JAMES RUSE AGRICULTURAL HIGH SCHOOL

TERM 1 ASSESSMENT 2000

YEAR 12 2/3 UNIT

Time Allowed: 85 minutes
Instructions:
Start each question on a new page
Silent approved calculators may be used
All questions are of equal value
Marks may not be awarded for poorly arranged work
Standard integral tables are at the end of this paper

QUESTION 1:

(a) Evaluate $\sum_{r=1}^{3}(2r^2 - 7r + 5)$

(b) Integrate with respect to x:

(i) $\dfrac{x+2}{x}$

(ii) $2\sec^2 2x$

(iii) $(x^2 - 2)^2$

(c) Evaluate exactly:

(i) $\displaystyle\int_1^2 \frac{dx}{x^2}$

(ii) $\displaystyle\int_0^2 (x + e^{-x})dx$

QUESTION 2:

(a) A polygon has 25 sides, the lengths of which form an arithmetic sequence. If the smallest side is 8cm and the next larger side is 11cm, find the:

(i) length of the longest side

(ii) perimeter of the polygon in metres.

(b) The sketch shows the curve $y = x^{\frac{1}{3}}$ in the first quadrant

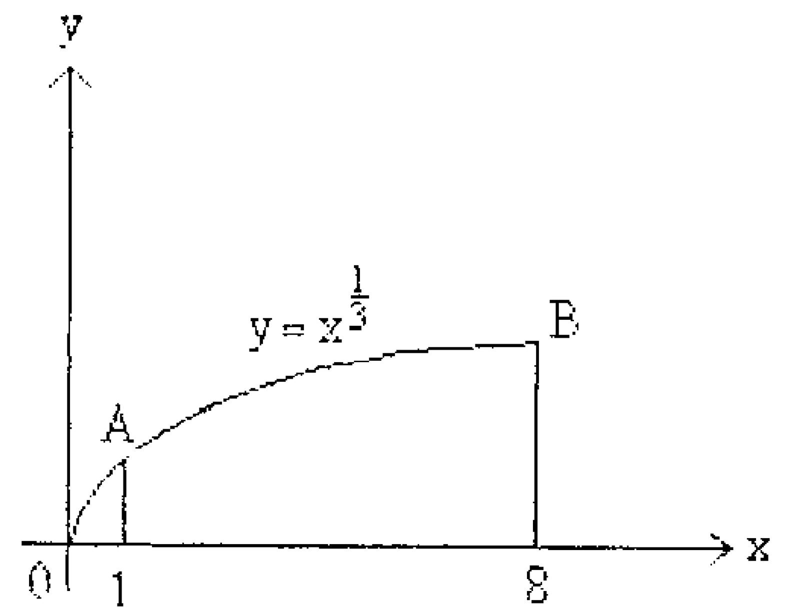


Diagram not to scale

(i) Determine the area bounded by the arc AB and the following lines:

(α) the x-axis, $x = 1$, $x = 8$

(β) the y-axis, $y = 1$, $y = 2$

(ii) the arc AB is rotated about the y-axis. Find the volume of the solid so generated.

QUESTION 3:

(a)

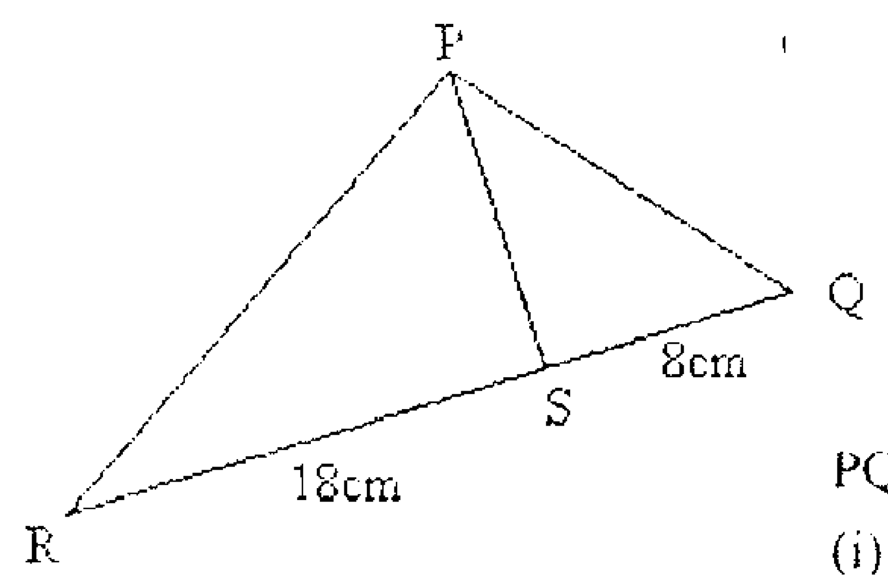


Diagram not to scale

$PQ \perp PR$, $PS \perp RQ$

(i) Prove $\Delta PQS ||| \Delta PRS$

(ii) Find the length of PS.

(b) (i) Sketch the curve $y = \frac{3-x}{3+x}$. Clearly label any asymptotes and x and y intercepts.

(ii) Using the graph, or otherwise, find all solutions to $\frac{3-x}{3+x} > 0$

(c)

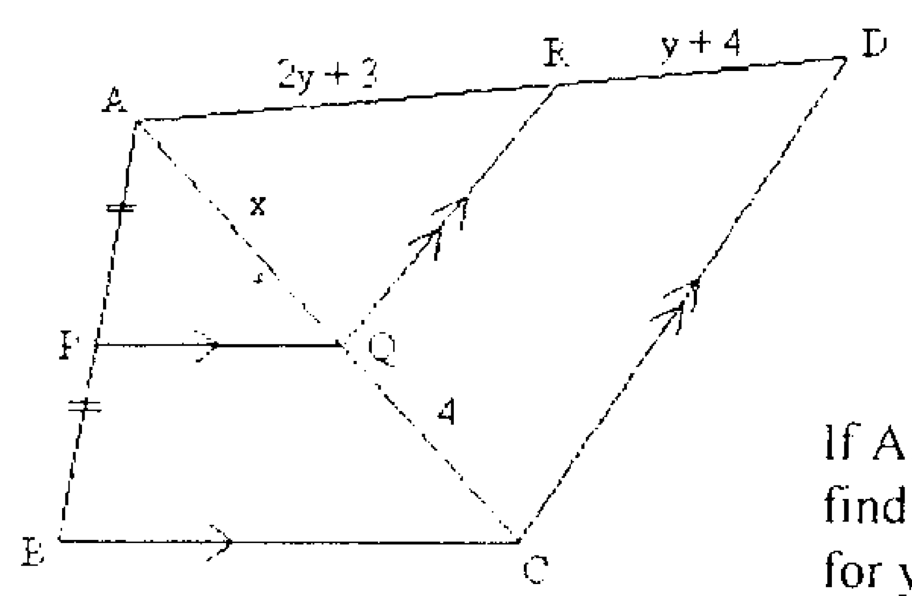


Diagram not to scale

If AP = PB, PQ || BC and QR || CD, find the values of x and y giving reasons for your answer.

QUESTION 4:

(a) Draw a clear sketch of the region which satisfies all three inequalities

$x^2 + (y-1)^2 \leq 1$, $y - x > 1$ and $y \geq 0$.

(b) Evaluate exactly:

$$\int_1^2 \frac{3}{x+1}\,dx$$

(c) A man borrowed \$2000 on the 1st January 1995. He agreed to pay back \$300 on 1st January in each succeeding year and to add to the debt 6 per cent per annum interest on the amount owing the year just completed. Find:

(i) the amount still owing immediately after 1st January 1996.

(ii) Show that after n repayments, the amount owing (A_n) is given by

$$A_n = 5000 - 3000(1.06)^n$$

(iii) the number of years necessary to free himself of the debt.

QUESTION 5:

(a) Use Simpson's rule with 5 function values to estimate the volume of the solid formed by rotating the curve $\frac{1}{2x^2+3}$ about the x-axis between x = -2 and 2 correct to 2 decimal places.

(b) Consider the sequence $a_1, a_2, a_3, \ldots$ with general term a_n. Given that the sum of the first n terms of the sequence is

$$S_n = \frac{n}{3n+1}$$

(i) determine S_1, S_2 and S_3

(ii) determine a_1, a_2 and a_3

(iii) determine a_n

(iv) determine the sum to infinity of the sequence

THIS IS THE END OF THE PAPER

2 UNIT TERM 1 2000 YR 12

ANSWERS

QUESTION 1: (12 marks)

(a) $\sum_{r=1}^{3}(2r^2-7r+5)=(2-7+5)+(8-14+5)+(18-21+5)$

$=0-1+2$

$=1$ [2]

(b)(i) $\int\left(\frac{x+2}{x}\right)dx = \int\left(1+\frac{2}{x}\right)dx$ [2]

$= x+2\ln x+C$

(ii) $\int 2\sec^2 2x\,dx = 2\times\frac{1}{2}\tan 2x + C$

$=\tan 2x + C$ [2]

(iii) $\int(x^2-2)^2dx = \int(x^4-4x^2+4)dx$

$=\frac{x^5}{5}-\frac{4x^3}{3}+4x+C$ [2]

(c)(i) $\int_1^2\frac{dx}{x^2} = \int_1^2 x^{-2}dx$

$=\left[\frac{-1}{x}\right]_1^2$

$=-\frac{1}{2}+1$

$=\frac{1}{2}$ [2]

(ii) $\int_0^2(x+e^{-x})dx = \left[\frac{x^2}{2}-e^{-x}\right]_0^2$

$=(2-e^{-2})-(0-e^0)$

$=3-\frac{1}{e^2}$ [2]

QUESTION 2: (12 marks)

(a) $a=8$cm

$a+d=11$cms

$\therefore d=3$cm

(i) $T_{25}=a+24d$

$=8+24\times3$

$\therefore$ Longest side $=80$cm [2]

(ii) $S_{25}=\frac{25}{2}(8+80)$

$=1100$cms

$=11$m [3]

(b)(i) $A=\int_1^8 x^{\frac{1}{3}}dx$

(α) $=\left[\frac{3}{4}x^{\frac{4}{3}}\right]_1^8$

$=\left[\frac{3}{4}\times8^{\frac{4}{3}}\right]-\left[\frac{3}{4}\times1\right]$

$=\left[\frac{3}{4}\times16\right]-\left[\frac{3}{4}\right]$

$=11\frac{1}{4}\ u^2$ [3]

(β) $A=\int_1^2 y^3dy$

$=\left[\frac{y^4}{4}\right]_1^2$

$=\frac{16}{4}-\frac{1}{4}$

$A=3\frac{3}{4}\ u^2$ [2]

(ii) $V=\pi\int_1^2 y^6dy$

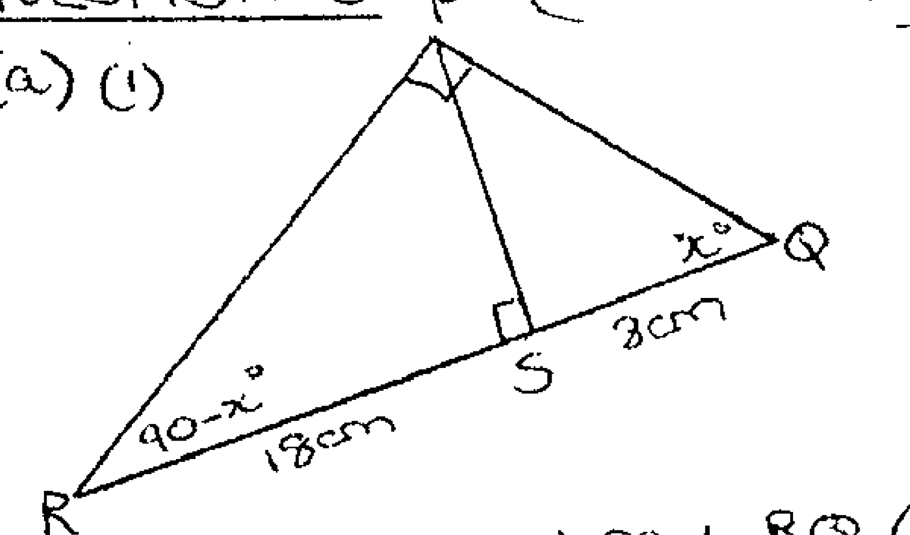
$=\pi\left[\frac{y^7}{7}\right]_1^2$

$=\pi\left[\frac{2^7}{7}-\frac{1}{7}\right]$

$=\frac{127\pi}{7}\ u^3$ [2]

QUESTION 3: (12 marks)

(a)(i)



$PQ\perp PR$ and $PS\perp RQ$ (given)

$\therefore \angle RPQ=\angle RSP=\angle PSQ=90°$

Let $\angle PQS = x°$

In $\triangle PRQ$

$\angle PRS=90-x°$ (angle sum of $\triangle$)

In $\triangle PSQ$

$\angle SPQ=90-x°$ (angle sum of $\triangle$)

In $\triangle RSP$

$\angle RPS=x°$ (angle sum of $\triangle$)

$\therefore \triangle PQS\ |||\ \triangle PRS$ (equiangular) [3]

QUESTION 3 (cont.): ②

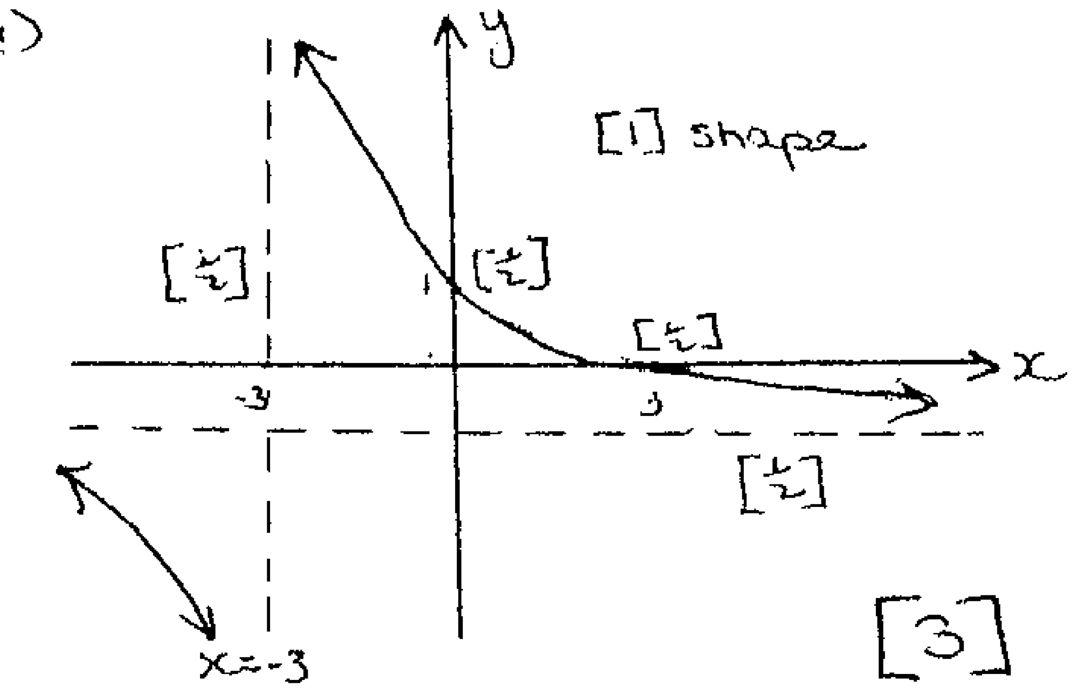
(ii) $\triangle PQS\ |||\ \triangle PRS$ (proven in (i))

$\therefore \frac{18}{PS}=\frac{PS}{8}$ (corresp. sides in similar $\triangle$'s are in proportion)

$PS^2=144$

$PS=12$cm [2]

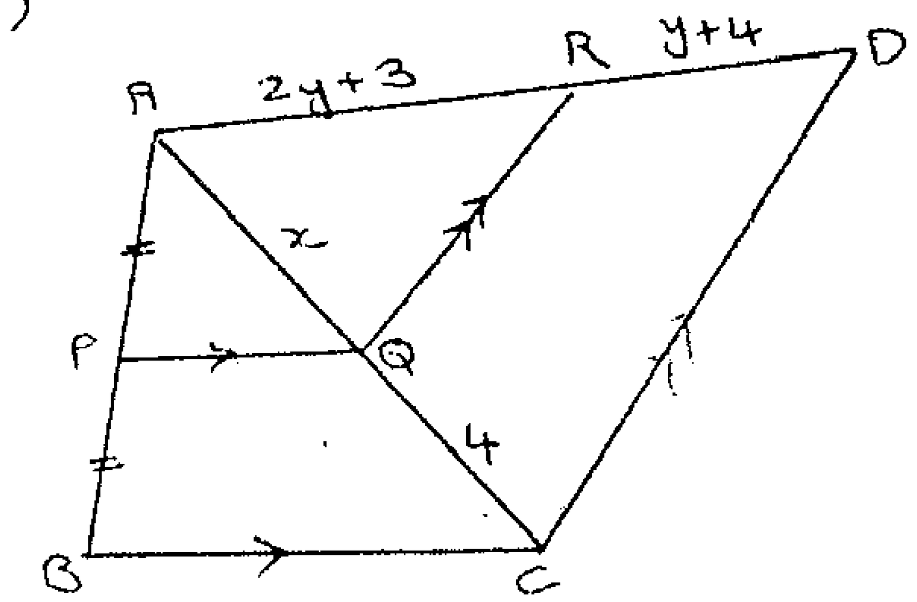
(b)(i)



[3]

(ii) $\frac{3-x}{3+x}>0$ for $-3<x<3$ [1]

(c)



$AP=PB$ (given)

$AQ=QC$ (equal intercepts on parallel lines)

$\therefore x=4$ [1]

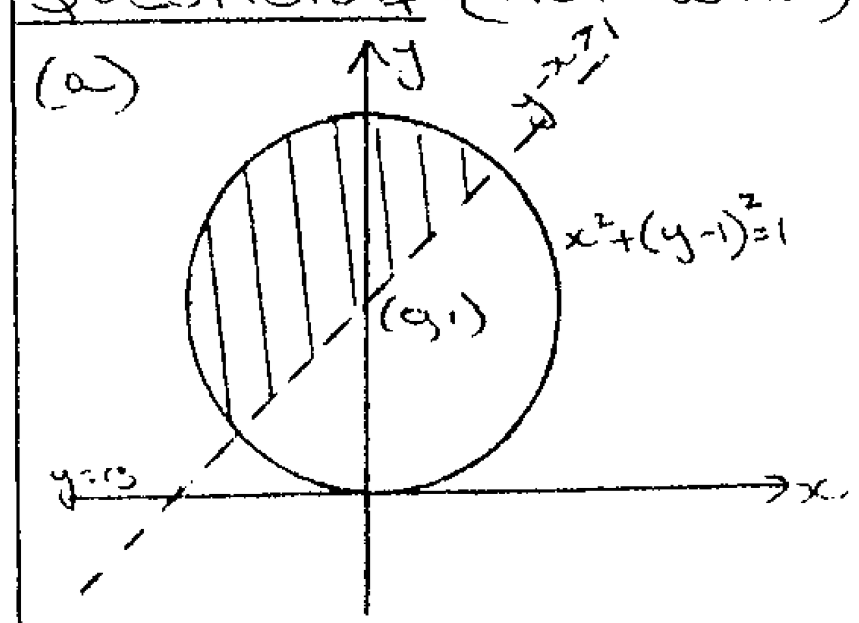
Similarly $AR=RD$

$\therefore 2y+3=y+4$

$y=1$ [2]

QUESTION 4: (12 marks)

(a)



[4]

(b) $\int_1^2\frac{3}{x+1}dx = 3\int_1^2\frac{1}{x+1}dx$

$=3\left[\ln(x+1)\right]_1^2$

$=3(\ln3-\ln2)$

$=3\ln1.5$ [2]

(c)(i) $A=\$2000\times1.06-\$300$

$\therefore A=\$1820$ [1]

(ii) $Amt=\$2000\times1.06^n-300\left(\frac{1(1.06^n-1)}{1.06-1}\right)$

$=\$2000\times1.06^n-5000(1.06^n-1)$

$=\$2000\times1.06^n-5000\times1.06^n+5000$

$\therefore A = 5000-3000(1.06)^n$ [3]

(iii) Amt owing $=0$

$\therefore 0=5000-3000\times1.06^n$

$3000\times1.06^n=5000$

$1.06^n=\frac{5}{3}$

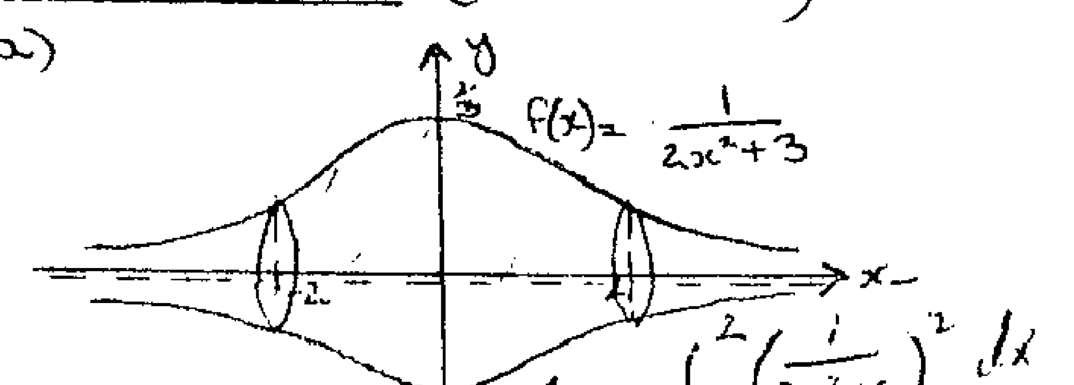
$n=\frac{\ln\frac{5}{3}}{\ln1.06}$

$n=8.7669291\ldots$

$\therefore n=9$ years [2]

QUESTION 5: (12 marks)

(a)



$V=\pi\int_{-2}^{2}\left(\frac{1}{2x^2+3}\right)^2dx$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{11}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{11}$
$[f(x)]^2$	$\frac{1}{121}$	$\frac{1}{25}$	$\frac{1}{9}$	$\frac{1}{25}$	$\frac{1}{121}$

$V=\pi\left[\frac{0+2}{6}\left(\frac{1}{121}+\frac{4}{25}+\frac{1}{9}\right)+\frac{2-0}{6}\left(\frac{1}{9}+\frac{4}{25}+\frac{1}{121}\right)\right]$

$=\frac{2\pi}{3}(0.279375573\ldots)$

$=0.5851228\ldots$

$V \doteqdot 0.59$ cubic units (2 d.p.) [3]

Question 5 (cont.)

(b) (i) $S_1 = \frac{1}{4}$

$S_2 = \frac{2}{7}$ [1] need 2 correct to get [$\frac{1}{2}$]

$S_3 = \frac{3}{10}$

(ii) $a_1 = S_1 = \frac{9}{4}$

$a_2 = S_2 - S_1$ [1] need 2 correct to get [$\frac{1}{2}$]

$\therefore a_2 = \frac{1}{28}$

$a_3 = S_3 - S_2$

$= \frac{1}{70}$

(iii) $a_n = S_n - S_{n-1}$

$= \left[\frac{n}{3n+1}\right] - \left[\frac{n-1}{3n-2}\right]$

$= \frac{n}{3n+1} - \frac{n-1}{3n-2}$

$\therefore a_n = \frac{1}{(3n+1)(3n-2)}$ [2]

(iv) $S = \lim_{n \to \infty} \left[\frac{n}{3n+1}\right]$

$= \lim_{n \to \infty} \left[\frac{1}{3+\frac{1}{n}}\right]$

$= \frac{1}{3}$

$S_\infty = \frac{1}{3}$ [2]