FINAL MARK

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

HALF YEARLY EXAMINATION

2014

ANSWERS COVER SHEET

Name: ____

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
1-3	/3	√			√				√
4	/1	√							√
5,6	/2	√	√		√				√
7	/1	√			√				√
8,9	/2	√	√		√				√
10	/1	√	√						√
Total M.C.	/10								
11	/15	√	√						√
	/15								
12	/15	√			√				√
	/15								
13a	/3	√							√
b	/5	√						√	√
c	/3	√	√		√				√
d	/7	√		√	√				√
	/18								
14ab	/7	√						√	√
c	/4	√	√					√	√
d	/4	√	√		√				
	/15								
15a	/4	√	√		√				√
b	/11	√	√		√	√			√
	/15								
16a	/8	√		√	√				√
b	/4				√				
	/12								
TOTALS	/100	/100	/46	/15	/64	/11		/16	/100

HSC Outcomes **Mathematics**

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts.

H2 constructs arguments to prove and justify results.

H3 manipulates algebraic expressions involving logarithmic and exponential functions.

H4 expresses practical problems in mathematical terms based on simple given models.

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6 uses the derivative to determine the features of the graph of a function.

H7 uses the features of a graph to deduce information about the derivative.

H8 uses techniques of integration to calculate areas and volumes.

H9 communicates using mathematical language, notation, diagrams and graphs.

GIRRAWEEN HIGH SCHOOL

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2014

MATHEMATICS

Time allowed - Two hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are on laminated sheet provided.
- Board-approved calculators may be used.
- Answers to multiple choice questions should be written on your working paper.
- Each of questions 11-16 is to be returned on a *separate* piece of paper clearly marked Question 11 , Question 12 , etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

PART A: Questions 1 – 10 (Multiple Choice) *Write the letter corresponding to the correct answer on your working pages.*

Question 1

$$\sum_{n=1}^{15} 5n - 3) =$$

(A) 72 **(B)** 77 **(C)** 483 **(D)** 555

Question 2 The limiting sum of a series with common ratio $\frac{4}{5}$ is 15. The THIRD term of the series is

(A) 3 **(B)** $\frac{12}{5}$ **(C)** $\frac{48}{25}$ **(D)** $\frac{192}{125}$

Question 3 The probability of selecting a red King from a standard deck of cards (4 of each kind, 13 of each suit, 52 in total) is

(A) $\frac{1}{52}$ **(B)** $\frac{1}{26}$ **(C)** $\frac{1}{13}$ **(D)** $\frac{1}{4}$

Question 4 $(2,7)$ is the midpoint of $(-1,5)$ and

(A) $(\frac{1}{2},6)$ **(B)** $(5,9)$ **(C)** $(-4,3)$ **(D)** $(\frac{3}{2},3)$

Question 5 The derivative of $y = \ln(x^2 + 1)$ is

(A) $\frac{1}{x^2+1}$ **(B)** $\frac{x}{x^2+1}$ **(C)** $\frac{2x}{x^2+1}$ **(D)** $\frac{x^2}{x^2+1}$

Question 6 The derivative of $y = e^{-x^2}$ is

(A) e^{-x^2} **(B)** e^{-2x} **(C)** $-x^2e^{-x^2}$ **(D)** $-2xe^{-x^2}$

Question 7 $\int \sqrt{5x-1}\,.\,dx =$

(A) $\frac{1}{5}(5x-1)\sqrt{5x-1}$ **(B)** $\frac{15}{2}(5x-1)\sqrt{5x-1}$ **(C)** $\frac{2}{15}\sqrt{5x-1}$ **(D)** $\frac{2}{15}(5x-1)\sqrt{5x-1}$

Question 8 The quadratic equation $-2x^2 + 4x = 11$ has

(A) No real roots **(B)** Two equal roots **(C)** Rational roots **(D)** Irrational roots

Question 9 A parabola has focus $(0,0)$ and directrix $y = 4$. Its equation is

(A) $x^2 = 8(2-y)$ **(B)** $y^2 = 8(x-2)$ **(C)** $x^2 = 8(y-2)$ **(D)** $y^2 = 8(2+x)$

Question 10 $y = ln(3x-1)$ has a vertical asymptote at

(A) $x = 0$ (B) $x = 1$ (C) $x = \frac{1}{3}$ (D) $x = 3$

PART B: Show all necessary working on your answer pages.

Question 11 ***(15 Marks)*** **Marks**

(a) Find $\frac{dy}{dx}$ if

(i) $y = x^2e^{-x}$ **2**

(ii) $y = \frac{lnx}{x^2+1}$ **2**

(iii) $y = (lnx)^3$ **3**

(iv) $y = \sqrt[3]{e^{2x}}$ **2**

(b) Find $\int_0^1 e^{3x} - 2x.dx$ **3**

(c) Differentiate $y = e^{x^3}$. Hence find $\int x^2e^{x^3}.dx$ **3**

Question 12 ***(15 Marks)***

(a) From a standard deck of cards (4 of each kind, 13 of each suit, 52 cards altogether) three cards are drawn *without replacement* to see how many Kings are drawn.

(i) Draw a probability tree of this experiment. **3**

(ii) Find the probability of drawing two Kings. **2**

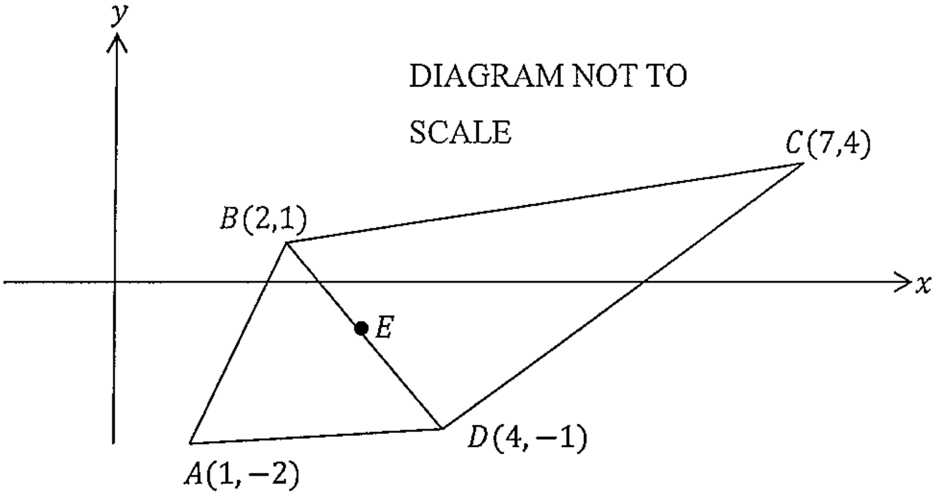
(iii) Find the probability of drawing *at least* two Kings. **2**

Question 12 continues on the next page

Question 12 *(continued)* **Marks**

(b) $A(1,-2), B(2,1), C(7,4)$ and $D(4,-1)$ are points on the number plane.

E is the midpoint of BD. *(See diagram.)*



(i) Find the equation of the line AC. **2**

(ii) Show that $AC \perp BD$. **2**

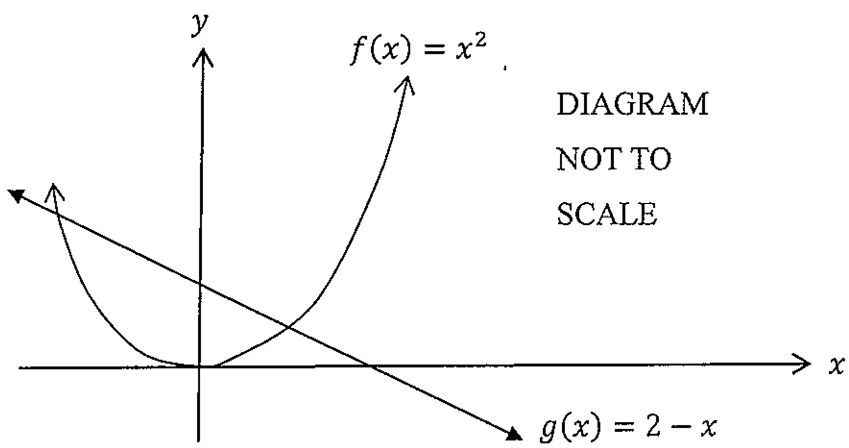
(iii) Find the coordinates of E and show that it lies on the line AC. **3**

(iv) Hence or otherwise show that $ABCD$ is a kite. **1**

Question 13 *(18 marks)*

(a) Find the locus of the set of points that are equally distant from the point $(1,-2)$ and the line $y = 4$. **3**

(b) On the graph below are graphs (not to scale) of the functions $f(x) = x^2$ and $g(x) = 2 - x$.



(i) Find the x coordinates of the points of intersection of the two curves. **2**

(ii) Find the area enclosed by the two curves. **3**

Question 13 continues on the next page

Question 13 *(continued)* **Marks**

(c) Find the equation of the tangent to $y = 3e^{2x}$ at the point where $x = 1$. **3**

(d) Stephanie is training for a swimming race. She swims $2km$ in the first week and each week she swims $300m$ more than the previous week.

(i) How far does she swim in the tenth week? **2**

(ii) How many weeks is it before she swims $10km$ in a week? **2**

(iii) How many weeks will it be before she has accumulated at least $50km$ of training? **3**

Question 14 *(15 Marks)*

(a) A surveyor wishes to find the approximate area of this beachside property. *(See diagram).*

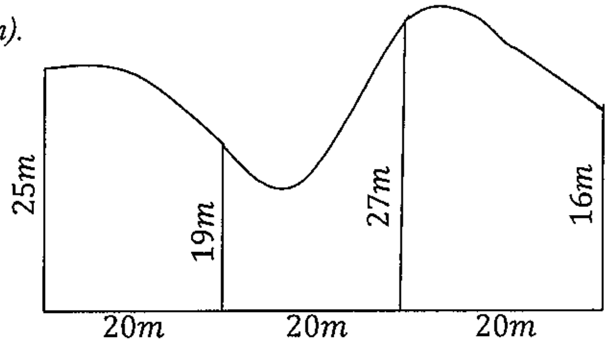


DIAGRAM NOT TO SCALE

Find the approximate area using three application of the trapezoidal rule. **3**

(b) (i) Use two application of Simpson's Rule (four sub intervals) to find the area enclosed by the curve $y = 2x^2 + x - 1$, the x axis and the lines $x = 1$ and $x = 3$. **3**

(ii) The area worked out using Simpson's Rule is exactly the same as the area which would be found using integration. Why is this the case? **1**

(c) Find the volume of the solid of revolution formed by rotating the curve $y = \ln(x)$ about the y axis between $y = 1$ and $y = 3$. **4**

(d) If $y = 5e^{3x} - 2e^{-x}$, show that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$. **4**

Question 15 *(15 Marks)*

(a) (i) If $y = xe^x$ find $\frac{dy}{dx}$. **2**

(ii) Hence find $\int xe^x . dx$ **2**

Question 15 continues on the next page

Question 15 *(continued)* **Marks**

(b) For the curve $y = e^{-x^2}$

(i) Explain why $y = e^{-x^2}$ has no x intercepts. **1**

(ii) Find the y intercept of $y = e^{-x^2}$ **1**

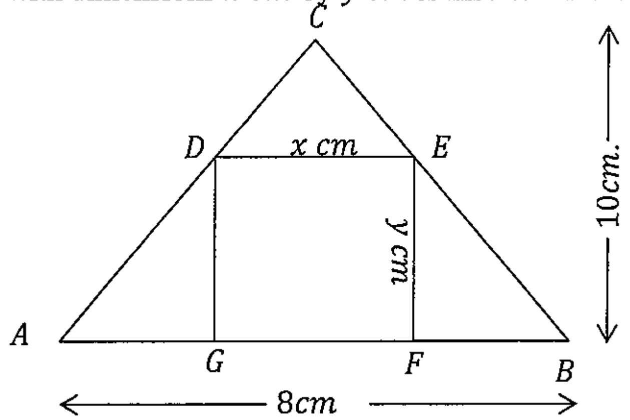
(iii) Find any stationary points for $y = e^{-x^2}$ and determine their nature. **4**

(iv) Find any points of inflexion for $y = e^{-x^2}$. **3**

(v) Sketch the graph of $y = e^{-x^2}$ showing all relevant features. **2**

Question 16 *(12 marks)*

(a) A scalene triangle has a breadth of $8cm$ and a height of $10cm$. A rectangle with dimensions $x\ cm$ by $y\ cm$ is inserted in the triangle as shown.

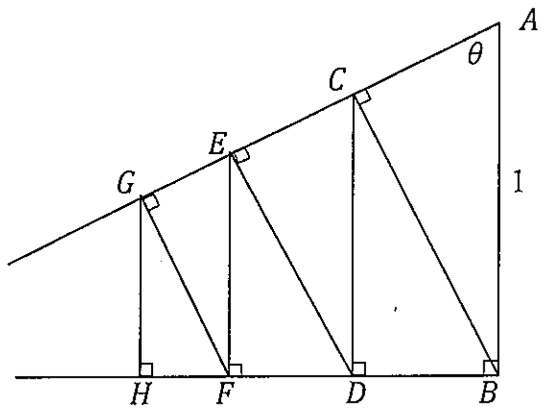


(i) Show that the area of the triangle ABC in terms of x and y is given by $Area = \frac{1}{2}x(10 - y) + \frac{1}{2}y(8 - x) + xy$ **2**

(ii) Show that $y = \frac{1}{4}(40 - 5x)$ **2**

(iii) Find the maximum area of the rectangle. **4**

(b) On the diagram below, $AB = 1$ and $\angle CAB = \theta$ as shown.



(i) Show that $CB = sin\theta$ and $CD = sin^2\theta$ **2**

(ii) Show that the total distance

$AB + BC + CD + DE + EF + FG + GH + HI + \cdots = sec\theta(sec\theta + tan\theta)$ **2**

HERE ENDETH THE EXAMINATION!

Year 12 2 Unit Maths
Midyear 2014
— Solutions —

(1) D (2) C (3) B (4) B (5) C (6) D (7) D (8) A (9) A
(10) C

Q. (11)(a)(i) $y = x^2 e^{-x}$

$\frac{dy}{dx} = u'v + v'u$

$= 2xe^{-x} - e^{-x} \times x^2$ 1

$= 2xe^{-x} - x^2 e^{-x}$ 1 $\underline{2}$

or $= \underline{xe^{-x}(2-x)}$

(ii) $y = \frac{\ln x}{x^2+1}$

$\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$

$= \frac{\frac{1}{x}(x^2+1) - 2x\ln x}{(x^2+1)^2}$ $\times x$ 1 $\underline{2}$

$= \frac{x^2+1 - 2x^2\ln x}{x(x^2+1)^2}$ 1

(iii) $y = [\ln x]^3$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= 3[\ln x]^2 \times \frac{1}{x}$ 1 $\underline{3}$

$= \frac{3[\ln x]^2}{x}$ 1

p.2

Q. (11)(a)(iv) $y = \sqrt[3]{e^{2x}}$

$\therefore y = e^{\frac{2}{3}x}$ 1

$\frac{dy}{dx} = \frac{2}{3}e^{\frac{2}{3}x}$ 1 $\underline{2}$

(b) $\int_0^1 e^{3x} - 2x \, dx$

$= \left[\frac{1}{3}e^{3x} - x^2\right]_0^1$ 1

$= \left(\frac{1}{3} \times e^{3\times1} - 1^2\right) - \left(\frac{1}{3} \times e^{3\times0} - 0^2\right)$ 1

$= \frac{e^3}{3} - 1 - \frac{1}{3}$

$= \frac{e^3 - 4}{3}$ 1 $\underline{3}$

$\doteqdot 5.36$ (2DP)

(c) $y = e^{x^3}$

By the chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= e^{x^3} \times 3x^2$

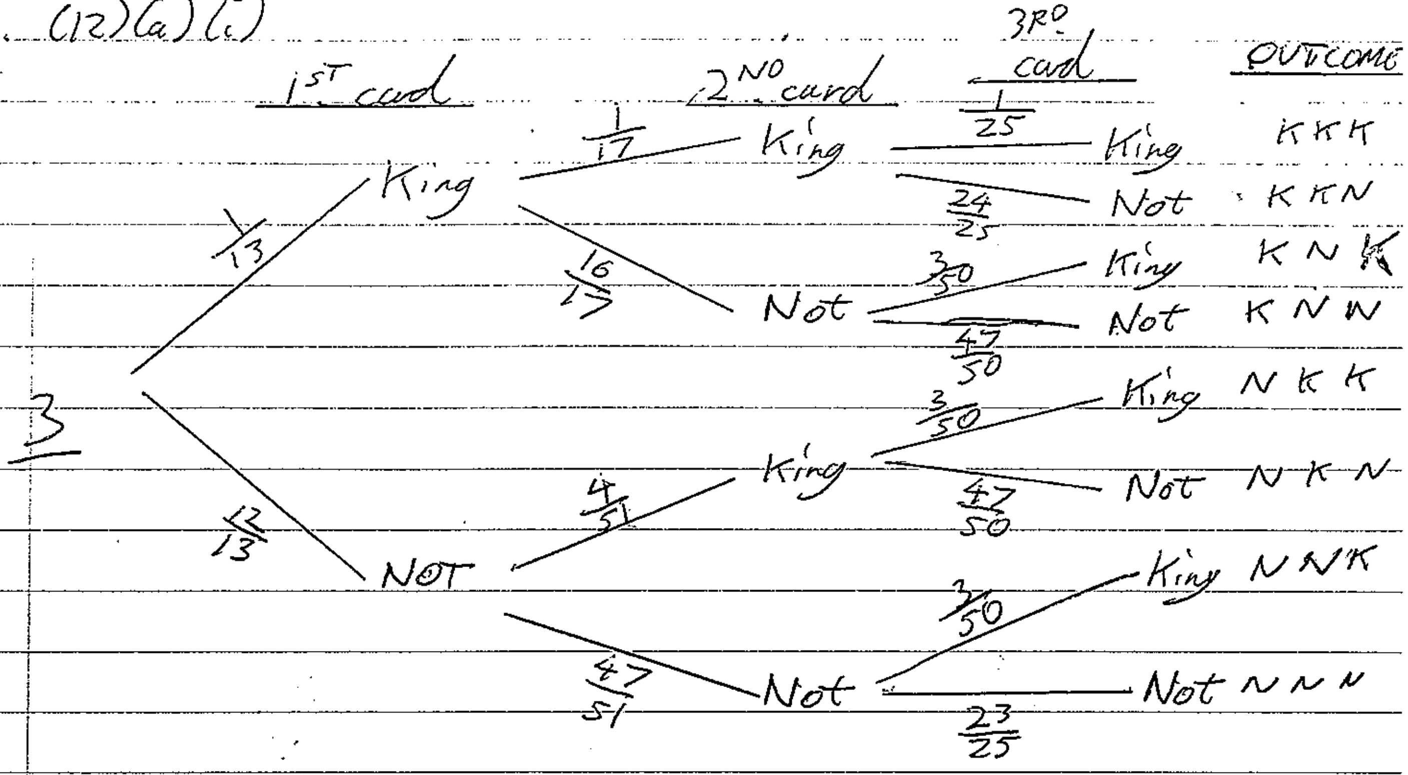
$= 3x^2 e^{x^3}$ 1 $\underline{3}$

Hence $\int x^2 e^{x^3} \, dx$ 1

$= \frac{1}{3}\int 3x^2 e^{x^3} \, dx$

$= \frac{1}{3}e^{x^3} + c$ 1

Q. (12)(a)(i)



(ii) P (2 Kings)

$$= \frac{1}{13} \times \frac{1}{17} \times \frac{24}{25} + \frac{1}{13} \times \frac{16}{17} \times \frac{3}{50} + \frac{12}{13} \times \frac{4}{51} \times \frac{3}{50}$$ |

$$+ \frac{12}{13} \times \frac{47}{51} \times \frac{3}{50}$$

$$= \frac{72}{5525}.$$ | 2

(iii) $P(3 \text{ kings}) = \frac{1}{13} \times \frac{1}{17} \times \frac{1}{25}$

$$= \frac{1}{5525}.$$ |

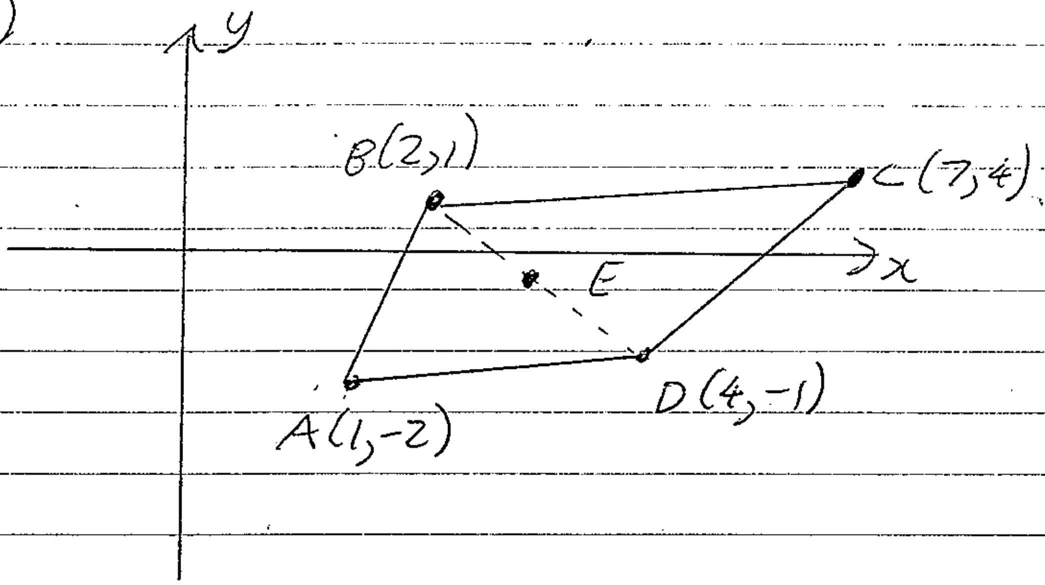
Total Probability of at least 2 Kings

= Pr (2 Kings) + Pr (3 Kings) 2

$$= \frac{72}{5525} + \frac{1}{5525}$$ |

$$= \frac{73}{5525}.$$

Q. (12)(b)



(i) Equation of line AC:

$$mAC = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4+2}{7-1}$$
$$= 1.$$

1

$\therefore$ Equation of AC is $y - y_1 = m(x - x_1)$ 2
$$y+2 = 1(x-1)$$ 1
$$y+2 = x-1$$
$$y = x-3.$$

(ii) $$mBD = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1-1}{4-2}$$
$$= -1.$$

1

$mAC \times mBD = 1 \times -1$
$= -1.$ 1 2

$\therefore AC \perp BD.$

PTO →

Q. (12)(b) [continued]:

(iii) E is midpoint of BD

$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2+4}{2}, \frac{1-1}{2}\right)$ 1

$E = (3, 0)$. 1

Show E lies on AC:

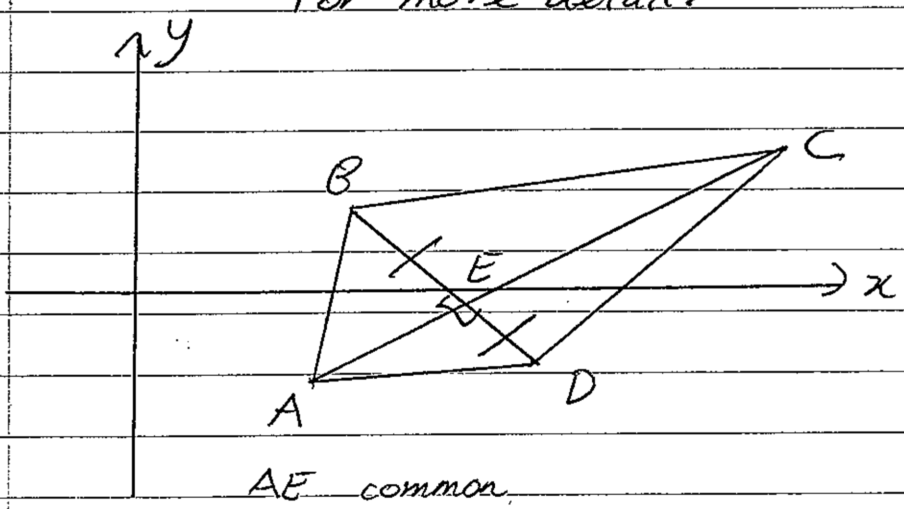
$y = x - 3$ $\underline{3}$

$0 = 3 - 3$ 1

True $\Rightarrow$ E lies on AC.

(iv) As diagonal AC bisects diagonal BD at right angles, ABCD is a kite. $\underline{1}$ Sufficient.

For more detail:



AE common.

$\triangle ABE \equiv \triangle ADE$ [SAS].

$\therefore AB = AD$ [matching sides, $\triangle ABE \equiv \triangle ADE$].

$\angle BAE = \angle DAE$ [" $\angle$'s " "].

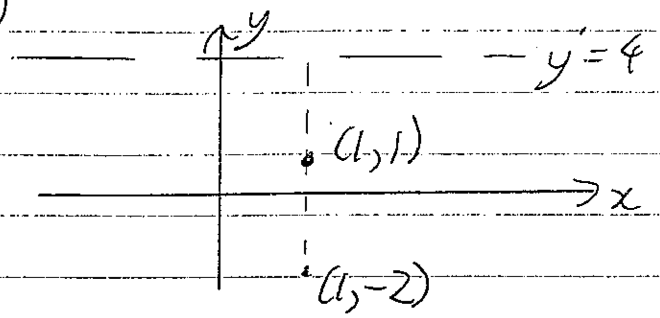
$\therefore \triangle ABC \equiv \triangle ADC$ [SAS].

$BC = DC$ [matching sides, $\triangle ABC \equiv \triangle ADC$].

$\therefore$ ABCD is a kite.

p.6

Q. (13)(a)



Note: Equidistant from POINT [S] & LINE [D]

$\therefore$ Parabola

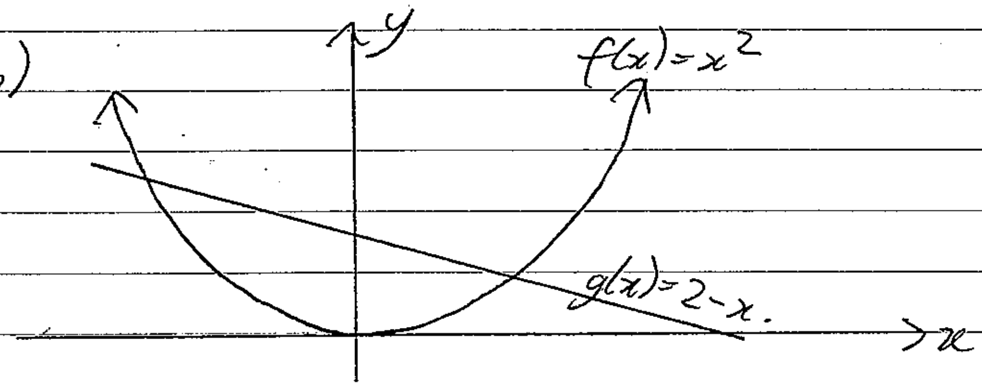
$S = (1, -2)$ directrix $= y = 4$. Vertex $= (1, 1)$ 1

$a = -3$. 1

$\therefore$ By $(x-p)^2 = 4a(y-q)$

$(x-1)^2 = -12(y-1)$ 1 3

(b)



(i) $x^2 = 2 - x$. 1

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$. 1 2

(ii) Area $= \int_{-2}^{1} (2-x) - x^2 \, dx$

$= \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_{-2}^{1}$ 1

$= \left(2 \times 1 - \frac{1}{2} \times 1^2 - \frac{1}{3} \times 1^3\right) - \left(2 \times -2 - \frac{1}{2} \times (-2)^2 - \frac{1}{3} \times (-2)^3\right)$ 1 3

$= \frac{7}{6} - -\frac{10}{3} = \frac{9}{2}$ square units 1

Q. (13)(c) $y = 3e^{2x}$

Where $x = 1$, $y = 3 \times e^{2 \times 1}$
$= 3e^2$. 1

m of tangent: $\frac{dy}{dx} = 6e^{2x}$

At $x = 1$, $\frac{dy}{dx} = 6e^{2 \times 1}$
$= 6e^2$. 1

3

$y - y_1 = m(x - x_1)$
$y - 3e^2 = 6e^2(x - 1)$ 1
$y - 3e^2 = 6e^2x - 6e^2$
$y = 6e^2x - 3e^2$
OR $6 \times e^2x - y - 3e^2 = 0$

(d)(i) Stephanie is swimming
$2000 + 2300 + 2600 + \ldots$
Arithmetic progression: $a = 2000$ $d = 300$] 1 2
By $T_n = a + (n-1)d$
10^{th} week $= 2000 + 9 \times 300$
$= 4700m$. 1

(ii) By $T_n = a + (n-1)d$
$10\,000 = 2000 + (n-1) \times 300$ 1
$8000 = 300n - 300$
$8300 = 300n$ 2
$27\frac{2}{3} = n$. 1
It will be 28 weeks before she breaks 10km in a week.

(iii) By $S_n = \frac{n}{2}[2a + (n-1)d]$
$50\,000 = \frac{n}{2}[2 \times 2000 + (n-1) \times 300]$ 1
$50\,000 = 2000n + 150n^2 - 150n$

PTO →

Q. (13)(d)(iii) [continued].

$$150n^2 + 1850n - 50\,000 = 0$$

$$3n^2 + 37n - 1000 = 0$$

By $n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$n = \frac{-37 \pm \sqrt{37^2 - 4 \times 3 \times -1000}}{2 \times 3}$$

$$n = \frac{-37 - \sqrt{13269}}{6} \quad \text{or} \quad n = \frac{-37 + \sqrt{13\,269}}{6}$$

$n = \cancel{-6.769}$ or $n = 13.07..$

→ No negative weeks

3

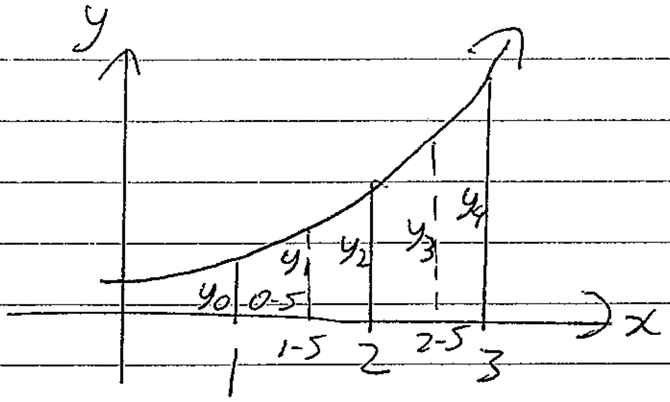
∴ It will be 14 weeks before Stephanie accumulates 50km of training.

Q. (14) (a) $A \doteqdot \frac{h}{2}\left(y_0+2(y_1+y_2)+y_3\right)$ 1 or appropriate table or other way of setting out trapezoidal rule.

$= \frac{20}{2}\left(25+2\times(19+27)+16\right)$ 1 $\underline{3}$

$= 1330 m^2$ 1

(b)



$h = 0.5 \quad y_0 = 2\times1^2+1-1 = 2$

$y_1 = 2\times1.5^2+1.5-1 = 5$

$y_2 = 2\times2^2+2-1 = 9$

$y_3 = 2\times2.5^2+2.5-1 = 14$

$y_4 = 2\times3^2+3-1 = 20$ 1

$A = \frac{h}{3}\left(y_0+y_4+4(y_1+y_3)+2y_2\right)$

$= \frac{0.5}{3}\left[2+20+4(5+14)+2\times9\right]$ 1 $\underline{3}$

$= \frac{58}{3}$ square units. 1

(ii) Simpson's Rule is based on approximating the curve using a parabola. As $y=2x^2+x-1$ is a parabola it will be completely accurate. $\underline{1}$

PTO →

Q. (14)(c) $y = \ln x$ about y axis

$\therefore x = e^y$

$$V = \pi \int_{y=a}^{y=b} x^2 \, dy$$

$$= \pi \int_1^3 (e^y)^2 \, dy$$

$$= \pi \int_1^3 e^{2y} \, dy$$

$$= \pi \left[\frac{1}{2} e^{2y} \right]_1^3$$

$$= \frac{\pi}{2} \left[e^6 - e^2 \right]$$

4

$\doteqdot$ 622.1 cubic units

(d) If $y = 5e^{3x} - 2e^{-x}$,

$$\frac{dy}{dx} = 15e^{3x} + 2e^{-x}$$

$$\frac{d^2y}{dx^2} = 45e^{3x} - 2e^{-x}$$

4

LHS:

$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y$$

$$= 45e^{3x} - 2e^{-x} - 2(15e^{3x} + 2e^{-x}) - 3(5e^{3x} - 2e^{-x})$$

$$= 45e^{3x} - 2e^{-x} - 30e^{3x} - 4e^{-x} - 15e^{3x} + 6e^{-x}$$

$= 0$

$=$ RHS QED.

p.11

Q. (15)(a)(i) $y = xe^x$

$\frac{dy}{dx} = u'v \quad + v'u$

$= e^x \quad + e^x x'$ 1

$= e^x \quad + xe^x.$ 1 $\underline{2}$

(ii) ∵ As, if $y = xe^x$, $\frac{dy}{dx} = e^x + xe^x$

∴ $\int e^x + xe^x \, dx = xe^x$ 1

$\int e^x \, dx + \int xe^x \, dx = xe^x$ $\underline{2}$

$\int xe^x \, dx = xe^x - \int e^x \, dx$

$\underline{\int xe^x \, dx = xe^x - e^x + C.}$ 1

(b) (i) $y = e^{-x^2}$ ⇒ e to the power of ANY real no. >0 $\underline{1}$

∴ $y > 0$ for all real x.

(ii) y intercept: $x = 0$

$y = e^{-0^2}$

$= 1.$ $\underline{1}$

y intercept = (0, 1)

(iii) $y = e^{-x^2}$

$\frac{dy}{dx} = -2xe^{-x^2}$ 1

$\frac{dy}{dx} = 0 \Rightarrow -2xe^{-x^2} = 0$

As $e^{-x^2} \neq 0$, $-2x = 0$ $\underline{4}$

$x = 0$ 1

Nature of turning point: $\frac{d^2y}{dx^2} = -2e^{-x^2} + -2x \times -2xe^{-x^2}$

$= (4x^2 - 2)e^{-x^2}$

At $x = 0$, $\frac{d^2y}{dx^2} = (4 \times 0^2 - 2)e^{-0^2}$

$= -2.$ 1

→ $\underline{\text{Local MAXIMUM at } (0, 1)}$ 1 [actually GLOBA[L]

Q. (15)(b)(iv) Points of inflexion:

$\frac{d^2y}{dx^2} = 0$ & changes sign.

$(4x^2-2)e^{-x^2} = 0$

$4x^2-2 = 0$ [as $e^{-x^2} \neq 0$].

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \frac{\pm 1}{\sqrt{2}}$ 1

At $x = \frac{\pm 1}{\sqrt{2}}$, $y = e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$

$= e^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{e}}$. 1

Possible points of inflexion at $\left(\pm\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$ 3

Noting that $\frac{d^2y}{dx^2} < 0$ for $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

[from testing turning point].

Testing left of $x = -\frac{1}{\sqrt{2}}$.

At $x = -1$, $\frac{d^2y}{dx^2} = (4x^2-2)e^{-x^2}$

$= (4\times(-1)^2-2)e^{-1^2}$

$= \frac{2}{e} > 0$ 1

Testing right:

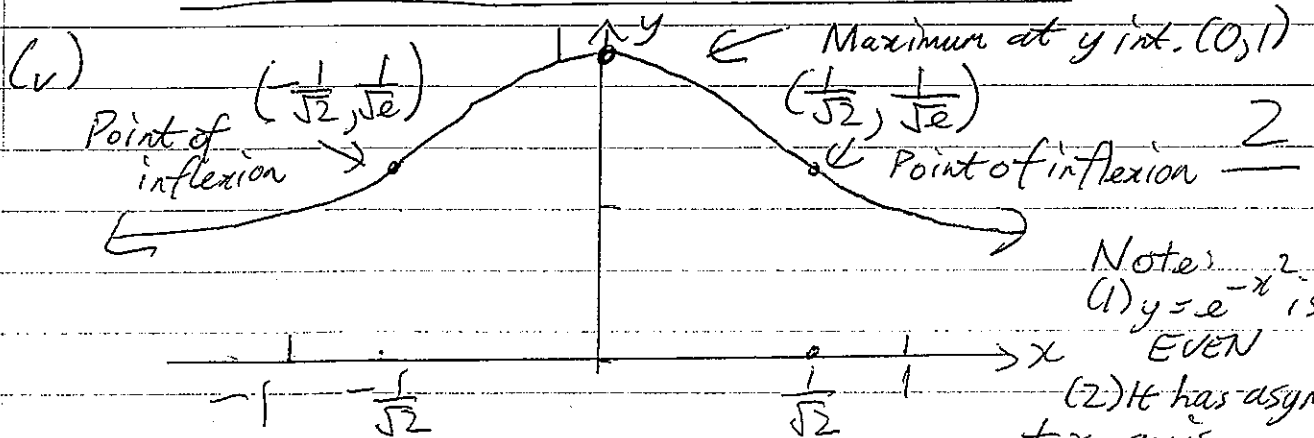
At $x = 1$, $\frac{d^2y}{dx^2}$

$= (4\times 1^2-2)e^{-1^2}$

$= \frac{2}{e} > 0$.

$\therefore$ Points of inflexion at $\left(\pm\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$.

(v)



2

Note:
(1) $y = e^{-x^2}$ is EVEN
(2) It has asymptote at x axis.

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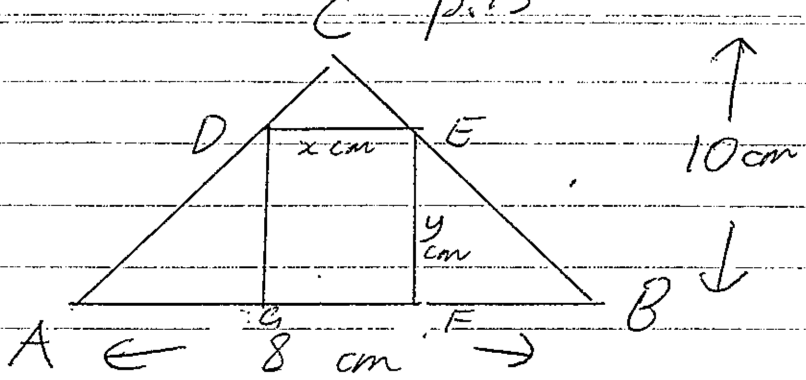
Q. (16)(a)



(i) Area top triangle DEC $= \frac{1}{2} Bh$

$= \frac{1}{2} x (10 - y)$. 1

Combined area $\triangle ADG$ and $\triangle EFB$

$= \frac{1}{2}(8 - x) \times y$. 1 2

Area rectangle DEFG $= xy$.

$\therefore$ Total area $\triangle ABC$

$= \frac{1}{2} x (10 - y) + \frac{1}{2} y (8 - x) + xy$

(ii) Using Area $\triangle ABC = \frac{1}{2} Bh$

$= \frac{1}{2} \times 8 \times 10$

$= 40 \text{ cm}^2$.

$\frac{1}{2} x (10 - y) + \frac{1}{2} y (8 - x) + xy = 40$ [from (i)] 1

$5x - \cancel{\frac{1}{2} xy} + 4y - \cancel{\frac{1}{2} xy} + \cancel{xy} = 40$

$5x + 4y = 40$ 2

$4y = 40 - 5x$.

$\therefore y = \frac{1}{4}(40 - 5x)$ 1

(iii) Maximum rectangle area

Area $= xy$

$A = \frac{1}{4} x (40 - 5x)$ 1

$A = 10x - \frac{5}{4} x^2$.

PTO →

Q. (16)(a)(iii) [continued]:

Finding maximum area.

$$\frac{dA}{dx} = 10 - \frac{5}{2}x.$$

Finding where $\frac{dA}{dx} = 0$

$$10 - \frac{5}{2}x = 0$$

$$x = 4.$$

Testing to see if maximum: $\frac{d^2A}{dx^2} = -\frac{5}{2}$

$\therefore$ Maximum area when $x = 4$.

Note: Could do $A = 10x - \frac{5}{4}x^2$

$$= 10 \times 4 - \frac{5}{4} \times 4^2$$

$$= \underline{20\text{ cm}^2}$$

OR $y = \frac{1}{4}(40 - 5x)$

$= \frac{1}{4}(40 - 5 \times 4)$ or mark here.

$= 5\text{ cm}.$

Area $= xy$

$= 4 \times 5$

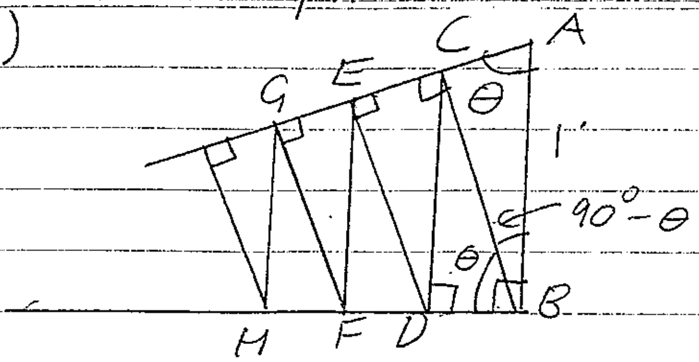
$= \underline{20\text{cm}^2}.$

$\therefore$ <u>Maximum area of rectangle = 20cm²</u>

4

p.15

Q. (16)(b)



(i) Using $\triangle ABC$, $\sin\theta = \dfrac{CB}{AB}$

$= \dfrac{CB}{1}$

$\therefore \underline{\sin\theta = CB.}$ (1) 1

$\angle CBA = 90° - \theta$ [$\angle$ sum $\triangle ABC$].

$\therefore \angle CBD = \theta$ (adjacent $\angle$'s).

Using $\triangle CBD$

$\sin\theta = \dfrac{CD}{CB}$

$\sin\theta = \dfrac{CD}{\sin\theta}$ 1 2

$\times$ BS by $\sin\theta$

$\underline{\sin^2\theta = CD.}$

$\therefore \underline{CB = \sin\theta,\ CD = \sin^2\theta.}$

(ii) $AB + BC + CD + DE + \dots$ 3

$= 1 + \sin\theta + \sin^2\theta + \sin^3\theta + \dots$ 1

By the limiting sum of a GP, $a = 1$, $r = \sin\theta$

$AB + BC + \dots = \dfrac{1}{1 - \sin\theta} \cdot \dfrac{\times\, 1 + \sin\theta}{\times\, 1 + \sin\theta}$ 1

$= \dfrac{1 + \sin\theta}{1 - \sin^2\theta}$

$= \dfrac{1 + \sin\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} + \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\cos\theta} \times \dfrac{\sin\theta}{\cos\theta}$

$= \sec^2\theta + \sec\theta\tan\theta$

$= \sec\theta(\sec\theta + \tan\theta)$ 1

$=$ RHS QED

END OF SOLUTIONS!