# JAMES RUSE AGRICULTURAL HIGH SCHOOL
## Mathematics (2 unit)
## Year 12 Term 1 Assessment 2001

**TIME ALLOWED: 85 Minutes**
**INSTRUCTIONS**:

- Start each question on a new page
- All questions are of equal value
- Each question is to be **handed in separately**
- Marks may not be awarded for poorly arranged work.
- Standard integral tables are at the end of the paper.

## QUESTION 1: (12 marks) Start this question on a new page

a) Integrate with respect to $x$:

i) $\cos\left(\frac{\pi}{2}-x\right)$

ii) $\frac{1}{2(3x+1)^2}$

iii) $\frac{x}{1+3x^2}$

b) Evaluate:

i) $\int_1^2 \frac{3x^2-5x+7}{x}dx$

ii) $\int_0^{\frac{1}{4}} \left(\frac{\pi}{2}+e^{-4x}\right)dx$

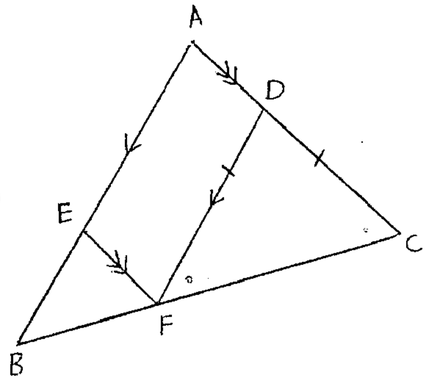
c) John receives \$5 pocket money each week when he is 5 years old, \$6 when he is 6, \$7 when he is 7 and so on. If he saves up all his pocket money for a car, how much will he have on his 17th birthday? (Do not include interest)

## QUESTION 2: (12 marks) Start this question on a new page

a) Using Simpson's Rule with 5 function values to give an estimate of $\int_0^2 3^x dx$.

(Give answer correct to 3 significant figures)

b) In $\Delta ABC$, DF=DC, DF||AB, EF||AC




i) Prove $\Delta ABC$ is isosceles (giving reasons)

ii) If $\angle BAC$ is 3 times the size of $\angle ACB$, find $\angle ACB$.

c) On separate graphs sketch the following curves, showing their intercepts with the coordinate axes, and any asymptotes if they exist:

i) $x^2+y^2-6x+8y=0$

ii) α) $y=\frac{4-x}{x+1}$

β) Using the graph, or otherwise, find all solutions to $\frac{4-x}{x+1}\le 0$

## QUESTION 3: (12 marks) Start this question on a new page

a) Find the limiting sum of the infinite series $\frac{1}{10^2}+\frac{2}{10^4}+\frac{4}{10^6}+\ldots\ldots\ldots\ldots$

b) F.B.Jones invests \$5000 at an interest rate of 12.5% p.a. for 4 years compounded six monthly.

i) Find, to the nearest dollar. The interest earned.
ii) Find, to the nearest 0.1%, the annual interest rate that would bring the same return in 3 years (ie. If \$5000 was invested for 3 years and compounded monthly).

c) i) On the same graph, sketch $y=(x+1)^2+3$ and $y=6x$, showing clearly any points of intersection and the $x$/y intercepts.

iii) Find the exact area bounded by these two graphs and the line $x=-1$.

## QUESTION 4: (12 marks) Start this question on a new page

a) Evaluate $\sum_{r=1}^{n} 2(3^r)$

b) i) Prove $\Delta PQT \equiv \Delta PSR$ (giving reasons)

ii) Find length RS

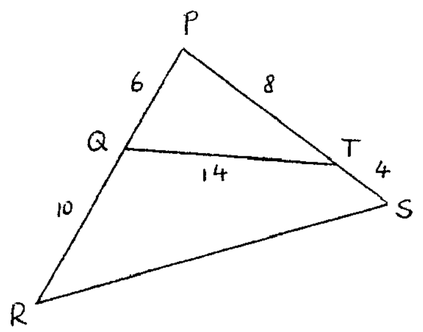
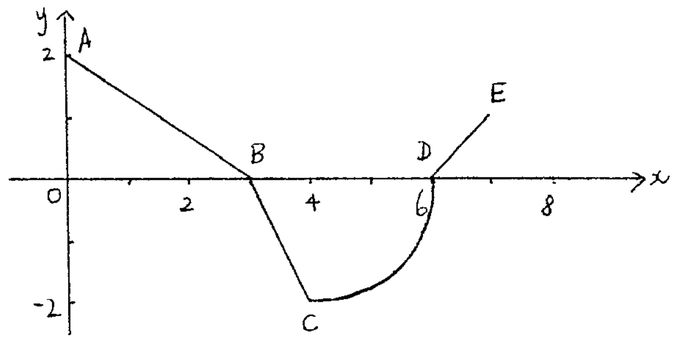



Diagram not to scale

c) The graph of the function y= f($x$) consists of the line segments AB, BC, DE and a quarter circle CD as shown below.

i) Evaluate $\int_0^7 f(x)dx$

ii) For what values of $x$ in the domain $0<x<7$ is the function not differentiable.



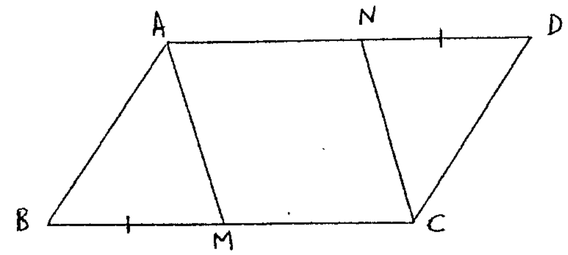

d)The cost of a new car is \$C. Its market value falls annually in geometric progression and is $\$\frac{C}{10}$ at the end of 10 years.

i) Find the common ratio of the progression, correct to 4 decimal places.

ii) If the new car costs \$35 000, find the market value at the end of 5 years (to nearest dollar)

## QUESTION 5: (12 marks) Start this question on a new page

a) ABCD is a parallelogram. BM=ND
Prove AMCN is a parallelogram



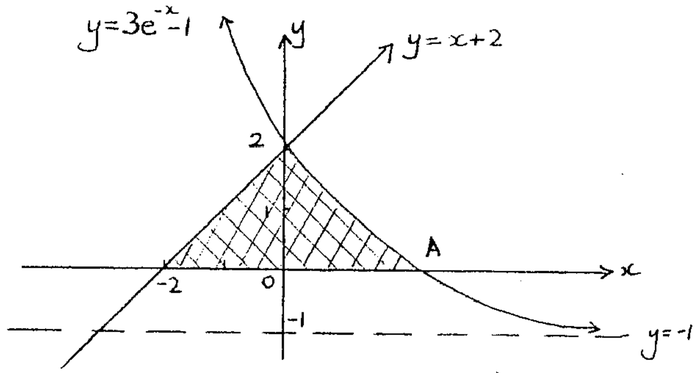

b) Sabrina bought a VCR for \$2000. She paid a deposit of \$150, and paid the balance off in equal monthly installments at 15% p.a. interest. If the loan was paid off after 24 months and Sabrina did not have to make any repayments for the first 3 months. Find,

i) the amount owing after that 3 months,
ii) an expression for the amount owing after 4 months where M, is the monthly repayment,
iii) derive an expression for the amount owing after n months,
iv) the amount of each monthly repayment (to nearest dollar).

c) The shaded area below shows the area bounded by y=$x$+2, $y=3e^{-x}-1$ and the x-axis.

iii) Show that the point A has coordinate ($ln$ 3, 0).

iv) Find the volume if the shaded area is rotated about the x-axis.




END OF PAPER

YR 12 TERM 1

# SOLUTIONS TO 2001 (2U) MATHEMATICS

## Question (1)

a) (i) $\int \cos\left(\frac{\pi}{2}-x\right) dx$

$= -\sin\left(\frac{\pi}{2}-x\right) + C$

OR $= -\cos x + C$

(ii) $\int \frac{1}{2(3x+1)^2} dx$

$= \frac{1}{2}\int (3x+1)^{-2} dx$

$= \frac{1}{2} \cdot \frac{1}{3} \frac{(3x+1)^{-1}}{-1} + C$

$= -\frac{1}{6}(3x+1)^{-1} + C$

OR $-\frac{1}{6(3x+1)} + C$

(iii) $\int \frac{x}{1+3x^2} dx$

$= \frac{1}{6}\ln(1+3x^2) + C$

b)(i) $\int_1^2 \frac{3x^2-5x+7}{x} dx$

$= \int_1^2 \left(3x - 5 + 7x^{-1}\right) dx$

$= \left[\frac{3x^2}{2} - 5x + 7\ln x\right]_1^2$

$= \left(6 - 10 + 7\ln 2\right) - \left(\frac{3}{2} - 5\right)$

$= 7\ln 2 - \frac{1}{2}$

(ii) $\int_0^{\frac{1}{4}} \left(\frac{\pi}{2} + e^{-4x}\right) dx$

$= \left[\frac{\pi}{2}x - \frac{1}{4}e^{-4x}\right]_0^{\frac{1}{4}}$

$= \left(\frac{\pi}{8} - \frac{1}{4e}\right) - \left(-\frac{1}{4}\right)$

$= \frac{\pi}{8} - \frac{1}{4e} + \frac{1}{4}$

c) $5\times52 + 6\times52 + 7\times52 + \cdots + 16\times52$

$= 52\left(5+6+7+8+\cdots+16\right)$

$= 52\left(\frac{12}{2}(5+16)\right)$

$= \$\,6552$

NB: (2) each, total = 12 marks

## Question 2

a)

| $x$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 |
|---|---|---|---|---|---|
| $f(x)$ | $3^0$ | $3^{\frac{1}{2}}$ | 3 | $3^{\frac{3}{2}}$ | $3^2$ |

$f(x) = 3^x$

$\therefore \int_0^2 3^x dx = \frac{1-0}{6}\left[3^0 + 4\times3^{\frac{1}{2}} + 3\right] + \frac{2-1}{6}\left[3 + 4\times3^{\frac{3}{2}} + 3^2\right]$

$= \frac{1}{6}\left[4 + 12 + 4\left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)\right]$

$= 7.29\ (3sf)$ (2)

b)(i) Let $\angle ACF = x$

$\angle DFC = x$ (angles opp. equal sides are equal)

$\angle ADF = 2x$ (ext. angle of $\Delta DFC$)

$\angle EAD = 180 - 2x$ (cointerior angles supplementary, $AE \parallel DF$)

$\angle ABC = 180 - (180 - 2x + x)$ (angle sum of $\Delta ABC$)

$= x$

$\therefore \Delta ABC$ is isosceles ($\angle ACB = \angle ABC = x$) (3)

(ii) $x + x + 3x = 180$, $\therefore 5x = 180$, $x = \frac{180}{5} = 36°$ (1)

c) (i) $x^2 - 6x + y^2 + 8y = 0$

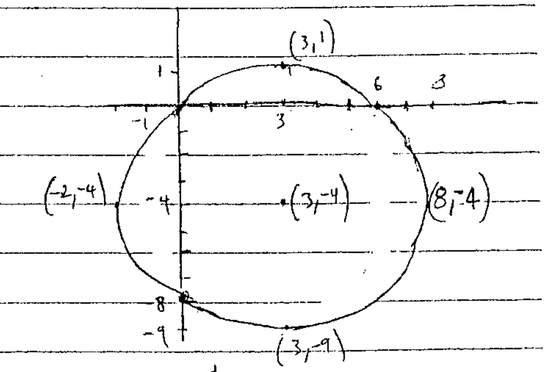
$x^2 - 6x + 9 + y^2 + 8y + 16 = 25$

$(x-3)^2 + (y+4)^2 = 25$

Centre $= (3, -4)$

Radius $= 5$

(2)



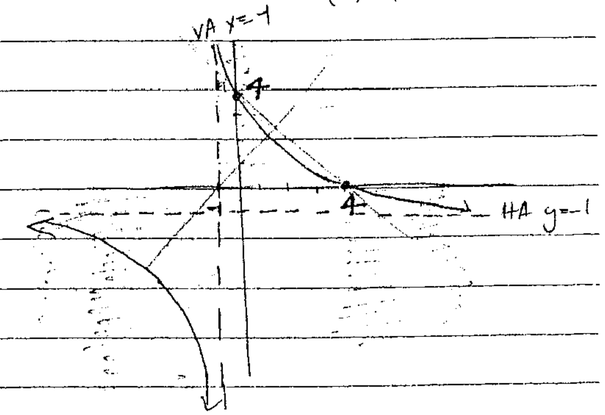

(ii) $y = \frac{4-x}{x+1}$

α) VA: $x = -1$

HA: $y = -1$

$y$ int $(x=0)$: $y = 4$

$x$ int $(y=0)$: $x = 4$




$x$/$y$ intercept ($\frac{1}{2}$ each) = 1

VA & HA ($\frac{1}{2}$ each) = 1

Shape = 1

(3)

β) $\frac{4-x}{x+1} \le 0 \Longrightarrow x < -1$ or $x \ge 4$ ($\frac{1}{2}$ each) (1)

## Question 3

a) $a = \frac{1}{10^2}$ $r = \frac{2}{10^2}$ $\therefore S = \frac{\frac{1}{10^2}}{1 - \frac{2}{10^2}} = \frac{1}{98}$ (2)

b) (i) Interest $= 5000(1.0625)^8 - 5000 = \$\,3121$ (2)

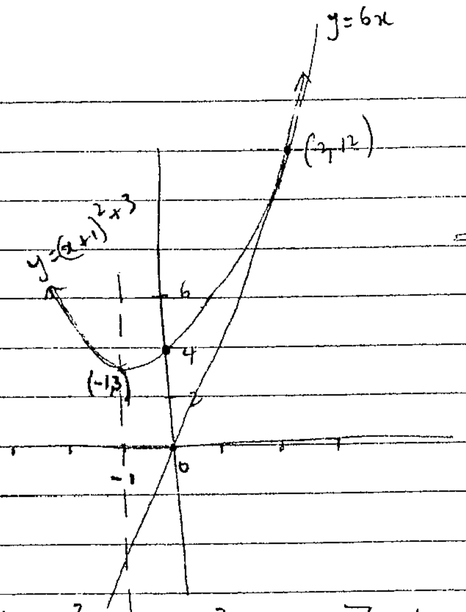
(ii) $(5000 + 3121) = 5000\left(1 + \frac{R}{100}\right)^6$

$\left(\sqrt[6]{\frac{8121}{5000}} - 1\right)100 = R$

$\therefore R = 8.4\%$ per six months

$\therefore R = 16.8\%$ per annum (2)

c) (i)




$y = (x+1)^2 + 3$
$y = 6x$
$\Rightarrow$ (2, 12) is pt of intersection

(3) $\Rightarrow$ 1 for (2,12)
x & y intercepts ($\frac{1}{2}$ each) = 1.
correct graphs ($\frac{1}{2}$ each) = 1

(ii) $A = \int_{-1}^{2} \left[ (x+1)^2 + 3 - 6x \right] dx$ 1

$= \int_{-1}^{2} (x^2 - 4x + 4)\, dx$

$= \left[ \frac{x^3}{3} - 2x^2 + 4x \right]_{-1}^{2}$ 1 (3)

$= \left( \frac{8}{3} - 8 + 8 \right) - \left( \frac{-1}{3} - 2 - 4 \right)$

$= 9\ u^2$ 1

Question 4

a) $T_n = 2(3^n)$ $a = 6$ $r = 3$

$S_n = \frac{6(3^n - 1)}{3-1} = 3(3^n - 1)$ (2)

b) (i) In $\Delta PQT$ & $PSR$: $\frac{PT}{PR} = \frac{8}{16} = \frac{1}{2}$, $\frac{PQ}{PS} = \frac{6}{12} = \frac{1}{2}$ } 1

$\angle P$ is common 1

$\therefore \Delta PQT \,|||\, \Delta PSR$ (2 adj sides in proportion & included angles equal) 1

(ii) $\frac{QT}{RS} = \frac{1}{2}$ (corresponding sides in same ratio)

ie $\frac{14}{RS} = \frac{1}{2} \Rightarrow RS = 28$ 1

c) (i) $\int_0^7 f(x)\, dx = \left(\frac{1}{2} \times 3 \times 2\right) - \left(\frac{1}{2} \times 1 \times 2\right) - \frac{1}{4}\pi \times 2^2 + \frac{1}{2} \times 1 \times 1$ 1

$= 3 - 1 - \pi + \frac{1}{2}$

$= \frac{5}{2} - \pi$ 1 (2)

(ii) B, C, D (1)

d) (i) Let Cost = C

Let $V_n$ = value after n years.

$V_1 = C \times \frac{1}{r}$

$V_2 = C \times \left(\frac{1}{r}\right)^2$

$\vdots$

$V_{10} = C \times \left(\frac{1}{r}\right)^{10}$

But $V_{10} = \frac{C}{10} \Rightarrow \frac{C}{r^{10}} = \frac{C}{10}$ 1

$r^{10} = 10$ (2)

$r = \sqrt[10]{10}$

$r = 1.2589$ (4 dp.) 1

(ii) If C = 35000

$V_5 = 35000 \left(\frac{1}{1.2589}\right)^5 = \$11069$ (1)

Question 5

a) $AN = AD - ND$

$MC = CB - BM$

Since $AD = CB$ (opp sides of parm ABCD equal)

& $ND = BM$ (given) (2)

Then: $AN = MC$.

Also, $AN \parallel MC$ (opp. sides of parm ABCD are parallel)

$\therefore$ ANCM is parm (one pair of opp. sides equal & parallel)

b) (i) After deposit, amount owing = 2000 − 150
= #1850

Let $A_n$ = amount owing after n months.

$A_3 = 1850(1.0125)^3$ (1)

$= \underline{\underline{1920}}$ (nearest dollar) OR #1920.24

(ii) $A_4 = 1850(1.0125)^4 - M$ (1)

OR $1920.24 \times (1.0125) - M \doteq 1944.25 - M$

(iii) $A_5 = \left(1850(1.0125)^4 - M\right) 1.0125 - M$

$= 1850(1.0125)^5 - M(1 + 1.0125)$

$\therefore A_6 = 1850(1.0125)^6 - M(1 + 1.0125 + 1.0125^2)$ (2)

$\vdots$

$A_n = 1850(1.0125)^n - M(1 + 1.0125 + 1.0125^2 + \cdots + 1.0125^{n-4})$

(iv) But $A_{24} = 0$

$0 = 1850(1.0125)^{24} - M(1 + 1.0125 + 1.0125^2 + \cdots + 1.0125^{20})$

$M = \dfrac{1850(1.0125)^{24}}{1 + 1.0125 + 1.0125^2 + \cdots + 1.0125^{20}}$

$= \dfrac{1850(1.0125)^{24}}{\left(\dfrac{1.0125^{21} - 1}{0.0125}\right)}$ (7)

$= \#104.53$

c) (i) $y = 3e^{-x} - 1$

$y = 0$

$3e^{-x} - 1 = 0$

$e^{-x} = \frac{1}{3}$ (2)

$-x = \ln\frac{1}{3}$ $\quad \therefore A = (\ln 3, 0)$

$x = -\ln\frac{1}{3}$

$\underline{x = \ln 3}$

(ii) $V = V_1 + V_2$

$= \pi\int_{-2}^{0} (x+2)^2\,dx + \pi\int_{0}^{\ln 3} \left(3e^{-x} - 1\right)^2 dx$

$= \frac{1}{3}\pi \times 2^2 \times 2 + \pi\int_{0}^{\ln 3} \left(9e^{-2x} - 6e^{-x} + 1\right) dx$

$= \frac{8\pi}{3} + \pi\left[\frac{-9}{2}e^{-2x} + 6e^{-x} + x\right]_0^{\ln 3}$

$= \frac{8\pi}{3} + \pi\left[\left(\frac{-9}{2}e^{-2\ln 3} + 6e^{-\ln 3} + \ln 3\right) - \left(\frac{-9}{2} + 6\right)\right]$

$= \frac{8\pi}{3} + \pi\left(-\frac{1}{2} + 2 + \ln 3 + \frac{9}{2} - 6\right)$

$= \frac{8\pi}{3} + \pi(\ln 3)$ (3)