JAMES RUSE AGRICULTURAL HIGH SCHOOL
TERM 1 ASSESSMENT 1999
YEAR 12 2/3 UNIT

Time allowed: 85 minutes

All questions are to be attempted.
Approved calculators may be used.
All questions are of equal value.
Each question to be handed in separately.

QUESTION 1

(a) Find the area of the triangle bounded between the x,y axes and the line $3x - 8y = 12$.

(b) Integrate with respect to x:

(i) $f(x) = \dfrac{x}{\sqrt{x}}$

(ii) $f(x) = \dfrac{1}{2x-1}$

(iii) $f(x) = \cos 4x$

(c) An amount of \$1500 is invested at a rate of 4.5% per annum with the interest compounded monthly. How long will it take for the investment to double its value. (Answer to the nearest month.)

QUESTION 2 (Start a new page)

(a) The limiting sum of a geometric series is $10\frac{4}{5}$ and the first term is 18. Find the fifth term of the series.

(b) Evaluate:

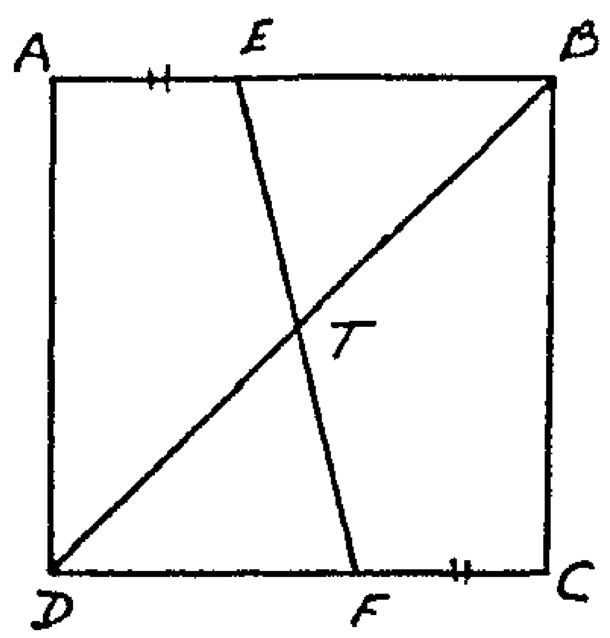
(i) $\displaystyle\int_0^4 2\pi \, dx$

(ii) $\displaystyle\int_1^2 \frac{x-4}{x^2} \, dx$

(iii) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}} e^{\sqrt{x}} \, dx$

QUESTION 3 (Start a new page)

(a) Find the area between the curve $y = x^2 - 2x$ and the line $y = 2x$.

(b) 

ABCD is a square with $AE = CF$, and EF and BD intersect at T. Prove that T bisects BD.

COPY THE DIAGRAM ONTO YOUR ANSWER SHEET

(c) (i) Draw a neat sketch of the curve with equation $y = \sqrt{4-x}$.

(ii) Find the area between the curve and the positive x and y axes.

QUESTION 4 (Start a new page)

(a) (i) A person contributes \$1000 annually in a superannuation fund, commencing at the beginning of 1985. If interest is paid at a rate of 6% per annum, find the value of the investment at the end of year 2010.

(ii) How many years must the contributor be in the fund in order for the investment to be worth \$100,000.

(b) Use Simpson's Rule with three function values to approximate the area between the x – axis, the curve $y = e^x \ln(x+1)$, and the lines $x = 0$ and $x = 2$. (Answer correct to two decimal places.)

(c) The sum of the first n terms of an arithmetic sequence is given by the formula: $S_n = 2n^2 - n$. Find:

(i) the first term.
(ii) the sum of the first six terms.
(iii) a formula for the nth term.

QUESTION 5 (Start a new page)

(a) (i) Draw a neat sketch of the curve with equation $y = \dfrac{x}{x-2}$, clearly showing all asymptotes and x,y intercepts.

(ii) Using the graph, or otherwise, find all solutions to $\dfrac{x}{x-2} > 0$.

(b) Find the volume generated when the area between the x-axis, the curve $y = \sqrt{\sin 2x}$, and the line $x = \frac{\pi}{8}$ is rotated about the x-axis.

(c) How many multiples of seven are there between the numbers 500 and 14,495.

(d) Draw a neat sketch of the curve with equation $y = \sqrt{8 + 2x - x^2}$.

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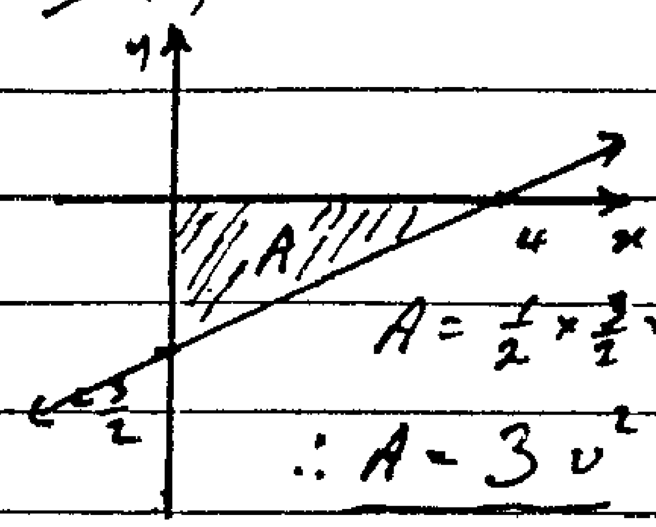
TERM I ASSESSMENT

YEAR 12 2/3 Unit 1999

SOLUTIONS

1 (a)



$A = \frac{1}{2} \times \frac{3}{2} \times 4$

$\therefore A = 3\ u^2$

(b)(i) $\int \frac{x}{\sqrt{x}}\,dx$

$= \int \sqrt{x}\,dx$

$= \frac{2}{3} x\sqrt{x} + C$

(ii) $\int \frac{dx}{2x-1}$

$= \frac{1}{2}\ln(2x-1) + c$

(iii) $\int \cos 4x\,dx$

$= \frac{1}{4}\sin 4x + C$

(c) Find t for $\$1500\left(1 + \frac{4.5}{1200}\right)^t = \$3000$

$\therefore 1.00375^t = 2$

$\therefore t = \frac{\ln 2}{\ln 1.00375}$

$t \doteqdot 186$ months

2 (a)

$S_\infty = 10\frac{4}{5}$

$a = 18$

$S_\infty = \frac{a}{1-r}$

$\therefore 10\frac{4}{5} = \frac{18}{1-r}$

$\therefore \frac{54}{5} = \frac{18}{1-r}$

$r = -\frac{2}{3}$

$T_5 = ar^4$

$= 18\left(-\frac{2}{3}\right)^4$

$= 18 \times \frac{16}{81}$

$\therefore T_5 = \frac{32}{9}\ \left(= 3\frac{5}{9}\right)$

(b)(i) $\int_0^4 2\pi\,dx$

$= \left[2\pi x\right]_0^4$

$= 2\pi(4-0)$

$= 8\pi$

(ii) $\int_1^2 \frac{x-4}{x^2}\,dx$

$= \int_1^2 \left(\frac{1}{x} - \frac{4}{x^2}\right) dx$

$= \left[\ln x + \frac{4}{x}\right]_1^2$

$= (\ln 2 + 2) - (\ln 1 + 4)$

$= \ln 2 - 2$

(iii) $\int_1^4 \frac{1}{\sqrt{x}} e^{\sqrt{x}}\,dx$

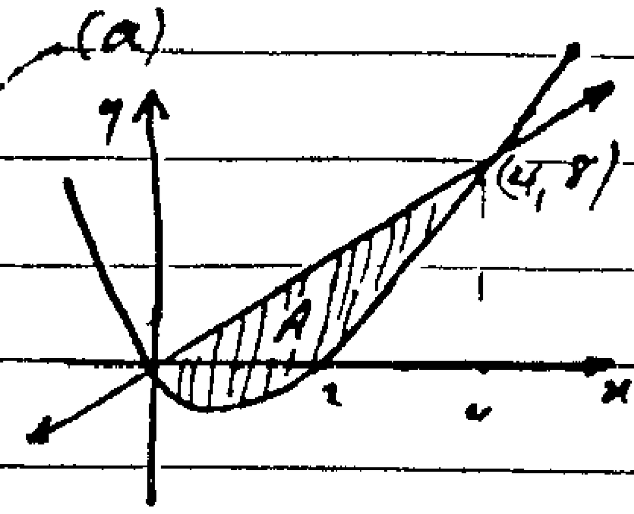
$= 2\int_1^4 \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\,dx$

$= 2\left[e^{\sqrt{x}}\right]_1^4$

$= 2\,(e^2 - e)$

$= 2e(e-1)$

3 (a)



$y = x^2 - 2x$

$y = 2x$

$\therefore x^2 - 4x = 0$

$x = 0, 4$

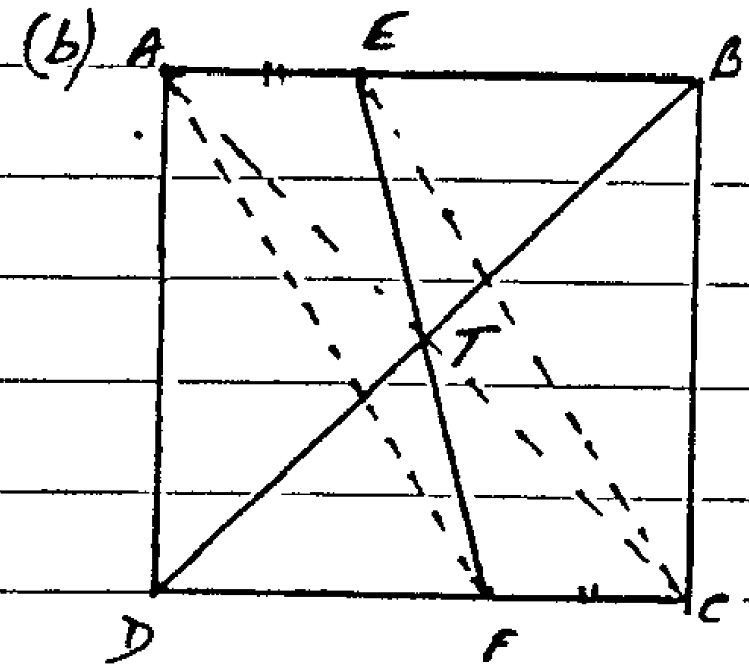
$A = \int_0^4 [2x - (x^2 - 2x)]\,dx$

$= \int_0^4 (4x - x^2)\,dx$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= 32 - \frac{64}{3}$ $\quad \therefore A = 10\frac{2}{3}\ u^2$

(b)



Join AF, EC and AC.

AECF is a para (AE, CF equal and parallel)

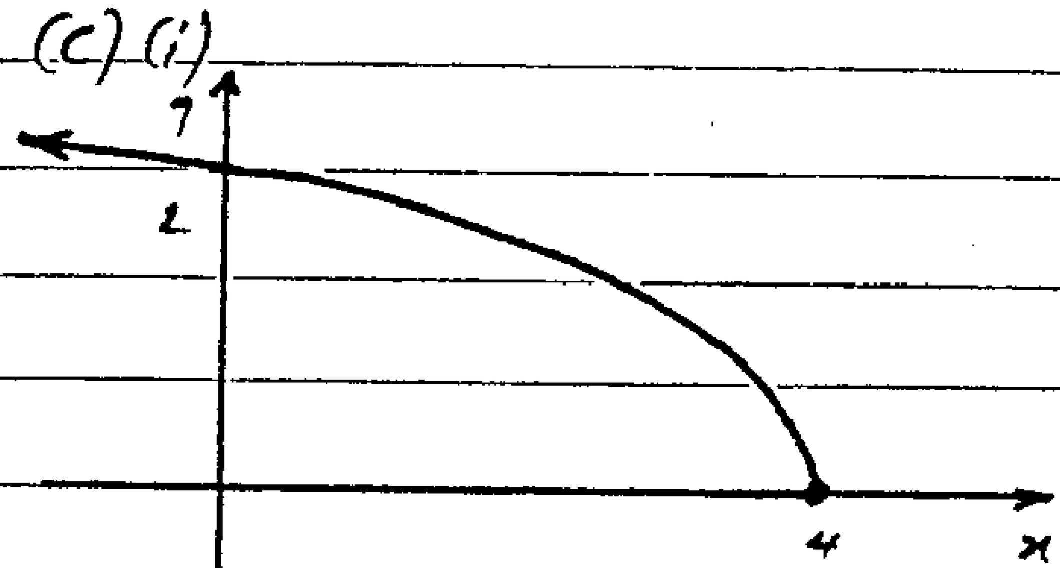
Now AC bisects EF at T (diagonals para AECF bisect each other)

Similarly AC bisects BD at T.

$\therefore$ EF bisects BD at T.

[Also proven by eg; $\Delta TEB \equiv \Delta TFD \Rightarrow BT = TD$]

(c)(i)



(ii) $A = \int_0^4 \sqrt{4-x}\,dx$

$= \left[-\frac{2}{3}(4-x)^{\frac{3}{2}}\right]_0^4$

$= \left(-\frac{2}{3} \times 0\right) - \left(-\frac{2}{3} \times 4^{\frac{3}{2}}\right)$

$\therefore A = 5\frac{1}{3}\ u^2$

4 (a)(i) Since beginning of 1985 to end of 2010 is 26 years:

$I = \$1000\left[1.06^{26} + 1.06^{25} + 1.06^{24} + \dots + 1.06\right]$

$= \$1000 \times 1.06\left[\frac{1.06^{26} - 1}{.06}\right]$

$\therefore I \doteqdot \$62\,706$

We must find t such that

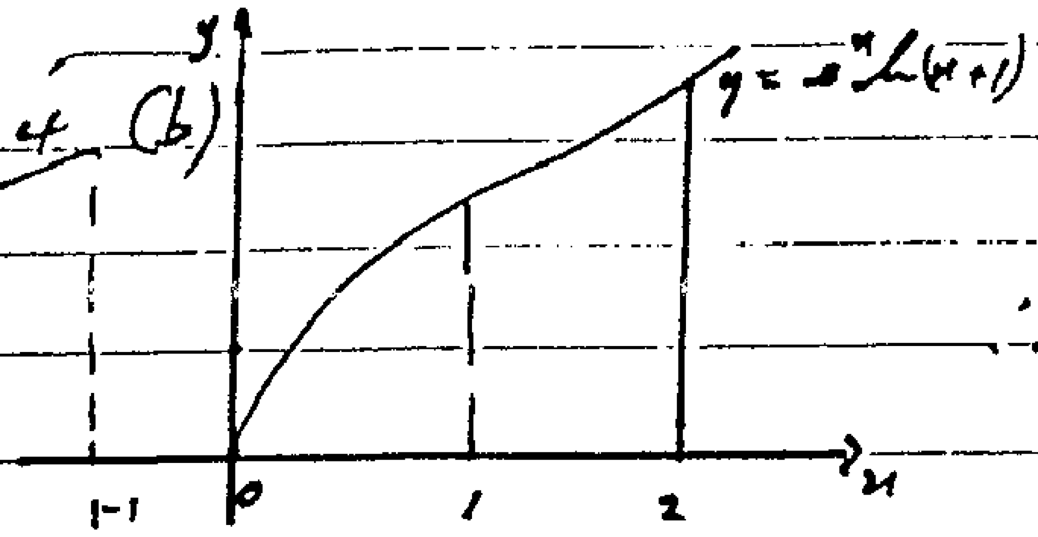
$\$1000\left[1.06^t + 1.06^{t-1} + \dots + 1.06\right] = \$100,000$

$\therefore 1.06\left[\frac{1.06^t - 1}{.06}\right] = 100$

$1.06^t = \frac{100 \times 0.06}{1.06} + 1$

$\therefore t \doteqdot 32.5$ years

4. (b)



$A \doteqdot \left(\frac{2-0}{6}\right)\left[e^0 \ln 1 + 4 . e^1 \ln 2 + e^2 \ln 3\right]$

$\therefore A \doteqdot 5.22\ u^2$

(c) $T_n = S_n - S_{n-1}$

$= (2n^2 - n) - \left[2(n-1)^2 - (n-1)\right]$

$= (2n^2 - n) - (2n^2 - 4n + 2 - n + 1)$

$\therefore T_n = 4n - 3$ (iii)

(i) $T_1 = 1$

(ii) $S_6 = \frac{6}{2}\left[2.1 + 5.4\right]$

$= 3\ (2 + 20)$

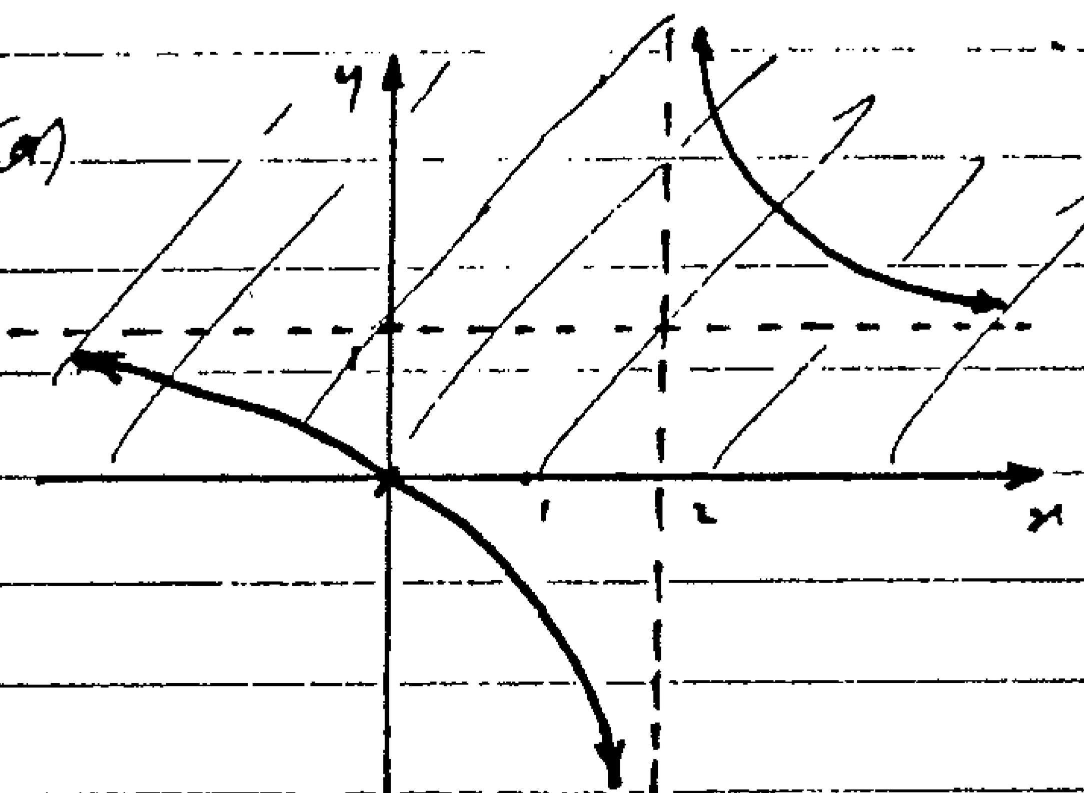
$\therefore S_6 = 66$

OR (iii) $T_n = 1 + (n-1)4$

$\therefore T_n = 4n - 3$

5 (a)

(i)



(ii) From the graph, the solution to $\frac{x}{x-2} > 0$ is $x < 0$ and $x > 2$

(b) $V = \pi \int_0^{\frac{\pi}{8}} f(x)^2\, dx$

$= \pi \int_0^{\frac{\pi}{8}} \sin 2x\, dx$

$= \frac{\pi}{2}\left[-\cos 2x\right]_0^{\frac{\pi}{8}}$

$= \frac{\pi}{2}\left[-\frac{1}{\sqrt{2}} + 1\right]$

$\therefore V = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2} - 1\right) u^3$

(c) Between 500 and 14,495 we require terms in $\{504, 511, \text{——}, 14490\}$

$T_n = a + (n-1)d$

$\therefore 14490 = 504 + (n-1)7$

$\therefore n = 1999$

(d)

$y = \sqrt{8 + 2x - x^2}$

$= \sqrt{9 - (x-1)^2}$

Semi-circle centre at $(1, 0)$ and radius 3.