NAME ______________________________

GOSFORD HIGH SCHOOL

2016

Higher School Certificate

MATHEMATICS

Assessment Task 2

Time Allowed – 90 minutes

General Instructions

- Reading time 5 minutes
- Working time 90 minutes
- Write using black or blue pen
- Board approved calculators may be used
- For all questions show relevant mathematical reasoning and/or calculations
- Use the attached multiple choice answer sheet to answer questions 1 to 5
- Start each question from 6 to 9 on a new page
- Total marks 65

MULPTIPLE CHOICE 5 marks 1 mark each

Question 1

The equation $(cos\emptyset - 1)(2sin\emptyset - 1) = 0$ for $0 \leq \emptyset \leq 2\pi$ has

A) 2 solutions B) 3 solutions

C) 4 solutions D) 5 solutions

Question 2

$$\int \frac{1}{\sqrt{1-4x}} dx =$$

A) $-\frac{\sqrt{1-4x}}{2} + c$ B) $-\frac{\sqrt{(1-4x)^3}}{2} + c$

C) $2\sqrt{1-4x} + C$ D) $\frac{2\sqrt{(1-4x)^3}}{3} + C$

Question 3

Find the range of values of the function $y = f(x)$,

given that $f(x) = 4 - x^2$ in the domain $-2 \leq x \leq 4$

A) $y \leq 4$ B) $0 \leq y \leq 4$

C) $-12 \leq y \leq 0$ D) $-12 \leq y \leq 4$

Question 4

Given that $\frac{dy}{dx} = cosx - sin2x$ and that when $x = 0, y = 1 \cdot 5$, then

A) $y = -sinx - \frac{1}{2}cos2x + 2$

B) $y = sinx + \frac{1}{2}cos2x + 1$

C) $y = sinx - 2cos2x + 3.5$

D) $y = -sinx + 2cos2x - 0.5$

Question 5

If $\frac{d^2y}{dx^2} = (x + 2)^2(x - 1)$, then point(s) of inflexion occur at

A) $x = -2$ only

B) $x = -2$ and $x = 1$

C) $x = 1$ only

D) neither $x = -2$ or $x = 1$

Question 6 15 marks

(a) Factorise $8 + m^3$ (1)

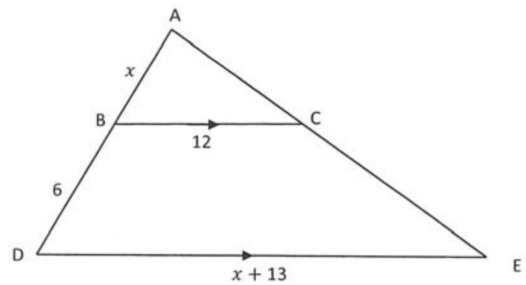
(b) Solve $|x - 3| = 2x$ (2)

(c) Find the vertex and the focus of the parabola $x^2 = 8y - 16$ (2)

(d) Find the perpendicular distance from the point $(1, -2)$ to the line with equation $3x + 4y - 6 = 0$ (2)

(e) A triangle has sides of AC = $4cm$, AB = $5cm$ and BC = $6cm$. Find angle BAC, correct to the nearest degree. (2)

(f)



(i) Prove ΔABC is similar to ΔADE (2)

(ii) Using the dimensions as shown on the diagram, find x (2)

(g) For what values of k is the quadratic expression $x^2 - 3kx + 9k$ positive definite. (2)

Question 7 15 marks

(a) Copy and complete the following function table for $f(x) = \frac{4}{x+1}$.

X	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x)$	4		$\frac{8}{3}$	$\frac{16}{7}$	2

(1)

If $J = \int_0^1 \frac{4}{x+1}\,dx$,

find the value of J using Simpson's rule and 5 function values (3)

(b) Find $\int x\sqrt{x}\,dx$ (1)

(c) Evaluate $\int_{-1}^{1} (x^2+1)^2\,dx$ (2)

(d) The two curves $y = x^2 + 3$ and $y = x^3 - 1$ intersect at the point $(2, 7)$.

(i) Draw the two curves on the same number plane and label the given point of intersection. (2)

(ii) Find the area of the region enclosed by the two curves and the y axis. (3)

(e) The arc of the curve $y = \frac{4}{x}$ between the points $(1, 4)$ and $(4, 1)$ is rotated about the y axis. Find the volume of the solid generated. (3)

Question 8 15 marks

(a) State the period and amplitude of the curve $y = 3 + cos\left(\frac{x}{2}\right)$ (2)

(b) What is the exact value of $sin\left(\frac{4\pi}{3}\right)$ (1)

(c) The area of a sector AOB of a circle, with centre O and radius 6cm, is 27cm^2.

i) Calculate the angle AOB in radians (1)

ii) Find the length of the minor arc AB (1)

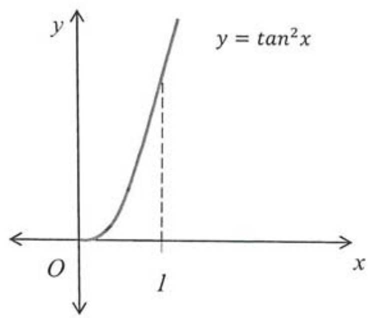
(d) Find $\frac{d}{dx}[tan^2 2x]$ (1)

(e) The curve $y = \sqrt{cos\pi x}$, where $0 \le x \le \frac{1}{2}$ is rotated about the x axis.

What is the volume of the solid of revolution thus generated? (3)

(f) What is the gradient of the tangent to the curve $y = \frac{x}{sin2x}$ at the point on the curve where $x = \frac{\pi}{4}$ (2)

(g) A portion of the graph of $y = tan^2x$ is shown below.
Find the area bounded by the curve $y = tan^2x$, the x axis and the ordinate at $x = 1$. (*Write your answer correct to 3 significant figures*) (4)



Question 9 15 marks

(a) Find the second derivative of $(x^2 + 1)^5$ (2)

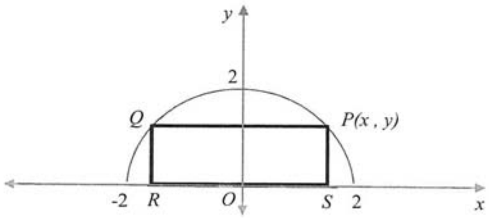
(b) (i) Find the co-ordinates of the stationary points on the curve

$y = x^3 - 6x^2 + 9x - 4$ (3)

(ii) Determine the nature of these stationary points (2)

(iii) Sketch the curve (2)

(c) A rectangle $PQRS$ is inscribed inside the semi-circle $y = \sqrt{4 - x^2}$ as shown in the diagram below. P and Q lie on the semi-circle.



(i) If P has co-ordinates (x, y) , show that the area of the rectangle (A)

is given by $A = 2x(4 - x^2)^{\frac{1}{2}}$. (1)

(ii) Show that $\frac{dA}{dx} = \frac{4(2-x^2)}{\sqrt{4-x^2}}$. (2)

(iii) Hence prove that the area of the rectangle is a maximum when $x = \sqrt{2}$. (2)

(iv) Find this maximum area (1)

MULTIPLE CHOICE ANSWER SHEET

STUDENT NAME/NUMBER ______________________

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D

Multiple Choice

1) $\cos\theta = 1$ or $\sin\theta = \frac{1}{2}$.

$\therefore \theta = 0, 2\pi \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

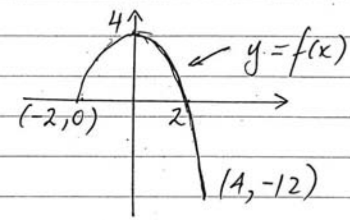
$\therefore$ 4 Solutions (C)

2) $\int (1-4x)^{-\frac{1}{2}}\,dx = \dfrac{(1-4x)^{\frac{1}{2}}}{\frac{1}{2}\times(-4)} + C$

$= \dfrac{\sqrt{1-4x}}{-2} + C$

$\therefore$ (A)

3)



Range is $-12 \le y \le 4$ (D)

4) $y = \sin x + \frac{1}{2}\cos 2x + C$

when $x = 0$, $y = 1.5$

$1.5 = \sin 0 + \frac{1}{2}\cos 0 + C$

$1.5 = 0.5 + C$

$1 = C$

$\therefore y = \sin x + \frac{1}{2}\cos 2x + 1$ (B)

5) $\dfrac{d^2y}{dx^2} = 0$ at $x = -2, 1$

when $x < -2$ $\dfrac{d^2y}{dx^2} < 0$ (say $x = -3$)

when $-2 < x < 1$ $\dfrac{d^2y}{dx^2} < 0$ (say $x = 0$)

when $x > 1$ $\dfrac{d^2y}{dx^2} > 0$ (say $x = 2$)

$\therefore$ Pt. of inflexion at $x = 1$ only (C)

Question 6 (1 mark)

a) $8 + m^3 = (2+m)(4 - 2m + m^2)$

b) $x - 3 = 2x$

$-3 = x$

But this value does not satisfy since $6 \neq -6$.

OR $-(x-3) = 2x$

$-x + 3 = 2x$.

$3 = 3x$

$1 = x$ 1 mark for 2 answers

Which satisfies since $2 = 2$

$\therefore x = 1$ only 1 mark

Question 6 (continued)

c) $x^2 = 8(y-2)$

$\therefore 4a = 8 \Rightarrow a = 2$

Vertex is $(0, 2)$ 1 mark

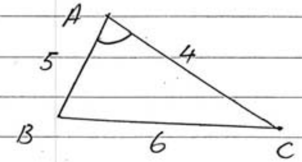
$\therefore$ Focus is $(0, 4)$ 1 mark.

d) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|1(3) + 4(-2) - 6|}{\sqrt{3^2+4^2}}$ 1 mark.

$d = \dfrac{11}{5}$ 1 mark for answer

e)



$\cos A = \dfrac{(5^2 + 4^2 - 6^2)}{(2\times 5\times 4)}$

$= \dfrac{5}{40}$

$= \dfrac{1}{8}$ 1 mark

$A = \cos^{-1}\left(\frac{1}{8}\right)$

$A = 83°$ 1 mark

f)(i) In Δ's ABC & ADC

$\hat{A}$ is common

$A\hat{B}C = A\hat{D}C$ (corresponding angles, BC // DE) 1 mark.

$\therefore \Delta ABC \;|||\; \Delta ADE$ (triangles are equiangular) 1 mark.

(ii) $\dfrac{x}{x+6} = \dfrac{12}{x+13}$ 1 mark.

(corresponding sides of similar triangles are in proportion)

$x(x+13) = 12(x+6)$

$x^2 + 13x = 12x + 72$

$x^2 + x - 72 = 0$

$(x+9)(x-8) = 0$ 1 mark.

$\therefore x = 8$ only since $x > 0$

g) Positive Definite if $\Delta < 0$ noting that coefficient of $x^2 > 0$

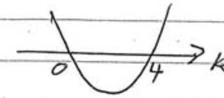
$\therefore b^2 - 4ac < 0$

$(-3k)^2 - 4(1)(9k) < 0$

$9k^2 - 36k < 0$ 1 mark.

$9k(k-4) < 0$

$\therefore 0 < k < 4$



1 mark

Question 7

a)

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x)$	4	3.2	$\frac{8}{3}$	$\frac{16}{7}$	2

1 mark

$$J \doteqdot \frac{h}{3}\left[f(0)+f(1)+2f\left(\tfrac{1}{2}\right)+ 4\left[f\left(\tfrac{1}{4}\right)+f\left(\tfrac{3}{4}\right)\right]\right]$$

1 mark for knowledge of correct Simpson's Rule.

$$\doteqdot \frac{0.25}{3}\left[4+2+2\times\frac{8}{3}+4\left(\frac{16}{5}+\frac{16}{7}\right)\right]$$ 1 mark.

$$\doteqdot \frac{1747}{630} \approx 2.8 \text{ (to 1 d.p)}$$ 1 mark (rounding permitted)

b) $$\int x\sqrt{x}\,dx = \int x^{\frac{3}{2}}\,dx.$$ ~~1 mark~~

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c.$$

$$= \frac{2x^{\frac{5}{2}}}{5} + c$$ 1 mark.

$$= \frac{2x^2\sqrt{x}}{5} + c$$

c) $$\int_{-1}^{1}(x^2+1)^2\,dx = 2\int_0^1 (x^4+2x^2+1)\,dx.$$

$$= 2\left[\frac{x^5}{5}+\frac{2x^3}{3}+x\right]_0^1$$ 1 mark.

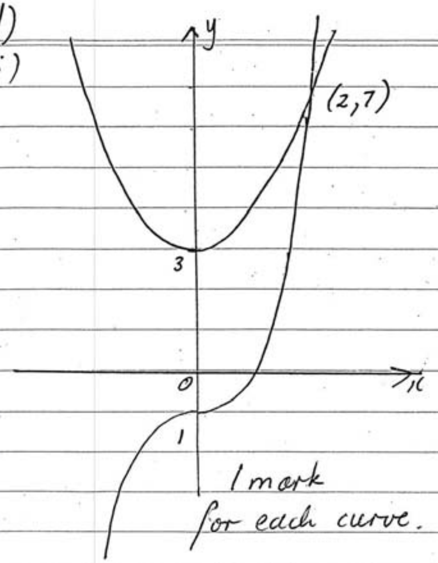
$$= 2\left[\frac{1}{5}+\frac{2}{3}+1\right]-0$$

$$= \frac{56}{15} = 3.7\dot{3}$$ 1 mark.

Question 7 (continued)

d)

(i)



1 mark for each curve.

(ii) $$\text{Area} = \int_0^2 (x^2+3)-(x^3-1)\,dx.$$ 1 mark.

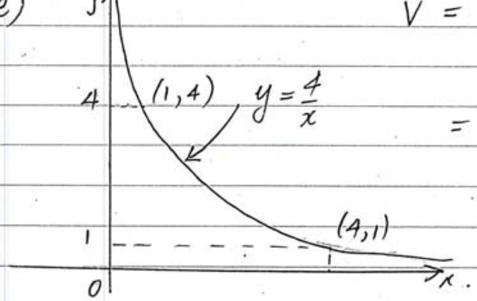
$$= \int_0^2 (x^2-x^3+4)\,dx.$$

$$= \left[\frac{x^3}{3}-\frac{x^4}{4}+4x\right]_0^2$$ 1 mark.

$$= \left[\frac{8}{3}-4+8-0\right].$$

$$= 6\tfrac{2}{3} \text{ sq. units.}$$ 1 mark.

e)



$$V = \pi\int_1^4 \left(\frac{4}{y}\right)^2 dy. \quad \text{since } x=\frac{4}{y}.$$ 1 mark.

$$= \pi\int_1^4 16y^{-2}\,dy.$$

$$= 16\pi\left[\frac{y^{-1}}{-1}\right]_1^4$$ 1 mark.

$$= -16\pi\left[\frac{1}{4}-1\right]$$

$$= -16\pi\times\left(\frac{-3}{4}\right)$$ 1 mark.

$$= 12\pi \text{ cubic units}$$

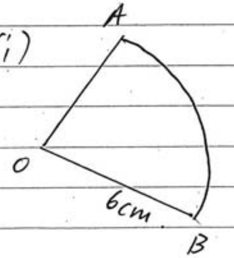
Question 8

a) $\text{Period} = \frac{2\pi}{1/2}$ Amplitude $= 1$

$= 4\pi$ (1 mark) (1 mark)

b) $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ (1 mark)

c) (i)



$A = 27 \text{ sq cm.}$

$\therefore \frac{1}{2} r^2 \theta = 27.$

$\frac{1}{2}(6)^2 \theta = 27.$

$\theta = \frac{27}{18}.$

$\theta = \frac{3}{2}$ radians. (1 mark)

(ii) $l_{AB} = r\theta.$

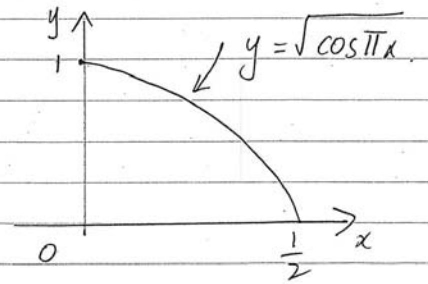
$= 6 \times \frac{3}{2}.$

$= 9 \text{cm}.$ (1 mark)

d) $\frac{d}{dx}\left[(\tan 2x)^2\right] = 2(\tan 2x)^1 \times 2\sec^2 2x.$ (1 mark)

$= 4 \tan 2x \sec^2 2x.$

e)



$V = \pi \int_0^{\frac{1}{2}} \cos \pi x \, dx.$ (1 mark)

$= \pi \left[\frac{1}{\pi} \sin \pi x\right]_0^{\frac{1}{2}}$ (1 mark)

$= \sin \frac{\pi}{2} - \sin 0$

$= 1$ sq unit. (1 mark)

Question 8 (continued)

f) $y = \frac{x}{\sin 2x}.$

$\frac{dy}{dx} = \frac{\sin 2x \cdot 1 + x \cdot 2\cos 2x}{\sin^2 2x}$ (1 mark)

$= \frac{\sin 2x + 2x \cos 2x}{\sin^2 2x}.$

$= \frac{1 + \frac{\pi}{2} \times 0}{1}$ at $x = \frac{\pi}{4}$

$= 1$ (1 mark)

$\therefore$ Gradient of required tangent is 1

g) $\text{Area} = \int_0^1 \tan^2 x \, dx.$ (1 mark)

$= \int_0^1 (\sec^2 x - 1)\, dx.$ (1 mark)

$= \left[\tan x - x\right]_0^1$ (1 mark)

$= \tan 1 - 1$

$= 0.557$ (1 mark)

Question 9

a) Let $y = (x^2+1)^5$

$$\frac{dy}{dx} = 5(x^2+1)^4 \times 2x.$$

$$= 10x(x^2+1)^4 \qquad \text{(1 mark)}$$

$$\frac{d^2y}{dx^2} = (x^2+1)^4 \times 10 + 10x \times 4(x^2+1)^3 \times 2x.$$

$$= 10(x^2+1)^4 + 80x^2(x^2+1)^3. \qquad \text{(1 mark)}$$

$$= 10(x^2+1)^3\left[(x^2+1) + 8x^2\right]$$

$$= 10(x^2+1)^3(9x^2+1).$$

b) (i) $y = x^3 - 6x^2 + 9x - 4$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\frac{d^2y}{dx^2} = 6x - 12.$$

For stationary points $\frac{dy}{dx} = 0$

$$\therefore\ 3x^2 - 12x + 9 = 0 \qquad \text{(1 mark)}$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0 \qquad \text{(1 mark)}$$

$$x = 1, 3$$

(1 mark)

$\therefore$ Stationary points at $(1,0)$ & $(3,-4)$

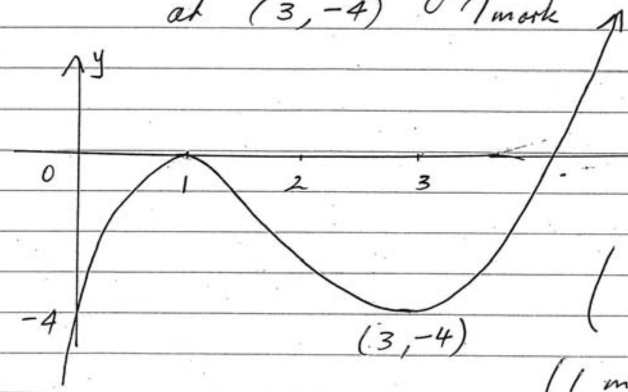
(ii) $\frac{d^2y}{dx^2} = -6 < 0$ when $x = 1$

$\therefore$ A maximum turning point exists at $(1,0)$ (1 mark)

Question 9 (b) (continued)

$\frac{d^2y}{dx^2} = 6 > 0$ when $x = 3$

$\therefore$ A minimum turning point exists at $(3,-4)$ 1 mark



(1 mark for correct shape.)

(1 mark for labelling)

(c) (i) $A = \text{Length} \times \text{Breadth}.$

$$= 2x \times y$$

$$A = 2x\sqrt{4-x^2}$$

$$A = 2x(4-x^2)^{\frac{1}{2}} \qquad \text{(1 mark)}$$

(ii) $\frac{dA}{dx} = (4-x^2)^{\frac{1}{2}} \times 2 + 2x \times \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \times (-2x)$

$$= 2(4-x^2)^{\frac{1}{2}} - 2x^2(4-x^2)^{-\frac{1}{2}}. \qquad \text{(1 mark)}$$

$$= 2(4-x^2)^{-\frac{1}{2}}\left[4 - x^2 - x^2\right]$$

$$= 2(4-x^2)^{-\frac{1}{2}}(4-2x^2) \qquad \text{(1 mark for show)}$$

$$= \frac{4(2-x^2)}{\sqrt{4-x^2}} \quad \text{as required.}$$

Question 7.(c) (continued)

(iii) For stationary points $\frac{dA}{dx} = 0$.

$\therefore \quad 2 - x^2 = 0$

$x = \sqrt{2}$ since $x > 0$ (1 mark)

when $x < \sqrt{2}$ $\quad \frac{dA}{dx} > 0$ $\quad \therefore$ increasing.
(say $x = 1$)

when $x > \sqrt{2}$ $\quad \frac{dA}{dx} < 0$ $\quad \therefore$ decreasing.
(say $x = 1.5$

$\therefore$ A maximum value of A occurs when $x = \sqrt{2}$ (1 mark)

(iv) Max. Area $= 2 \times \sqrt{2} \left(4 - (\sqrt{2})^2\right)^{\frac{1}{2}}$.

$= 2\sqrt{2} \times \sqrt{2}$

$= 4$ square units (1 mark)