NAME ______________________________

GOSFORD HIGH SCHOOL

2015

Higher School Certificate

MATHEMATICS

Assessment Task 2

General Instructions

- Reading Time 5 minutes
- Working Time 90 minutes
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided
- Use multiple choice answer sheet provided for Section 1 (Q 1 to 5)
- For Section 2 (Q 6 – 9) show relevant mathematical reasoning and/or calculations
- Total marks 65

Section 1 (5 marks)

Use the multiple choice answer sheet provided

1. Which of the following is NOT equal to $tan\emptyset$

(A) $sec^2\emptyset - 1$

(B) $\cot(90° - \emptyset)$

(C) $\tan(180° + \emptyset)$

(D) $\frac{sin\emptyset}{cos\emptyset}$

2. The range of the function $y = \sqrt{4 - x^2}$ is

(A) $y \geq 0$

(B) $-2 \leq y \leq 2$

(C) $y \leq 2$

(D) $0 \leq y \leq 2$

3. The equation $(2cosx + 1)sinx = 0$ for $0° \leq x \leq 360°$ has

(A) 2 solutions

(B) 3 solutions

(C) 4 solutions

(D) 5 solutions

4. The equation of a parabola with focus $(0, -1)$ and focal length 2 units has four possibilities. Which of the following could be the equation of this parabola.

(A) $x^2 = 8(y + 1)$

(B) $(y + 1)^2 = 8(x + 2)$

(C) $x^2 = -8(y + 1)$

(D) $y^2 = 8(x - 3)$

5. Consider the continuous function $y = f(x)$ in the interval $a \leq x \leq b$.
It is known that $f(x)$, $f'(x)$ and $f''(x)$ are all positive for all values of x in the given domain.

The **exact area** under the curve and above the x axis between the ordinates at $x = a$ and $x = b$ is defined to be (E) square units.

A student correctly uses the Trapezoidal rule to approximate the given area to be (M) square units.

Which one of the following statements is true

(A) E > M
(B) E < M
(C) E = M
(D) None of the above, as the number of strips used by the student would need to be known to make the above comparisons between E and M.

Section II

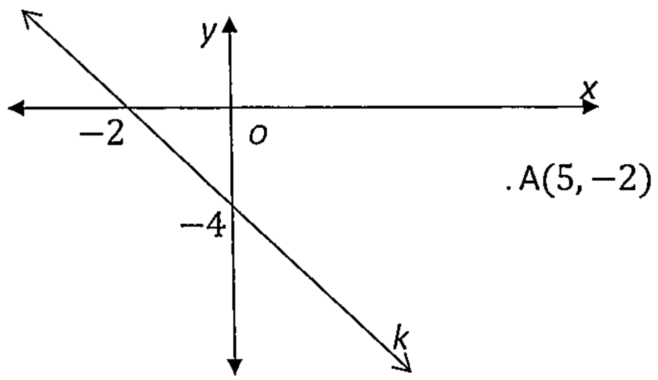
Question 6 (15 marks) Start a new page

a) Evaluate $\sqrt{\dfrac{3.4}{5.6\times10^{-9}}}$, writing your answer correct to 3 significant figures. (1)

b) Find integers a and b such that $(3-\sqrt{2})^2 = a - b\sqrt{2}$ (2)

c) Solve $3x^2 - x - 1 = 0$ writing your answers in exact form. (2)

d) The point A and the line k are shown on the number plane below



(i) Find the equation of the line k, writing your answer in general form (2)

(ii) Find the perpendicular distance from the point $A(5,-2)$ to the line k.
Leave your answer in surd form. (2)

e) $G(x) = 3x^2 - px + p - 3$, where p is a Real constant.

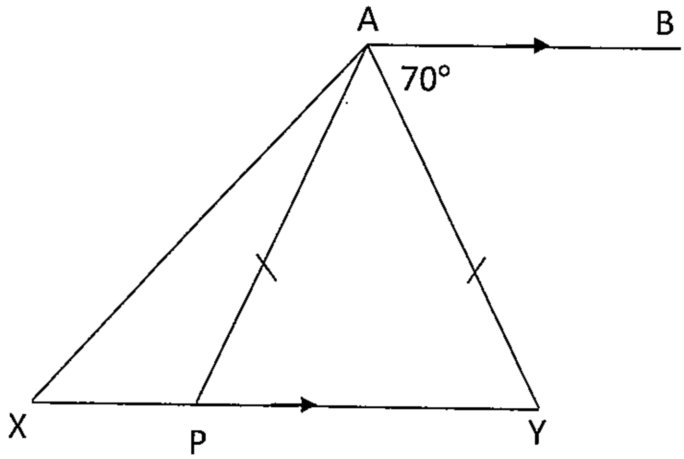
Find p if $G(x)$ has

(i) one root equal to 2 (1)

(ii) roots which are reciprocals (2)

(f) In the diagram $AB \parallel XY$, $\angle BAY = 70°$ and P lies on XY such that $AP = AY$.

Find $\angle PAY$, giving reasons. (3)



Question 7 (15 marks) Start a new page

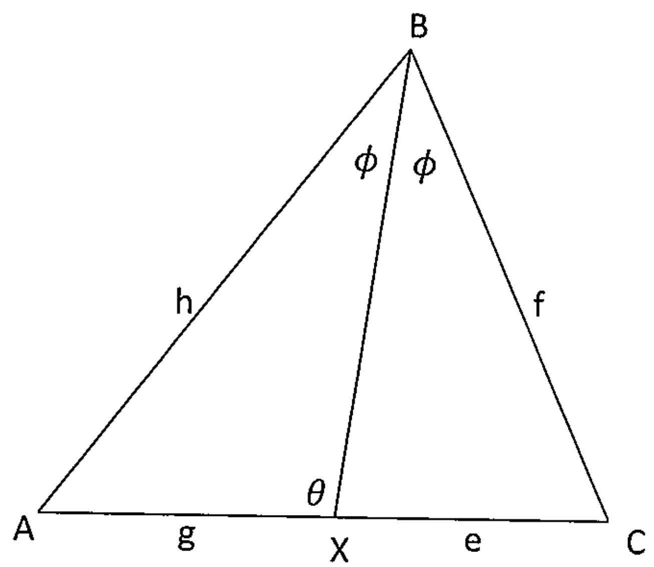
a) If $sec\beta = -\frac{7}{5}$ and $tan\beta > 0$, find the exact value of $sin\beta$ (2)

b) Show that $\frac{d}{dx}\left[x\sqrt{1-2x}\right] = \frac{1-3x}{\sqrt{1-2x}}$ (3)

c) Find the equation of the tangent to the parabola $y = x^2 - 2x + 4$ at the point where the tangent is parallel to the line $y = 2x + 1$ (3)

d) Given that m is a real number, show that the line $y = mx - 3m^2$ touches the parabola $x^2 = 12y$ (3)

e) In triangle ABC, $\angle BXA = \theta$, AX $= g$, CX $= e$, AB $= h$, BC $= f$ and BX bisects $\angle ABC$ such that $\angle ABX = CBX = \phi$.



(i) With consideration to ΔABX, show that $\frac{sin\phi}{sin\theta} = \frac{g}{h}$ (1)

(ii) Hence, and with similar consideration to ΔCBX, prove that $\frac{e}{f} = \frac{g}{h}$ (3)

Question 8 (15 marks) Start a new page

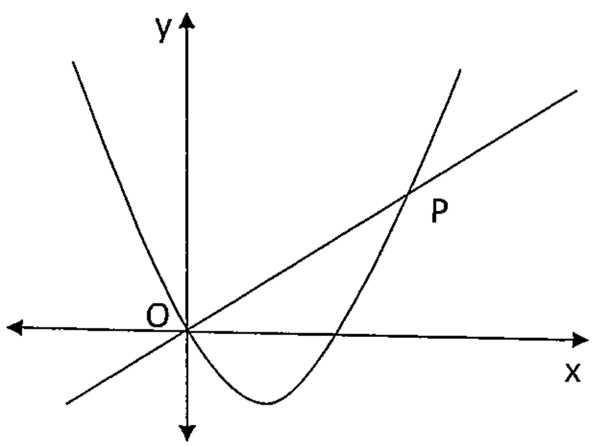
(a) Find $\int (1+2x)^3\,dx$ (1)

(b) Evaluate $\int_1^4 \frac{x+1}{\sqrt{x}}\,dx$ (3)

(c) The area bounded by the curve $y = x^2 + 1$, the coordinate axes and the line $x = 4$ is rotated about the x axis.

Find the volume of the solid generated. (4)

(d) The straight line $y = 3x$ and the parabola $y = 2x^2 - 5x$ meet at the points O and P as indicated on the number plane below.



(i) Find the coordinates of P (1)

(ii) Find the area enclosed between the line and the parabola (3)

Question 9 (15 marks) Start a new page

a) A curve is defined by the equation $y = 7 + 4x^3 - 3x^4$

i) Find the coordinates of the two stationary points on the curve. (2)

ii) Find all values of x for which $\frac{d^2y}{dx^2} = 0$ (2)

iii) Determine the nature of the stationary points. (3)

iv) Sketch the curve in the domain $-1 \leq x \leq 2$ (3)

b) **NOTE**

For this question students are given the following two formulae

Volume of the Cylinder $\boldsymbol{V = \pi r^2 h}$

Surface Area of the Cylinder $\boldsymbol{S = 2\pi r^2 + 2\pi r h}$

A closed cylinder, with radius *(r)* and height *(h)* has a volume *(V)* of 2156 cm^3.

(i) Show that the surface area(*S*) of the cylinder is given by $S = 2\pi r^2 + \frac{4312}{r}$. (1)

(ii) Hence, using differential calculus and *the approximation* $\boldsymbol{\pi = \frac{22}{7}}$,

find the radius (*r*) so that the surface area of the cylinder is a minimum. (4)

e) (i) Copy and complete the following table for $y = 2^x - 1$

x	0	1	2	3	4
y					

(1)

(ii) Use Simpson's Rule with 5 function values to evaluate

$\int_0^4 (2^x - 1)\,dx$ (2)

NAME ____

ANSWER SHEET FOR SECTION 1 - Multiple Choice

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D

Table of Standard Integrals

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \qquad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \qquad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \qquad x > 0$

e) (i) $y = 2^x - 1$

x	0	1	2	3	4
y	0	1	3	7	15

i) $\int_0^4 (2^x - 1)\,dx = \frac{1}{3}\left[0 + 15 + 2\times 3 + 4(1+7)\right]$

$= \frac{1}{3}\left[53\right]$

$= \frac{53}{3}$

Question 9

a) (i) $y = 7 + 4x^3 - 3x^4$

$\frac{dy}{dx} = 12x^2 - 12x^3$

For stationary points $\frac{dy}{dx} = 0$

$12x^2 - 12x^3 = 0$

$12x^2(1 - x) = 0$

$\therefore x = 0, 1$

Stat. pts at $(0,7)$ & $(1,8)$

(ii) $\frac{d^2y}{dx^2} = 24x - 36x^2$

$24x - 36x^2 = 0$

$12x(2 - 3x) = 0$

$x = 0, \frac{2}{3}$

(iii) when $x = 0$, $\frac{d^2y}{dx^2} = 0$

when $x < 0$, say $x = -1$, $\frac{d^2y}{dx^2} < 0$

when $x > 0$, say $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} > 0$

$\therefore$ Concavity change

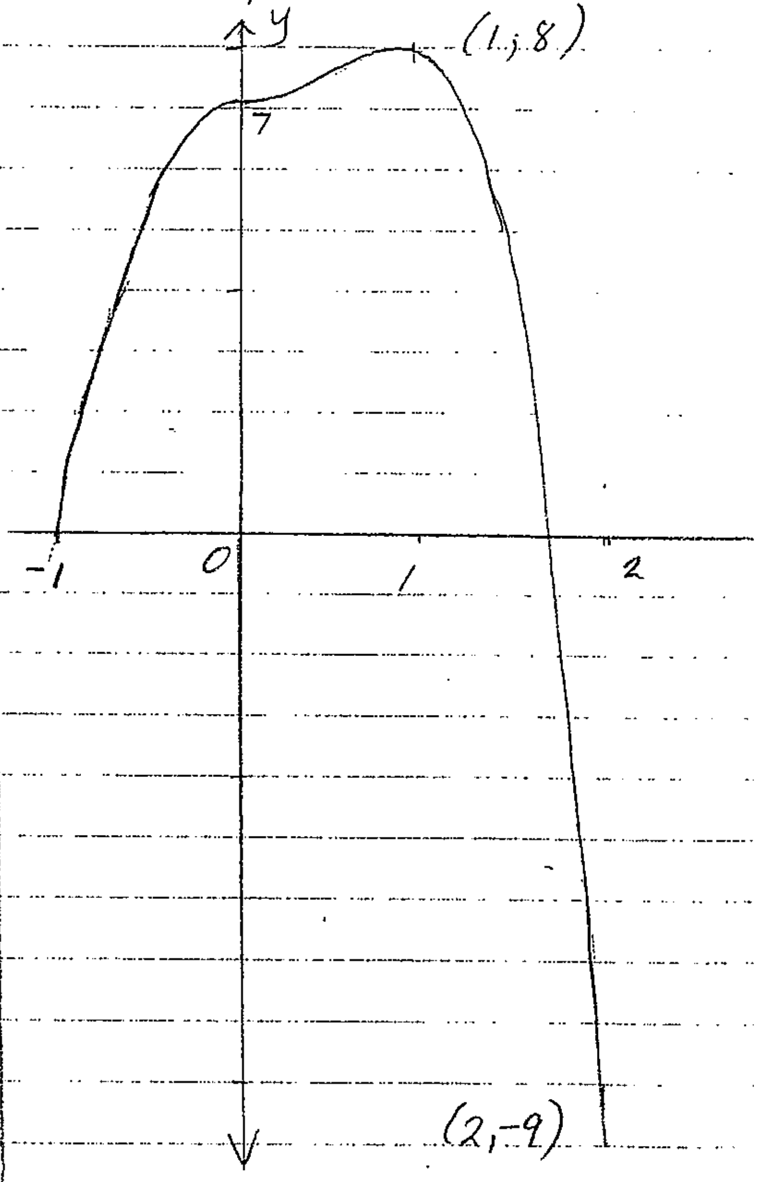
$\therefore$ $(0,7)$ is a horizontal point of inflexion

when $x = 1$, $\frac{d^2y}{dx^2} < 0$

$\therefore$ $(1,8)$ is a maximum turning point

(iv) $f(-1) = 0$

and $f(2) = -9$



Question 9 (continued)

(i) $2156 = \pi r^2 h$

$h = \dfrac{2156}{\pi r^2}$

$\therefore S = 2\pi r^2 + 2\pi r \cdot \dfrac{2156}{\pi r^2}$

$S = 2\pi r^2 + \dfrac{4312}{r}$

(ii) $S = 2\pi r^2 + 4312 r^{-1}$

$\dfrac{ds}{dr} = 4\pi r - 4312 r^{-2}$

$= 4\pi r - \dfrac{4312}{r^2}$

$\dfrac{d^2s}{dr^2} = 4\pi + 8624 r^{-3}$

$= 4\pi + \dfrac{8624}{r^3}$

$\therefore \dfrac{d^2s}{dr^2} > 0$ for all $r > 0$

For stat. pts $\dfrac{ds}{dr} = 0$

$\therefore 4\pi r - \dfrac{4312}{r^2} = 0$

$\therefore 4\pi r^3 - 4312 = 0$

$r^3 = \dfrac{4312}{4\pi}$

$r^3 = 343$ if $\pi = \dfrac{22}{7}$

$r = 7$

$\therefore$ Now since $\dfrac{d^2s}{dr^2} > 0$

$\therefore$ A minimum value of Surface Area (S) occurs when $r = 7$

oops!

8 c) $V = \pi \int_0^4 (x^2+1)^2\, dx$

$V = \pi \int_0^4 (x^4 + 2x^2 + 1)\, dx$

$= \pi \left[\dfrac{x^5}{5} + \dfrac{2x^3}{3} + x \right]_0^4$

$= \pi \left[\dfrac{4^5}{5} + \dfrac{2}{3} \times 4^3 + 4 - 0 \right]$

$= \dfrac{3772\pi}{15}$ cubic units

HSC Maths Ass Task 2
Solutions

Section I

(1) A (2) D (3) D
(4) B (5) B

Section II

Question 6

a) 24600 or 2.46×10^4

b) $(3-\sqrt{2})^2 = 9+2-6\sqrt{2}$
$= 11-6\sqrt{2}$
$\therefore a = 11,\ b = 6$

c) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{1 \pm \sqrt{(-1)^2-4(3)(-1)}}{6}$

$x = \dfrac{1 \pm \sqrt{13}}{6}$

d) (i) $m_k = \dfrac{4}{-2} = -2$
$\therefore$ Equation of line k is
$y = -2x - 4$
or $2x + y + 4 = 0$ in general form

(ii) $d = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

$d = \left| \dfrac{2(5) + 1(-2) + 4}{\sqrt{(2)^2 + (1)^2}} \right|$

$d = \left| \dfrac{12}{\sqrt{5}} \right|$

$d = \dfrac{12}{\sqrt{5}}$ or $\dfrac{12\sqrt{5}}{5}$

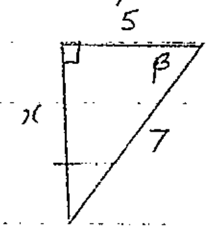
e) $G(2) = 0$
$\therefore 3(2)^2 - 2p + p - 3 = 0$
$\therefore p = 9$

(ii) Product of Roots = 1
$\therefore \dfrac{p-3}{3} = 1$
$\therefore p = 6$

f) $A\hat{Y}P = B\hat{A}Y = 70^\circ$
(alternate angles, $AB \parallel XY$)
$A\hat{P}Y = A\hat{Y}P = 70^\circ$
(equal angles opposite equal sides of an isosceles triangle)
$P\hat{A}Y = 180^\circ - (A\hat{Y}P + A\hat{P}Y)$
(angle sum of a triangle)
$\therefore P\hat{A}Y = 180^\circ - 2\times 70^\circ$
$= 40^\circ$

Question 7

a) $\cos\beta = -\frac{5}{7}$ & $\tan\beta > 0$
$\therefore \beta$ lies in 3rd quadrant



$x^2 = 7^2 - 5^2$
$x^2 = 24$
$x = 2\sqrt{6}$.
$\therefore \sin\beta = -\dfrac{2\sqrt{6}}{7}$

b) Let $y = x(1-2x)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = (1-2x)^{\frac{1}{2}}\cdot 1 + x\cdot\frac{1}{2}(1-2x)^{-\frac{1}{2}}\cdot(-2)$
$= (1-2x)^{\frac{1}{2}} - x(1-2x)^{-\frac{1}{2}}$
$= (1-2x)^{-\frac{1}{2}}\left[(1-2x) - x\right]$
$= \dfrac{1-3x}{\sqrt{1-2x}}$

c) $\dfrac{dy}{dx} = 2x - 2$

$\therefore 2x - 2 = 2$ at pt of contact

Question 7(c) continued

$\therefore x = 2$

when $x = 2$, $y = (2)^2 - 2(2) + 4 = 4$

$\therefore (2,4)$ is point of contact

$\therefore$ Equation of line is

$y - 4 = 2(x-2)$

$y - 4 = 2x - 4$

$y = 2x$

d) Solving simultaneously

$x^2 = 12(mx - 3m^2)$

$x^2 = 12mx - 36m^2$

$x^2 - 12mx + 36m^2 = 0$

$\Delta = b^2 - 4ac$

$= (-12m)^2 - 4(1)(36m^2)$

$= 144m^2 - 144m^2$

$= 0$

$\therefore$ One solution

$\therefore y = mx - 3m^2$ is a tangent to the curve for all real m

e) (i) $\frac{\sin\phi}{g} = \frac{\sin\theta}{h}$

$\therefore \frac{\sin\phi}{\sin\theta} = \frac{g}{h}$

(ii) $\frac{\sin\phi}{e} = \frac{\sin(180-\theta)}{f}$

$\frac{\sin\phi}{e} = \frac{\sin\theta}{f}$

$\frac{\sin\phi}{\sin\theta} = \frac{e}{f}$

$\therefore \frac{e}{f} = \frac{g}{h} \left(= \frac{\sin\phi}{\sin\theta}\right)$

Question 8

a) $\int (1+2x)^3 \, dx$

$= \frac{(1+2x)^4}{4 \times 2} + c$

$= \frac{(1+2x)^4}{8} + c$

b) $\int_1^4 \frac{x+1}{x^{\frac{1}{2}}} \, dx = \int_1^4 x^{\frac{1}{2}} + x^{-\frac{1}{2}} \, dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right]_1^4$

$= \left[\frac{2}{3} \times 8 + 2 \times 2 - \left(\frac{2}{3} + 2 \right) \right]$

$= \frac{20}{3}$

c) oops → see back page

d) For P, $3x = 2x^2 - 5x$

$2x^2 - 8x = 0$

$2x(x-4) = 0$

$x = 0, 4$

$\therefore P(4, 12)$

(ii) $A = \int_0^4 3x - (2x^2 - 5x) \, dx$

$= \int_0^4 8x - 2x^2 \, dx$

$= \left[4x^2 - \frac{2x^3}{3} \right]_0^4$

$= \left[64 - \frac{128}{3} \right]$

$= \frac{64}{3}$ sq. units