Name______________________________Teacher____________________

GOSFORD HIGH SCHOOL

2012

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK 2

MATHEMATICS

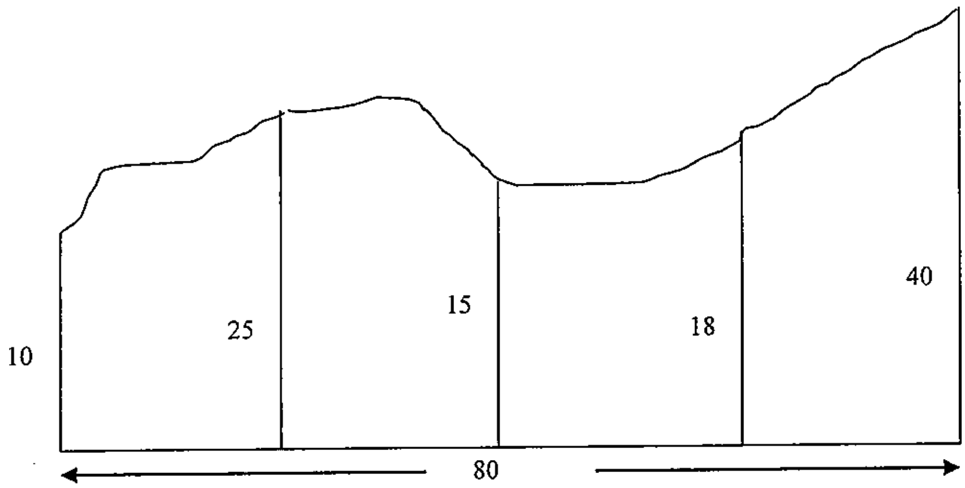
Duration- 90 minutes plus 5 minutes reading time

Multiple choice 6 questions worth **1 mark each.** (Write your answer on your own paper, **not** on the test paper)	/6
Applications of differentiation.	/22
Integration	/32
TOTAL	/60

Part A

Multiple choice. Write your answers on **your** own paper not on the question sheet.

Question 1.



The field drawn is to have its area approximated by applying Simpson's Rule. The value of h is:

(A) 80 (B) 40 (C) 20 (D) 10

Question 2.

For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

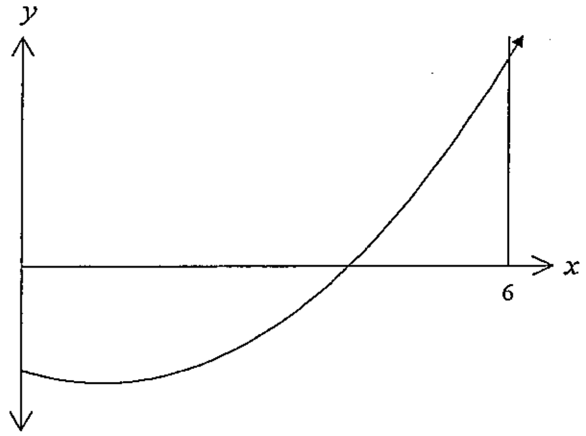
(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

Question 3.



What is the correct expression for the area bounded by the x-axis and the curve $y = x^2 - 2x - 8$ between $0 \le x \le 6$?

(A) $A = \int_0^5 x^2 - 2x - 8dx + \left|\int_5^6 x^2 - 2x - 8dx\right|$

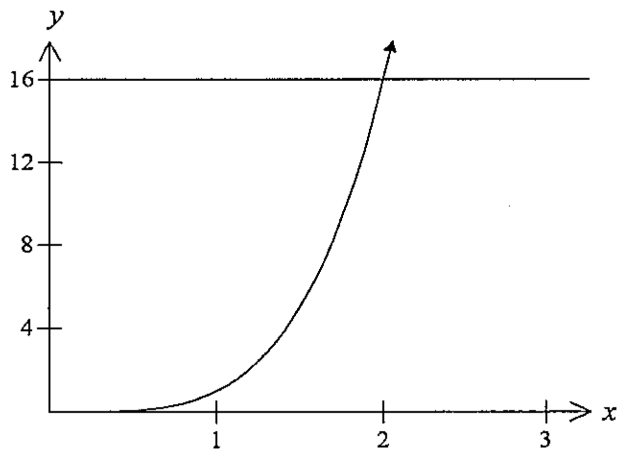
(B) $A = \int_0^4 x^2 - 2x - 8dx + \left|\int_4^6 x^2 - 2x - 8dx\right|$

(C) $A = \left|\int_0^5 x^2 - 2x - 8dx\right| + \int_5^6 x^2 - 2x - 8dx$

(D) $A = \left|\int_0^4 x^2 - 2x - 8dx\right| + \int_4^6 x^2 - 2x - 8dx$

Question 4.

A region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 16$.



Which of the following expressions is correct for the volume of the solid of revolution when this region is rotated about the y-axis?

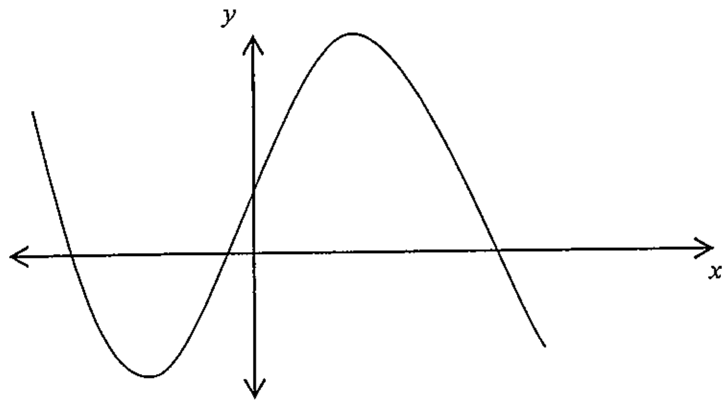
(A) $V = \pi\int_0^2 x^8 dx$

(B) $V = \pi\int_0^{16} x^8 dx$

(C) $V = \pi\int_0^2 y^{\frac{1}{2}} dy$

(D) $V = \pi\int_0^{16} y^{\frac{1}{2}} dy$

Question 5



The graph shows $y = f'(x)$. The graph of $y = f(x)$ has:

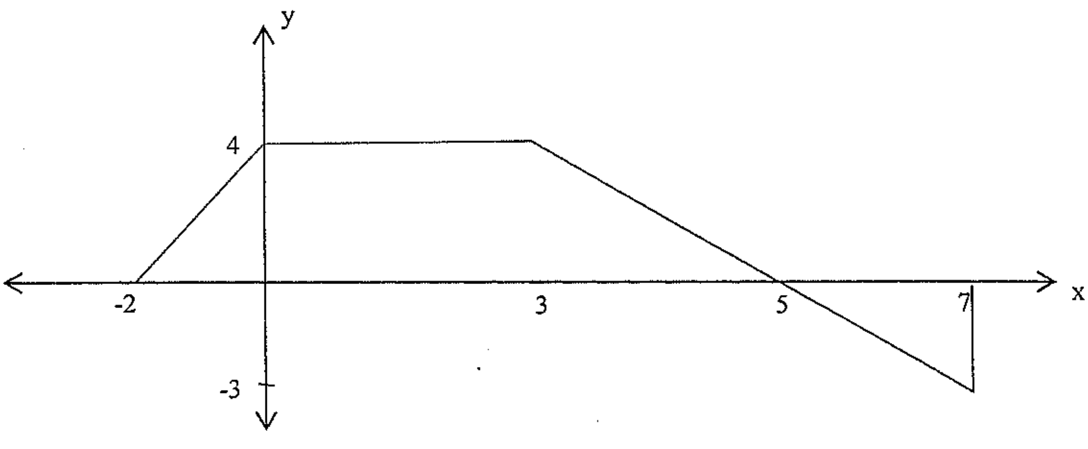
(A) A maximum and a minimum turning point.

(B) Two maximum and a minimum turning point

(C) Two minimum and a maximum turning point.

(D) Two maximum turning points and a horizontal point of inflection.

Question 6



The graph of $y = f(x)$ is given above. The value of $\int_{-2}^{7} f(x)\,dx$ is:

(A) 23
(B) 17
(C) 19
(D) 13

PART B

Question 1.

a) Find the following indefinite integrals.

i) $\int x^3 + 6x - 1 \, dx$ **(1)**

ii) $\int (3x-2)^5 \, dx$ **(1)**

iii) $\int \frac{x^3 + 5x^2 - 1}{x^2} \, dx$ **(2)**

iv) $\int x\sqrt{x} \, dx$ **(2)**

b) Evaluate

i) $\int_1^3 6x^2 - 1 \, dx$ **(2)**

ii) $\int_{-3}^3 \sqrt{9 - x^2} \, dx$ **(1)**

Question 2. (Start a new page)

a) A continuous curve $y = f(x)$ has the following properties over the domain $0 \le x \le 6$: $f(x) \ge 0, f'(x) \ge 0$ and $f''(x) \ge 0$. Sketch a curve satisfying these conditions over the given domain. **(1)**

b) If $\frac{d^2y}{dx^2} = 12x + 6$ and $\frac{dy}{dx} = 1$ at the point $(-1,-2)$ find the equation of the curve. **(3)**

c) The table shows the values of $x\log_e x$ for 5 values of x. By using Simpson's rule find an approximation (to 1 decimal place) for the area between the curve $y = x\log_e x$ and the x axis from $x = 1$ to $x = 5$. **(2)**

x	1	2	3	4	5
$x\log_e x$	0	1.4	3.3	5.5	8.1

d) i) Find the points of intersection for the curves $y = x^2$ and $x = y^2$ **(1)**

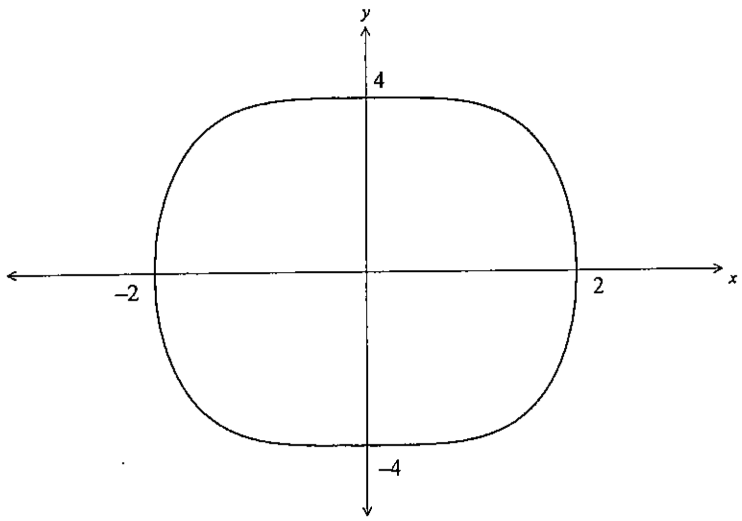
ii) Hence find the area between the two curves. **(2)**

Question 3 (Start a new page)

a) The region bounded by the x axis and the curve $y = \frac{1}{x^2}$ from $x = 1$ to $x = b$ is $\frac{1}{2}$ a square unit. What is the value of b? **(2)**

b) Find the area enclosed between the curve $y = x(1-x)$ and the x axis from $x = 0$ to $x = 2$ **(3)**

c) The sketch shows the curve $y^2 + x^4 = 16$.



The area enclosed within the curve is rotated about the x axis.
Find the volume of the solid of revolution so formed. **(2)**

d) By using calculus find the volume of the solid generated by rotating the line $2x + y = 1$ about the y axis from $y = -1$ to $y = 1$. **(2)**

Question 4 (Start a new page)

a) Use the trapezoidal rule with 5 function values to approximate $\int_0^1 \sqrt{1 - x^2}\, dx$ (give your answer correct to 2 decimal places) **(3)**

b)

i) On the same number plane sketch the curves $y = 2 - x^2$ and $y = x^2$. **(2)**

ii) Given that the two curves intersect at (1,1) in the first quadrant calculate the area of the region, **in the first quadrant,** between the two curves and the x axis. **(4)**
(correct to 3 decimal places)

Question 5. (Start a new page)
Consider the curve given by $y = 1 + 3x - x^3$ for $-2 \le x \le 3$.

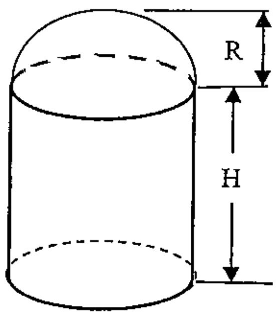
a) Find the turning points and determine their nature. **(3)**

b) What are the coordinates of any points of inflexion? **(2)**

c) Sketch the curve for $-2 \le x \le 3$. **(2)**

d) What are the maximum and minimum values of y for $-2 \le x \le 3$. **(2)**

Question 6. (Start a new page)



The diagram illustrates the shape of a new wheat silo, comprising a hemisphere on top of an open cylinder. It is to be erected on an existing concrete slab, which acts as the base of the silo. The volume is to be 576π cubic metres. The builder charges \$10 per square metre to build the cylinder and \$20 per square metre to build the hemisphere.

i) Given that the radius of the hemisphere is R metres and the height of the cylinder is H metres write down an expression for the volume of the silo in terms of H and R.

(you may use the formulae $V = \pi r^2 h, V = \frac{4}{3}\pi r^3$) **(1)**

ii) Show that the cost of construction, P dollars, is given by $P = 20\pi R(2R + H)$.

(you may use the formulae $A = 4\pi r^2, A = 2\pi rh$) **(2)**

iii) Hence prove that $P = \frac{80\pi R^2}{3} + \frac{11520\pi}{R}$. **(3)**

iv) Hence find the minimum cost of construction, to the nearest dollar. **(3)**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

MATHEMATICS : ASSESSMENT TASK 2. 2012

PART A

1) C 2) A 3) D
4) D 5) B 6) B

PART B

Q1) a) i) $\int x^3+6x-1\,dx$
$= \frac{x^4}{4}+3x^2-x+c$

ii) $\int (3x-2)^5\,dx$
$= \frac{(3x-2)^6}{18}+c$

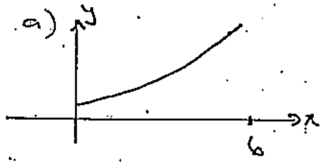
iii) $\int \frac{x^3+5x^2-1}{x^2}\,dx$
$= \int x+5-\frac{1}{x^2}\,dx$
$= \frac{1}{2}x^2+5x+\frac{1}{x}+c$

iv) $\int x\sqrt{x}\,dx$
$= \int x^{3/2}\,dx$
$= \frac{2x^{5/2}}{5}$
$= \frac{2\sqrt{x^5}}{5}+c$

b) $\int_1^3 6x^2-1\,dx$
$\left[2x^3-x\right]_1^3$
$= (54-3)-(2-1)$
$= 50$

ii) $\int_{-3}^{3}\sqrt{9-x^2}\,dx$
$= \frac{1}{2}\pi\times 3^2$
$= \frac{9\pi}{2}$

Q2) a)



b) $\frac{d^2y}{dx^2}=12x+6$

$\frac{dy}{dx} = 6x^2+6x+c$

$x=-1,\ \frac{dy}{dx}=1$
$1 = 6-6+c$
$\therefore c=1$
$\frac{dy}{dx}=6x^2+6x+1$

$y = 2x^3+3x^2+x+c_1$
passes through $(-1,-2)$
$-2 = -2+3-1+c_1$
$-2 = c_1$
$\therefore y = 2x^3+3x^2+x-2$

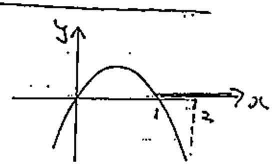
c) $A = \frac{h}{3}\{1^{st}+last+2\times evens+4\times odds\}$
$= \frac{1}{3}\{0+8.1+2\times3.3+4(1.4+5.5)\}$
$= \frac{1}{3}\{8.1+6.6+27.6\}$
$= \frac{1}{3}\times 42.3$
$= 14.1$

d) $y=x^2$ ... (1)
$x=y^2$ ... (2)
Sub (2) into (1)
$y=y^4$
$y^4-y=0$
$y(y^3-1)=0$
$y=0,1$
$\therefore x=0,1$
$\therefore$ pts. $(0,0)$ $(1,1)$

ii) $A=\int_0^1 \sqrt{x}-x^2\,dx$
$= \left[\frac{2x^{3/2}}{3}-\frac{x^3}{3}\right]_0^1$
$= \left(\frac{2}{3}-\frac{1}{3}\right)-0$
$= \frac{1}{3}$ square units

Q3) a) $\int_1^b \frac{1}{x^2}\,dx = \frac{1}{2}$
$\left[-\frac{1}{x}\right]_1^b = \frac{1}{2}$
$-\frac{1}{b}-(-1) = \frac{1}{2}$
$-\frac{1}{b} = -\frac{1}{2}$
$b=2$

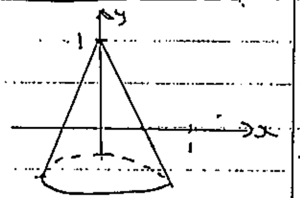
b)



$A=\int_0^1 x-x^2\,dx+\left|\int_1^2 x-x^2\,dx\right|$
$= \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1+\left|\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_1^2\right|$
$= \left(\frac{1}{2}-\frac{1}{3}\right)-0+\left|\left(2-\frac{8}{3}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)\right|$
$= \frac{1}{6}+\left|-\frac{5}{6}\right|$
$= 1$

c) $\pi\int_{-2}^{2} 16-x^4\,dx$
$= 2\pi\int_0^2 16-x^4\,dx$
$= 2\pi\left[16x-\frac{x^5}{5}\right]_0^2$
$= 2\pi\left[\left(32-\frac{32}{5}\right)-0\right]$
$= 2\pi\times\frac{128}{5}$
$= \frac{256\pi}{5}$ cubic units

d)



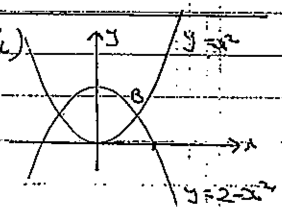
$= \pi\int_{-1}^{1}\left(\frac{1-y}{2}\right)^2 dy$
$= \frac{\pi}{4}\int_{-1}^{1} 1-2y+y^2\,dy$
$= \frac{\pi}{4}\left[y-y^2+\frac{y^3}{3}\right]_{-1}^{1}$
$= \frac{\pi}{4}\left[\left(1-1+\frac{1}{3}\right)-\left(-1-1-\frac{1}{3}\right)\right]$
$= \frac{\pi}{4}\left(\frac{1}{3}-\left(-2\frac{1}{3}\right)\right)$
$= \frac{\pi}{4}\left(\frac{8}{3}\right)$
$= \frac{2\pi}{3}$ cubic units

Q4. a) $\sqrt{1-x^2}$

x	0	.25	.5	.75	1
y	1	.968	.866	.661	0

$A \doteq \frac{h}{2}\{ends+2\times middle\}$
$= \frac{0.25}{2}\{0+1+2(2.495)\}$
$= 0.125\{5.99\}$
$\doteq 0.75$

b) i)

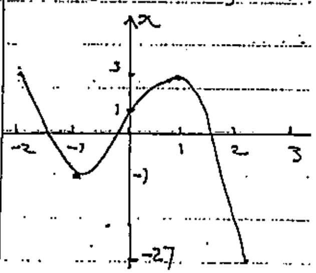


ii) $A=\int_0^1 x^2\,dx+\int_1^{\sqrt{2}} 2-x^2\,dx$
$= \left[\frac{x^3}{3}\right]_0^1+\left[2x-\frac{x^3}{3}\right]_1^{\sqrt{2}}$
$= \left(\frac{1}{3}-0\right)+\left(2\sqrt{2}-\frac{2\sqrt{2}}{3}\right)-\left(2-\frac{1}{3}\right)$
$= \frac{1}{3}+\frac{4\sqrt{2}}{3}-1\frac{2}{3}$
$= 0.552.$

Q5) a) $y=1+3x-x^3$
$y' = 3-3x^2$
$y'' = -6x$
turning pts. $y'=0$
$3-3x^2=0$
$3(1-x^2)=0$
$3(1-x)(1+x)=0$
$x=1$, $y=3$ or $x=-1$, $y=-1$
$f''(1)<0$ $f''(-1)>0$
$\therefore (1,3)$ is max $\therefore (-1,-1)$ [min]

b) possible point of inflexion $y''=0$
$-6x=0$
$x=0$
$y=1$
$f''(-1)>0$
$f''(1)<0$
$\therefore$ change in concavity
$\therefore$ pt of inflexion $(0,1)$

c) $x=-2$, $x=3$
$y=3$, $y=-17$



d) max 3, min -17

Q6)

i) $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \pi r^2 h$

$V = \frac{2}{3}\pi R^3 + \pi R^2 H$

ii) $A = \frac{1}{2}(4\pi R^2) + 2\pi R H$

$= 2\pi R^2 + 2\pi R H$

$\therefore P = 20 \times 2\pi R^2 + 10 \times 2\pi R H$

$= 20\pi R(2R + H)$

iii) from (i)

$576\pi = \frac{2}{3}\pi R^3 + \pi R^2 H$

$576 = \frac{2}{3}R^3 + R^2 H$

$1728 = 2R^3\ 3R^2 H$

$H = \frac{1728 - 2R^3}{3R^2}$

Sub into (ii)

$P = 20\pi R\left(2R + \frac{1728 - 2R^3}{3R^2}\right)$

$= 20\pi R\left(\frac{6R^3 + 1728 - 2R^3}{3R^2}\right)$

$= 20\pi R\left(\frac{4R^3 + 1728}{3R^2}\right)$

$= 20\pi R\left(\frac{4R}{3} + \frac{1728}{3R^2}\right)$

$P = \frac{80\pi R^2}{3} + \frac{11520\pi}{R}$

iv) $P = \frac{80\pi R^2}{3} + \frac{11520\pi}{R}$

$\frac{dP}{dR} = \frac{160\pi R}{3} - \frac{11520\pi}{R^2}$

$\frac{d^2P}{dR^2} = \frac{160\pi}{3} + \frac{23040\pi}{R^3}$

min $\frac{dP}{dR} = 0$

$\frac{160\pi R}{3} - \frac{11520\pi}{R^2} = 0$

$\frac{160\pi R}{3} = \frac{11520\pi}{R^2}$

$160\pi R^3 = 34560\pi$

$R^3 = 216$

$R = 6$ (min $\frac{d^2P}{dR^2} > 0$ for all

$\therefore$ min cost when $R = 6$

$\therefore P = \frac{80\pi \times 6^2}{3} + \frac{11520\pi}{6}$

$= 960\pi + 1920\pi$

$= 2880\pi$

$= 9047.78$

$= \$9048$