JAMES RUSE AGRICULTURAL HIGH SCHOOL

2 Unit Term 1 ASSESSMENT 1997

Time allowed: 85 minutes

QUESTION 1 (START A NEW PAGE)

(a) Integrate with respect to x:

(i) $4x^2 + \sqrt{x}$

(ii) $\cos 6x$

(iii) $(x^3 + 4)^2$

(b) Evaluate:

(i) $\int_0^1 e^{3x}\, dx$

(ii) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\, dx$

(c) Find the amount of interest if \$5000 is invested for 12 years at 9.95% interest compounded annually.

QUESTION 2 (START A NEW PAGE)

(a) Find:

(i) $\int (3x-2)^9\, dx$

(ii) $\int 2xe^{x^2}\, dx$

(iii) $\int \frac{\sin x}{2+\cos x}\, dx$

(b) On a fun run numbered flags are placed at regular intervals along the course. Flag number 1 is 100 metres from the start and all the rest are 350 metres apart.

(i) How far must a competitor run to reach flag number 5?

(ii) Write a formula for the distance, F_n, that a competitor must run to reach flag number n.

(iii) If a competitor runs 4 kilometres, which is the nearest flag and how far is the competitor from that flag?

QUESTION 3 (START A NEW PAGE)

(a) Daily measurements taken each evening at a lake indicate that the lake loses a constant volume of water equal to $\frac{1}{4}\%$ of its remaining volume each day.

(i) Find the approximate percent of the lake that remains after 30 days.

(ii) Find the approximate number of days before the lake is at 75% of its original capacity.

(b) Find the area bounded by $y = \cos\frac{1}{3}x$, the co-ordinate axes and the line $x = \pi$.

(c) Find the exact value of the area bounded by the curve $y = \sqrt{x}$, the y-axis and the line $y = 2$.

QUESTION 4 (START A NEW PAGE)

(a) The area bounded by $y = \sec x$, the co-ordinate axes and the line $x = \frac{\pi}{4}$ is rotated one revolution about the x-axis. Find the volume of this solid.

(b) The area bounded by the curve $y = \ln x$, the x-axis, the y-axis and the line $y = 3$ is rotated one revolution about the y-axis. Find the volume of this solid.

(c) (i) Find the co-ordinates of the points of intersection of the parabola $y = 4 - x^2$ and the line $y = x - 2$.

(ii) Draw a neat sketch showing the region bounded by the above curves.

(iii) Find the exact value of the area bounded by the above curves.

QUESTION 5 (START A NEW PAGE)

(a) The area bounded by $y = x$ and $y = 4x - x^2$ is rotated one revolution about the x-axis. Find the exact value of the volume of the solid formed.

(b) The probability that a car part will last 3 years or less is given by the following integral:

$$\text{Probability} = \int_0^3 f(x)\, dx \quad \text{where} \quad f(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}$$

Use Simpson's rule and 3 function values to find an approximation for the above integral.
(Give your answer to 2 decimal places)

(c) Samuel invests in a superannuation fund. He invests \$150 at the start of each month and the interest rate is 6% p.a. compounded on the balance in the fund at the end of each month.

(i) Find the value of the first deposit at the end of 10 years. (Give your answer to the nearest dollar)

(ii) Find the total value of all his monthly deposits at the end of 10 years. (Give your answer to the nearest dollar)

YEAR 12 1997 2 UNIT TERM 1 ASSESSMENT

QUESTION 1

(a) (i) $\frac{4}{3}x^3 + \frac{2}{3}x^{3/2} + c$

(ii) $\frac{1}{6}\sin 6x + c$

(iii) $\int (x^3+4)^2\,dx = \int x^6 + 8x^3 + 16\,dx$

$= \frac{1}{7}x^7 + 2x^4 + 16x + c$

(b) (i) $\left[\frac{1}{3}e^{3x}\right]_0^1 = \frac{1}{3}(e^3 - e^0)$

$= \frac{1}{3}(e^3 - 1)$

(ii) $\left[-\frac{1}{2}\cos 2x\right]_{-\pi/6}^{\pi/2} = -\frac{1}{2}\{\cos\pi - \cos(-\pi/3)\}$

$= -\frac{1}{2}(-1 - \frac{1}{2})$

$= 3/4$

(c) $I_{nt} = \$5000\left(1 + \frac{9.5}{100}\right)^{12} - 5000$

$= \$10,606.76$ (to nearest cent)

QUESTION 2

(a) (i) $\frac{(2x-3)^{10}}{20} + c$

(ii) $e^{x^2} + c$

(iii) $-\ln(2+\cos x) + c$

(b) (i) Dist $= 100 + 4 \times 350$ m

$= 1500$ m

(ii) $F_n = 100 + 350(n-1)$

$= 350n - 250$

(iii) $4000 = 350n - 250$

$350n = 4250$

$n = \frac{4250}{350}$

$\doteqdot 12.14$

$\therefore$ nearest flag is flag 12

$F_{12} = 350 \times 12 - 250$

$= 3950$

$\therefore$ dist to flag $= 50$ m.

QUESTION 3.

(a) (i) % left $= \left(1 - \frac{4}{100}\right)^{30}$

$= 0.9276$

Approx. % $= 92.76\%$

(ii) $0.75 = 0.9975^n$

$n \ln 0.9975 = \ln 0.75$

$n = \frac{\ln 0.75}{\ln 0.9975}$

$\doteqdot 114.9$

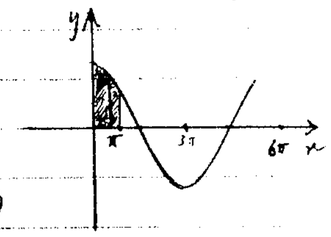
$\therefore$ approx. number of days $= 115$

(b) $A = \int_0^{\pi} \cos\frac{1}{3}x\,dx$

$= \left[3\sin\frac{1}{3}x\right]_0^{\pi}$

$= 3(\sin\frac{\pi}{3} - \sin 0)$

$= \frac{3\sqrt{3}}{2}$ u²

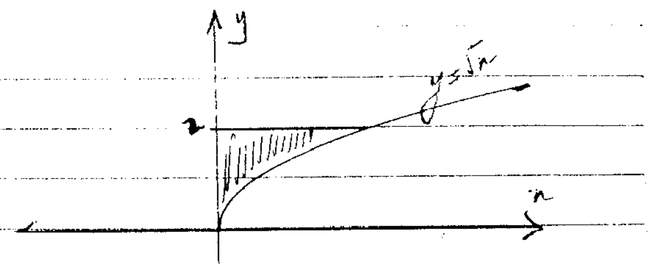


3(c) $A = \int_0^1 x\,dy$

$= \int_0^2 y^2\,dy$

$= \left[\frac{1}{3}y^3\right]_0^2$

$= 8/3\ u^2$



4(c)(iii) $A = \int_{-3}^{2} (4-x^2) - (x-2)\,dx$

$= \int_{-3}^{2} -x^2 - x + 6\,dx$

$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x\right]_{-3}^{2}$

$= \left(-\frac{8}{3} - \frac{4}{2} + 12\right) - \left(+\frac{27}{3} - \frac{9}{2} - 18\right)$

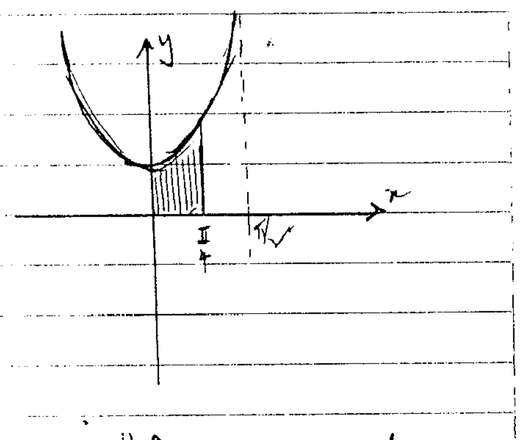
$A = 20\frac{5}{6}\ u^2$

QUESTION 4

(a) $V = \pi \int_0^{\pi/4} \sec^2 x\,dx$

$= \pi \left[\tan x\right]_0^{\pi/4}$

$= \pi \cdot 1\ u^3$

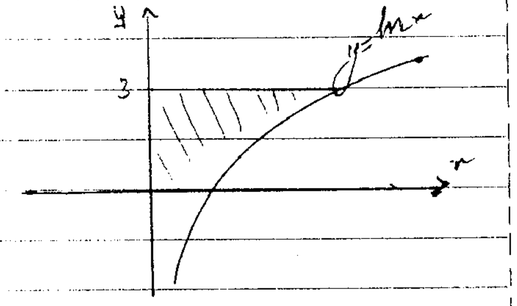


(b) $A = \pi \int_0^3 x^2\,dy$

$= \pi \int_0^3 e^{2y}\,dy$

$= \left[\frac{1}{2}e^{2y}\right]_0^3$

$= \frac{e^6 - 1}{2}\ u^3$



(c)(i) $y - x^2 = x - 2$

$x^2 + x - 6 = 0$

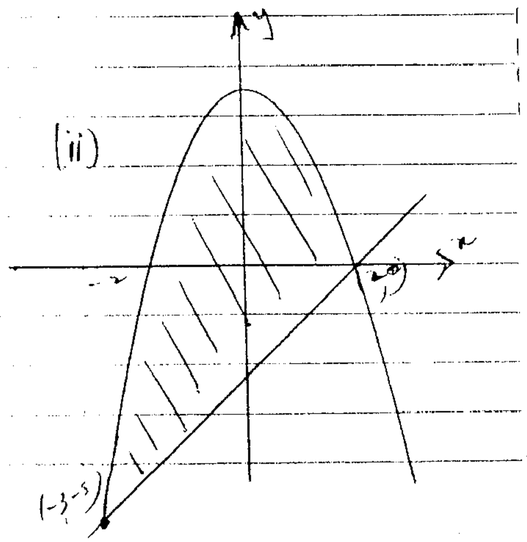
$(x+3)(x-2) = 0$

$x = -3\ 2$

$x = -3 \quad y = -5$

$x = 2 \quad y = 0$

pts are $(-3,-5)$ $(2,0)$

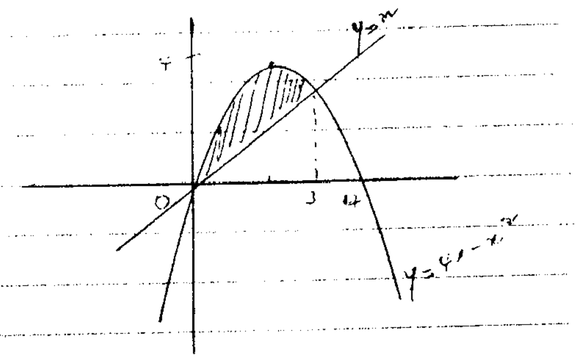


QUESTION 5

(a) $4x - x^2 = x$

$x^2 = 3x$

$x = 0, 3$



$V = \pi \int_0^3 (4x-x^2)^2 - (x)^2\,dx$

$= \pi \int_0^3 16x^2 - 8x^3 + x^4 - x^2\,dx$

$= \pi \int_0^3 x^4 + 15x^2 - 8x^3\,dx$

$= \pi \left[\frac{1}{5}x^5 - 2x^4 + 5x^3\right]_0^3$

$= \pi \left\{\left(\frac{243}{5} - 2(81) + 5(27)\right) - 0\right\}$

$V = \frac{108\pi}{5}\ u^3$

(b) $Prob \doteq \frac{3-0}{6}\left\{\frac{e^0}{\sqrt{2\pi}} + \frac{4 \cdot e^{-0.5 \times 1.5^2}}{\sqrt{2\pi}} + \frac{e^{-0.5\times 9}}{\sqrt{2\pi}}\right\}$

$= \frac{1}{2\sqrt{2\pi}}\left\{e^0 + 4e^{-1.125} + e^{-4.5}\right\}$

$Prob = 0.46$

4 (c). (i) Value $= \$150 \times \left(1 + \frac{0.5}{100}\right)^{120}$ $r = 0.5\%$ / mth

$\doteq \$273$ (to nearest $)

(ii) Total value $= \$150 \times 1.005^{120} + 150 \times 1.005^{119} + \cdots + 150 \times 1.005^{2} + 150 \times 1.005$

$= \$150\left(1.005 + 1.005^{2} + \cdots + 1.005^{120}\right)$

$= \$150 \times \dfrac{1.005\left(1.005^{120} - 1\right)}{0.005}$

$= \$24705.81$

$= \$24705$ (to nearest $)