YEAR 12

JAMES RUSE AGRICULTURAL HIGH SCHOOL
2 Unit Term 1 ASSESSMENT 1998

Time allowed: 85 minutes
All questions are of equal value.
Attempt all questions.
Calculators may be used.

QUESTION 1 (START A NEW PAGE)

(a) Integrate with respect to x:

2 (i) $\frac{x^2+3}{2x}$

2 (ii) $\cos\frac{1}{3}x$

2 (iii) $(\sqrt{x}+4)^2$

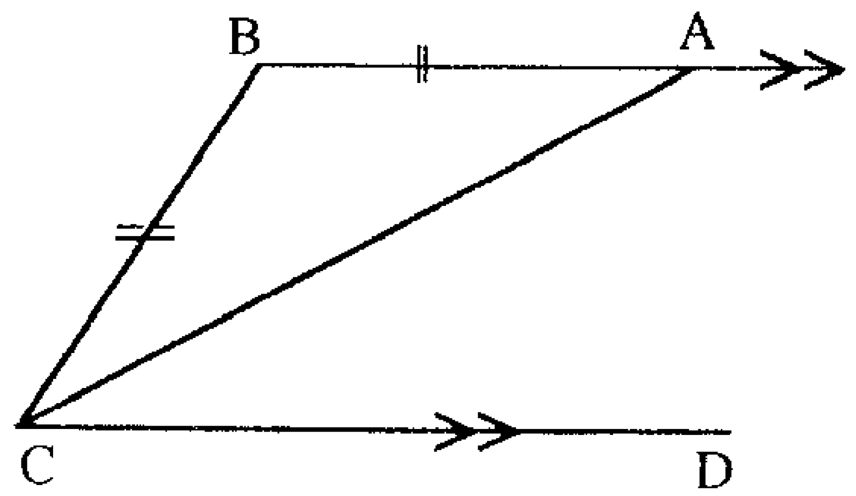
2 (b) Evaluate: (i) $\int_1^3 \frac{1}{2x-1}\,dx$.

2 (ii) $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \sec^2 2x\,dx$.

2 (c) Find the general term of the arithmetic sequence $\{101, 97, 93, \ldots\}$.

QUESTION 2 (START A NEW PAGE)

3 (a) Given $BA \parallel CD$ and $BA = BC$, prove that AC bisects $B\hat{C}D$.



3 (b) Find the volume of the solid formed when the area bounded by $y=\sqrt{4-x^2}$ and the x-axis for $-1 \le x \le 2$ is rotated one revolution about the x-axis.

(c) Sketch the following curves showing their intercepts with the co-ordinate axes and any asymptotes if they exist.

3 (i) $y=\frac{x+2}{x-1}$.

3 (ii) $x^2+y^2-6x+8y=0$.

QUESTION 3 (START A NEW PAGE)

(a) Mrs Jones decides to set up a small fund to pay for her Christmas holiday. Each weekend she keeps \$25 of her ironing money and deposits the \$25 in her fund at the start of that week. The fund receives 3.25% p.a. interest and the interest is compounded at the end of each week. (Assume 1 year = 52 weeks)

2 (i) Find the value of her first deposit at the end of 50 weeks. (Give answer to nearest cent)

3 (ii) Find the value of her fund at the end of 50 weeks. (Give answer to nearest dollar)

3 (b) (i) Show that the equation of the tangent to $y=e^{3x}$ at the point where $x=1$ is given by $y=e^3(3x-2)$.

1 (ii) Draw a diagram showing the area bounded by $y=e^{3x}$, the tangent in (i) and the y-axis.

3 (iii) Find the exact area of the region described in (ii).

QUESTION 4 (START A NEW PAGE)

3 (a) If $f'(x)=\cot x + x$ and $f\left(\frac{\pi}{2}\right)=0$, find an expression for f(x).

3 (b) Find the area bounded by $y=\sqrt{3-x}$ and the co-ordinate axes.

2 (c) (i) Draw a neat sketch showing the area bounded by the curve $y=1+\sqrt{x}$ and the x-axis for $0 \le x \le 4$.

4 (ii) Find the volume of the solid formed when the area in (i) is rotated one revolution about the y-axis.

QUESTION 5 **(START A NEW PAGE)**

(a) ABCD is a parallelogram and P is a point on the diagonal BD.

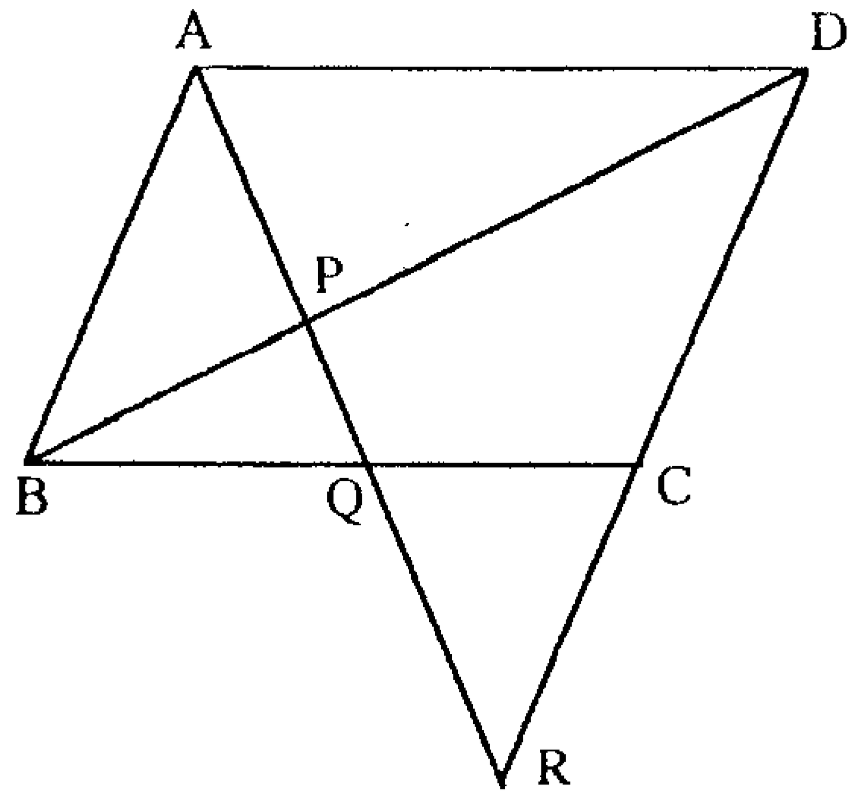
2 (i) Prove that ΔAPD and ΔQPB are similar.

If the ratio AP:PQ = 2:1

2 (ii) Prove that Q is the midpoint of BC.

2 (iii) Prove that ΔABQ and ΔRCQ are congruent.

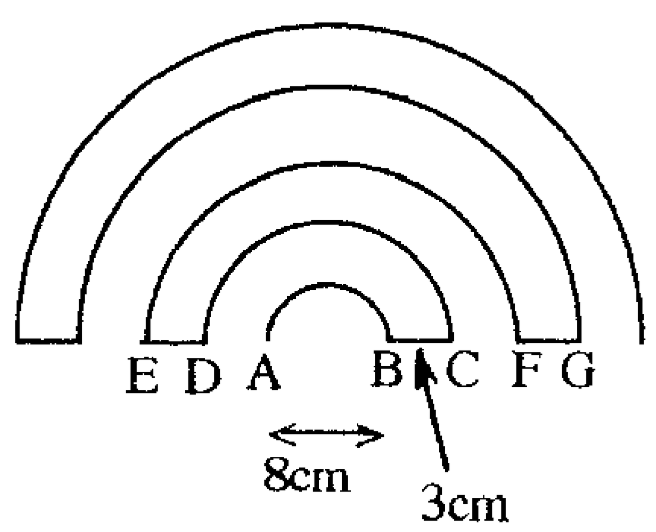
2 (iv) Prove that C is the midpoint of DR.



(b) A piece of wire (ABCDEF...) is used to make a pattern formed from many concentric semi-circular arcs (AB , CD , EF , ...) joined by short intervals (BC , DE , FG , ...). The diameter of the smallest semi-circle is 8cm and each interval is 3cm. (see diagram)

2 (i) What is the length of the 20th semi-circular arc?

2 (ii) How much wire is needed to form a pattern containing 20 semi-circular arcs?



THIS IS THE END OF THE PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

JAMES RUSE AGRICULTURAL H.S. 2UNIT TERM 1 ASSESSMENT 1998

QUESTION 1

(i) $\int \frac{1}{2}x + \frac{3}{2x}\, dx = \frac{1}{4}x^2 + \frac{3}{2}\ln x + c$

(ii) $\int \cos\frac{1}{3}x\, dx = 3\sin\frac{1}{3}x + c$

(iii) $\int x + 8\sqrt{x} + 16\, dx = \frac{1}{2}x^2 + \frac{16}{3}x^{3/2} + 16x + c$

(i) $\int_1^3 \frac{1}{2x-1}\, dx = \left[\frac{1}{2}\ln(2x-1)\right]_1^3$

$= \frac{1}{2}\ln 5 - \frac{1}{2}\ln 1$

$= \frac{1}{2}\ln 5$

(ii) $\int_{\pi/8}^{\pi/6} \sec^2 2x\, dx = \left[\frac{1}{2}\tan 2x\right]_{\pi/8}^{\pi/6}$

$= \frac{1}{2}\tan\frac{\pi}{3} - \frac{1}{2}\tan\frac{\pi}{4}$

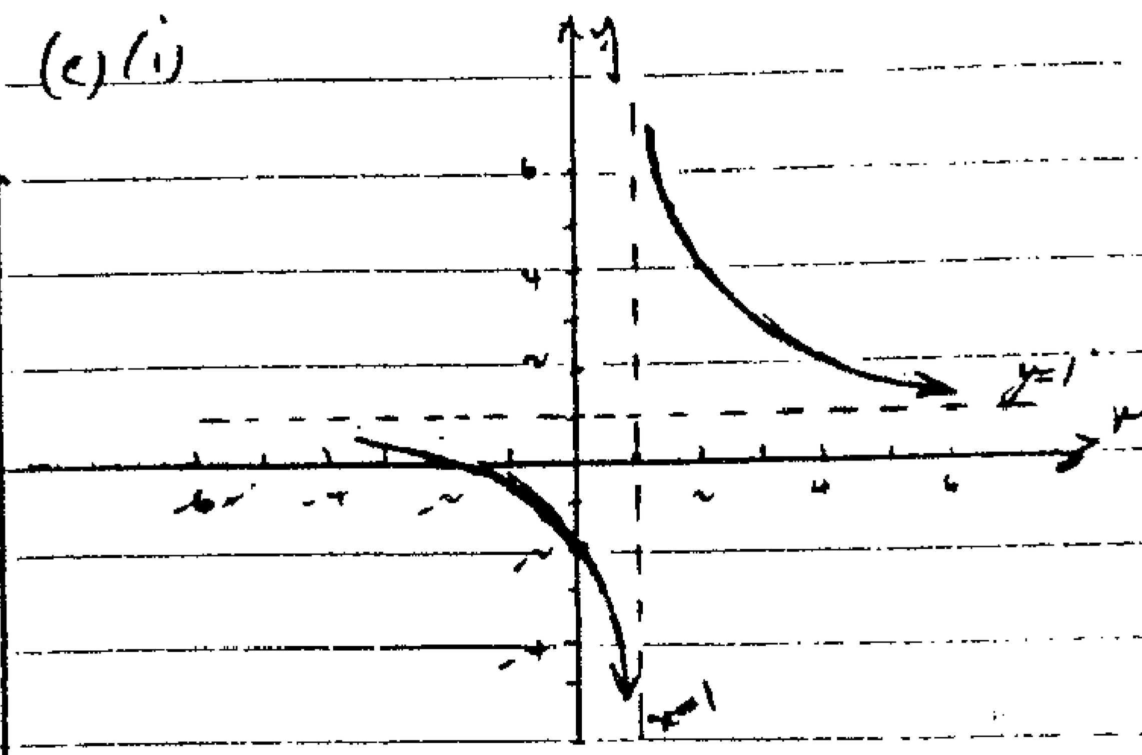
$= \frac{\sqrt{3}}{2} - \frac{1}{2}$

$a = 101 \quad d = -4$

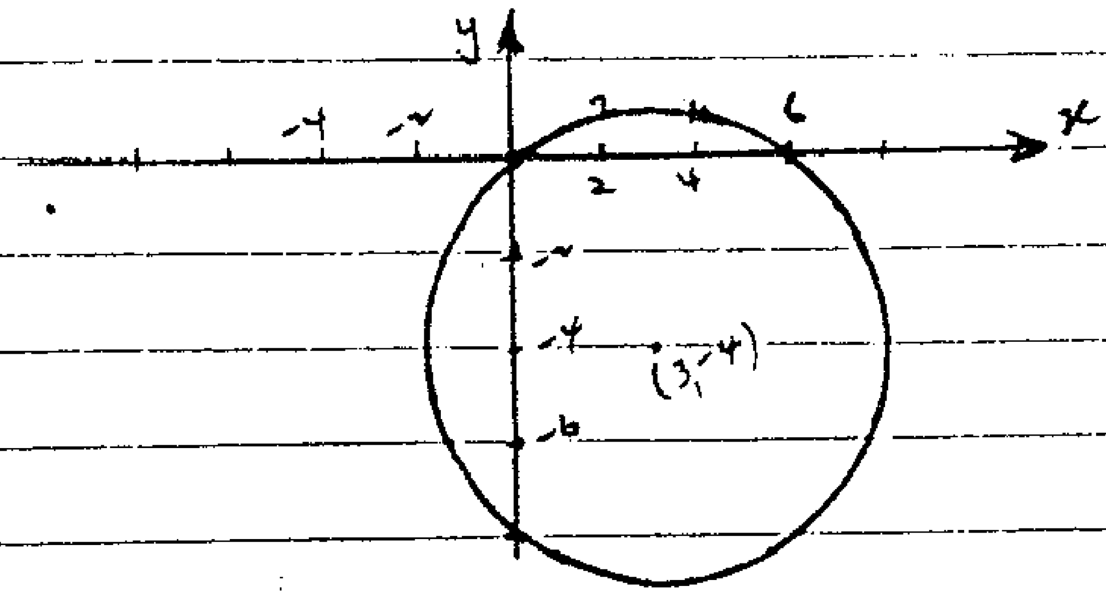
$T_n = 101 + (n-1)(-4)$

$= 105 - 4n$

(c)(i)



(ii) $(x-3)^2 + (y+4)^2 = 25$



QUESTION 2

Let $B\hat{A}C = x°$

$\therefore B\hat{C}A = x°$ (equal angles are opposite equal sides)

$A\hat{C}D = x°$ ($AB \parallel CD$, alternate angles)

$\therefore A\hat{C}B = A\hat{C}D$ (both $x°$)

$\therefore AC$ bisects $B\hat{C}D$ ($A\hat{C}B = A\hat{C}D$)

$V = \pi\int_{-1}^{2} y^2\, dx$

$= \pi\int_{-1}^{2} 4 - x^2\, dx$

$= \pi\left[4x - \frac{1}{3}x^3\right]_{-1}^{2}$

$= 9\pi$ cubic units

QUESTION 3

(a)(i) Value $= \$25 \times \left(1 + \frac{3.25}{5200}\right)^{50}$

$= \$25.79$

(ii) Value $= \$25\left(1.000625^{50} + 1.000625^{49} + \ldots + 1.000625\right)$

$= \$25 \times 1.000625 \frac{(1.000625^{50} - 1)}{1.000625 - 1}$

$= \$1270$

(b)(i) $y' = 3e^{3x}$

at $x=1$, $y' = 3e^3$

$y = e^3$

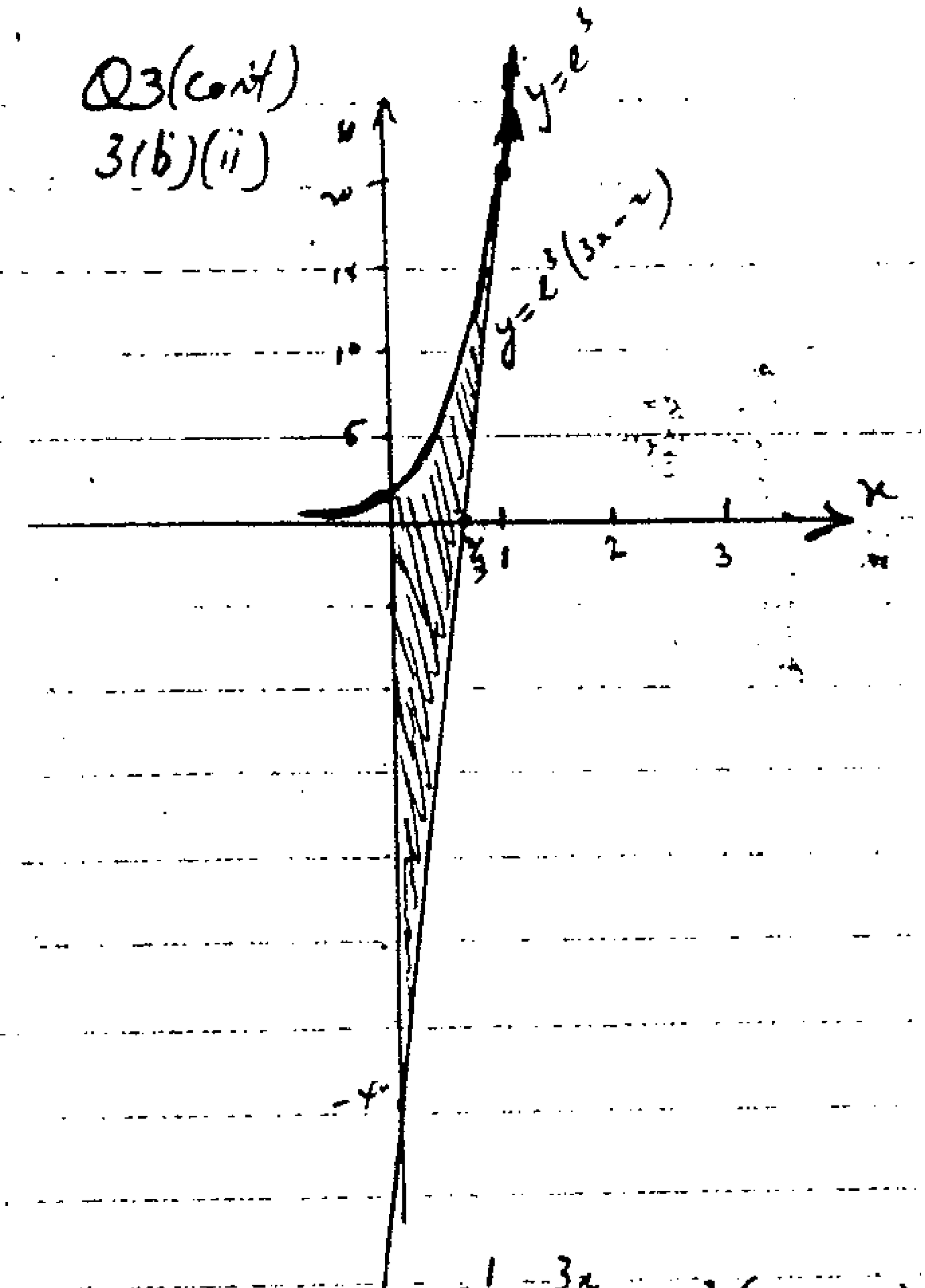
tangent is $y - e^3 = 3e^3(x-1)$

$y = 3e^3x - 2e^3$

$y = e^3(3x-2)$

Q3(cont)

3(b)(ii)



(iii) $A = \int_0^1 e^{3x} - e^3(3x-2)\, dx$

$= \left[\frac{1}{3}e^{3x} - \frac{3}{2}e^3x^2 + 2e^3x\right]_0^1$

$A = \frac{1}{6}(5e^3 - 2)$ square units

QUESTION 4

(a) $f'(x) = \frac{\cos x}{\sin x} + x$

$f(x) = \ln(\sin x) + \frac{1}{2}x^2 + c$

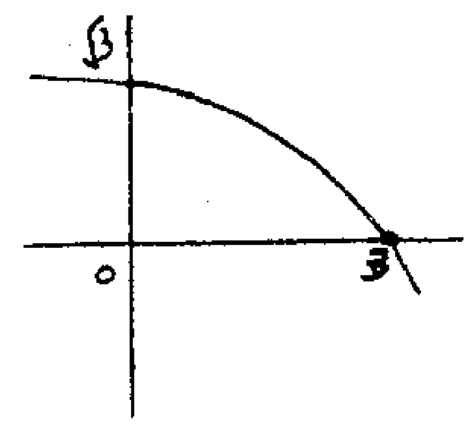
$f(\frac{\pi}{2}) = 0 \Rightarrow 0 = \ln(\sin\frac{\pi}{2}) + \frac{\pi^2}{8} + c$

$c = -\frac{\pi^2}{8}$

$\therefore f(x) = \ln(\sin x) + \frac{1}{2}x^2 - \frac{\pi^2}{8}$

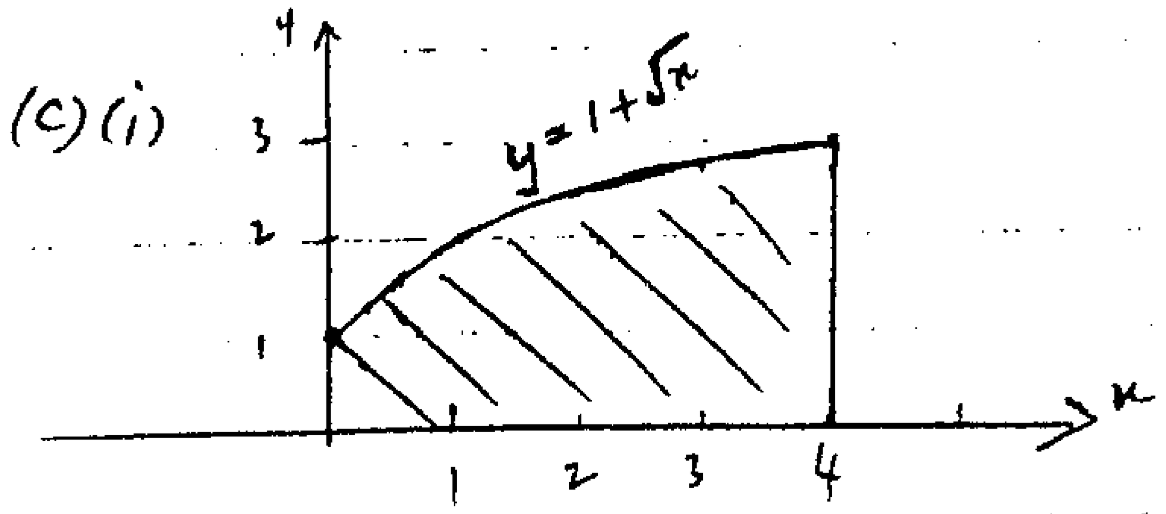
(b) $A = \int_0^3 (3-x)^{\frac{1}{2}}\, dx$

$= \left[-\frac{2}{3}(3-x)^{3/2}\right]_0^3$



$= 2\sqrt{3}$ square units.

(c)(i)



(ii) $V = \pi\int_0^3 16\, dy - \pi\int_1^3 (y-1)^4\, dy$

$= \pi\left[16y\right]_0^3 - \pi\left[\frac{1}{5}(y-1)^5\right]_1^3$

$= 48\pi - \frac{32}{5}\pi$

$= \frac{208}{5}\pi$ cubic units

QUESTION 5

(a)(i) In ΔAPD & ΔQPB

$A\hat{P}D = Q\hat{P}B$ (vertically opposite angles)

$A\hat{D}P = Q\hat{B}P$ ($AD \parallel BC$, opposite sides of parm, alternate angles)

$\therefore \Delta APD \,|||\, \Delta QPB$ (equiangular)

(ii) $\frac{BQ}{DA} = \frac{PQ}{AP}$ (ratios of corresponding sides)

$= \frac{1}{2}$

$\therefore BQ = \frac{1}{2}AD$

but $BC = AD$ (opposite sides of parm)

$\therefore BQ = \frac{1}{2}BC$

$\therefore Q$ is midpt of BC

(iii) In ΔABQ and ΔRCQ

$BQ = QC$ (proven in (ii))

5 (cont)

(iii) $A\hat{Q}B = C\hat{Q}R$ (vertically opposite angles)

$B\hat{A}Q = Q\hat{R}C$ (AB∥DR opposite sides of parm, alternate angles are equal)

$\therefore \Delta ABQ \equiv \Delta RCQ$ (AAS)

iv) $AB = CR$ (corresponding sides in congruent triangles)

$AB = DC$ (opposite sides of parm)

$\therefore DC = CR$ (both $= AB$)

$\therefore C$ is midpoint of DR.

b) (i) $\{4\pi, 7\pi, 10\pi, \ldots\}$ $a = 4\pi$, $d = 3\pi$

$\ell = 4\pi + 19 \times 3\pi$ $n = 19$.

length $= 61\pi$ cm

(ii) $\ell = \frac{20}{2}\{8\pi + 19(3\pi)\} + 19 \times 3$

$= 10(65\pi) + 57$ cm

length $= 650\pi + 57$ cm.