-3)^2}$ ③

e)(i) $\int \ldots dx$

$= x^3 - x^2 + x + c$ ③

(ii) $\int\sqrt{6x+5}\,dx$

$= \int (6x+5)^{\frac{1}{2}}\,dx$

$= \frac{(6x+5)^{\frac{3}{2}}}{(\frac{3}{2})(6)} + c$

$= \frac{(6x+5)^{\frac{3}{2}}}{9} + c$ ③

$= \frac{1}{9}(6x+5)\sqrt{6x+5} + c$

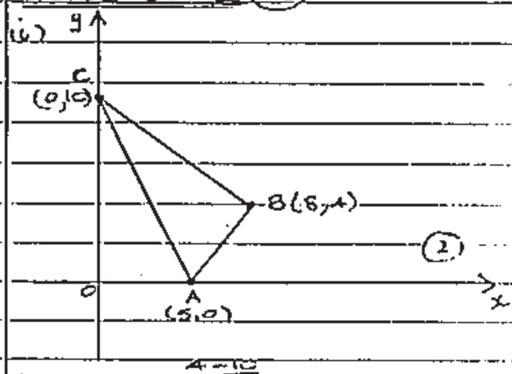
(iii) $\int_0^1 e^{2x}\,dx$

$= [\frac{1}{2}e^{2x}]_0^1$

$= \frac{1}{2}e^2 - \frac{1}{2}e^0 = \frac{1}{2}e^2 - \frac{1}{2}$ ③

Question 2 (13)

(i)



②

(ii) $m_{BC} = \frac{4-10}{8-0}$

$= \frac{-6}{8}$

$= -\frac{3}{4}$ ②

(iii) $m_{AB} = \frac{4-0}{8-5}$

$= \frac{4}{3}$

$m_{AB}\times m_{BC} = \frac{4}{3}\times -\frac{3}{4}$

$= -1$

$\therefore AB \perp BC$ ②

(iv) $d_{AB} = \sqrt{(8-5)^2+(4-0)^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{25}$

$= 5$ units ②

$AC = CA$ (common side) (H)

$OA = AB = 5$ units (given) (S)

$\therefore \triangle ACO \equiv \triangle ACB$ (RHS) ③

(vi) Area $ABCO = 2\times$ Area $\triangle AOC$

$= 2\times\frac{1}{2}\times 5\times 10$

$= 50$ units^2 ②

Question 3 (18)

a)(i) P(both present) $= \frac{3}{4}\times\frac{19}{20}$

$= \frac{57}{80}$ ①

(ii) P(at least 1 absent)

$= 1 -$ P(both present)

$= 1 - \frac{57}{80}$

$= \frac{23}{80}$ ②

b) $\int f(x)\,dx \doteq \frac{h}{3}\{y_0 + 4y_{odd} + 2y_{even} + y_n\}$

$= \frac{0.5}{3}\{8.6 + 4(11.9+39.8) + 2(23.7) + 56.7\}$

$= 53.25$ ③

c) $y = x^2+6x-2$

(i) $x^2+6x = y+2$

$x^2+6x+9 = y+11$

$(x+3)^2 = y+11$

$\therefore$ vertex $(-3, -11)$ ③

(ii) $4a = 1$

$a = \frac{1}{4}$

$\therefore$ focal length $= \frac{1}{4}$ unit ②

(iii) focus $= (-3, -11+\frac{1}{4})$

$= (-3, -10\frac{3}{4})$ ①

Question 4 (14)

a)(i) $T_6 = 58$ $\quad T_9 = 112$

$a+5d = 58 \quad a+8d = 112$

$a+5d = 58$

$a+8d = 112$

$3d = 54$

$d = 18$

$\therefore$ common difference is 18 ③

$a + 5(18) = 58$

$a + 90 = 58$

$a = -32$

$\therefore$ first term is -32

(ii) $S_n = \frac{n}{2}\{2a+(n-1)d\}$

$S_{100} = \frac{100}{2}\{2(-32)+99(18)\}$

$= 50(1718)$

$= 85\,900$ ②

b) $a = 6,\ r = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{6}{1-\frac{1}{3}}$

$= 6\times\frac{3}{2}$

$= 9$ ③

c) $y = e^{2x} + x$

$\frac{dy}{dx} = 2e^{2x} + 1$

when $x = 0$, $\frac{dy}{dx} = 2e^0 + 1$

$= 3$

$\therefore$ slope of tangent is 3

$y - 1 = 3(x-0)$

$y - 1 = 3x$

$3x - y + 1 = 0$

Question 5 (12)

a) $V = \pi\int x^2\,dy$

$y = x^2 - 1$

$x^2 = y + 1$

$\therefore V = \pi\int_{-1}^{3}(y+1)\,dy$

$= \pi\left[\frac{1}{2}y^2 + y\right]_{-1}^{3}$

$= \pi\{\frac{1}{2}(3)^2 + 3 - (\frac{1}{2}(-1)^2 - 1)\}$

$= \pi(\frac{9}{2} + 3 - \frac{1}{2} + 1)$

$= 8\pi$ units^3 ④

b) (i) $W = 36$

$\therefore x + y + x = 36$

$y = 36 - 2x$ (1)

(ii) $A = xy$

$A = x(36 - 2x)$

$A = 36x - 2x^2$ (2)

(iii) $\frac{dA}{dx} = 36 - 4x$

$\frac{d^2A}{dx^2} = -4$

stationary points occur when $\frac{dA}{dx} = 0$

ie $36 - 4x = 0$

$4x = 36$

$x = 9$

when $x = 9$, $\frac{d^2A}{dx^2} = -4 < 0$

$\therefore$ when $x = 9$, Area is a maximum. (5)

Question 6 (15)

a) $(x+3)^2 + 5(x+3) + 6 = 0$

let $m = x + 3$

$m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3$ or $m = -2$

$x + 3 = -3$ $\quad x + 3 = -2$

$x = -6$ $\quad x = -5$

$\therefore x = -6$ or $x = -5$ (4)

b) (i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{2}{3}$ (1)

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-1}{3}$ (1)

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{2}{3}\right)^2 - 2\left(-\frac{1}{3}\right)$

$= \frac{4}{9} + \frac{2}{3}$

$= \frac{10}{9}$ (2)

c) (i) $x^2 + x = 2 + x - x^2$

$2x^2 - 2 = 0$

$2(x^2 - 1) = 0$

$2(x+1)(x-1) = 0$

$x = -1$ or $x = 1$

$\therefore$ A is $(-1, 0)$ and B $(1, 2)$ (3)

(ii) Area $= \int_{-1}^{1} \{(2 + x - x^2) - (x^2 + x)\}\,dx$

$= \int_{-1}^{1} (2 + x - x^2 - x^2 - x)\,dx$

$= \int_{-1}^{1} (2 - 2x^2)\,dx$

$= \left[2x - \frac{2}{3}x^3\right]_{-1}^{1}$

$= \{2(1) - \frac{2}{3}(1)^3\} - \{2(-1) - \frac{2}{3}(-1)^3\}$

$= 2 - \frac{2}{3} + 2 - \frac{2}{3}$

$= 2\frac{2}{3}$ units^2 (4)

Question 5 (15)

a) (i) (α) $f(x) = 0$ at b, g, h (2)

(β) $f'(x) = 0$ at d, h (2)

(γ) $f''(x) = 0$ at ~~...~~ f (1)

(~~...~~)

(ii) (α) $f'(x) > 0$

$a < x < d$, $h < x < i$ (2)

(β) $f''(x) < 0$

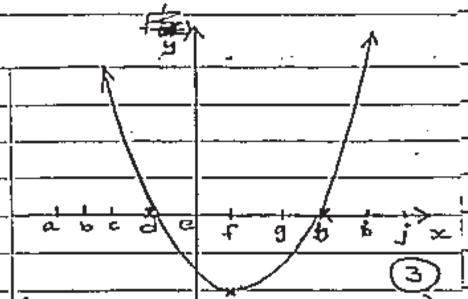
$a < x < f$ (~~...~~) (1)

(depends on answer ~~...~~)

(γ) decreasing

$d < x < h$ (1)

(iii)



(3)

~~...~~

b) $\int_1^e \frac{\log x}{x}\,dx = \frac{1}{2}\int_1^e \frac{2\log x}{x}\,dx$

$= \frac{1}{2}\left[(\log x)^2\right]_1^e$

$= \frac{1}{2}\{(\log e)^2 - (\log 1)^2\}$

$= \frac{1}{2}(1 - 0)$

$= \frac{1}{2}$ (3)