# Year 12 Term 1 Assessment 2005
# Mathematics (2 unit)

## Question 1: (12 marks) — Marks

(a) Find:

(i) $\int \left( x^2 + \frac{8}{x} \right) dx$. — 2

(ii) $\int \sin 3x \, dx$. — 2

(iii) $\int \left( e^{6x} + 4x\sqrt{x} \right) dx$. — 2

(b) Evaluate:

(i) $\int_{-2}^{4} \frac{6}{3x+8} \, dx$. — 2

(ii) $\int_{0}^{\pi} \sec^2\left( \frac{x}{3} \right) dx$. — 2

(c) Find the value of $x$ if $2x-1$, $x$ and $3x+2$ are successive terms in an Arithmetic Progression. — 2

## Question 2: (12 marks)

(a) On a number plane clearly indicate the region defined by the intersection of the inequalities $y \le 2 - x$ and $x^2 + y^2 \ge 16$. — 4

(b) Find the 100$^{th}$ term in the Arithmetic Progression { -29, -25, -21, … }. — 3

(c) (i) Sketch the curve $y = x^3 - 4x^2$ clearly showing any intercepts with the coordinate axes. (You are not required to show the position of the turning points) — 2

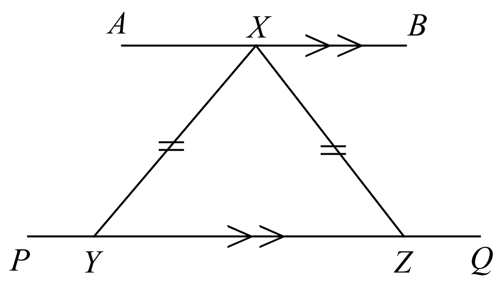
(ii) Find the area bounded by the curve $y = x^3 - 4x^2$ and the $x$-axis. — 3

## Question 3: (12 marks)

Marks

(a) Intervals $AB$ and $PQ$ are parallel and $XY = XZ$. **3**

Copy the diagram onto your answer sheet and prove that $\angle AXY$ and $\angle XZQ$ are supplementary.




(b) Does a Geometric Progression exist with a first term equal to 9 and a limiting sum equal to 15? Justify your answer. **3**

(c) (i) Find the points of intersection of the curves $y = 4 - x$ and $y = x^2 - 3x - 4$. **3**

(ii) Find the area bounded by the curves $y = 4 - x$ and $y = x^2 - 3x - 4$. **3**

## Question 4: (12 marks)

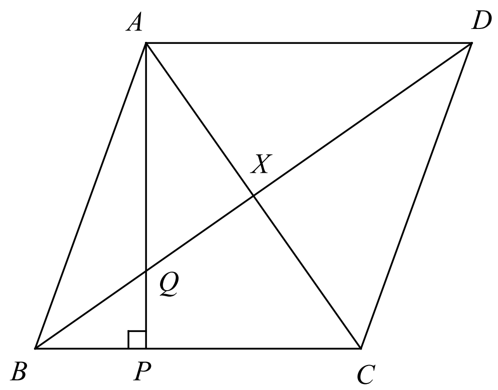
(a) (i) Sketch the curve $y = 2 - e^x$ clearly showing any intercepts with the coordinate axes. **2**

(ii) Find the volume of the solid formed when the area bounded by the curve $y = 2 - e^x$ and the coordinate axes is rotated one revolution about the $x$-axis. Write your answer in the form $\pi\left(A + B\log_e 2\right)$ where $A$ and $B$ are rational numbers. **3**

(b) Find the sum of the first 5 terms of the Geometric Progression $\{32, 24, 18, \ldots\}$. Write your answer as a mixed numeral in simplest form. **3**

(c) $ABCD$ is a rhombus whose diagonals intersect at $X$. $AP$ is perpendicular to $BC$ and intersects diagonal $BD$ at $Q$. **4**

Copy the diagram onto your answer sheet and prove that $\angle PAC = \angle ADB$.



## Question 5: (12 marks)

Marks

(a) (i) Sketch the curve $y = 8 - x^3$ clearly showing any intercepts with the coordinate axes. **2**

(ii) Find the volume of the solid formed when the area bounded by $y = 8 - x^3$ and the positive coordinate axes is rotated one revolution about the $y$-axis. **3**

(b) (i) Peter Perfect sets up a bank account to save for a holiday. He decides to deposit \$100 in the account at the start of each month commencing on February 1st 2005. Interest at a rate of 3% p.a. is paid at the end of each month on the balance in the account.

$(\alpha)$ Prove that the value $\$A_n$ of the account at the end of the $n^{th}$ month is given by the formula $A_n = 40100\left(1.0025^n - 1\right)$. **2**

$(\beta)$ How many months will Peter need to contribute to the account if he needs \$3000 for his holiday? Give your answer correct to the nearest month. **2**

(ii) In part (i) Peter decides to increase each monthly deposit by 2% with the first increase in March 2005.

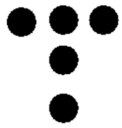
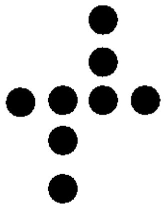
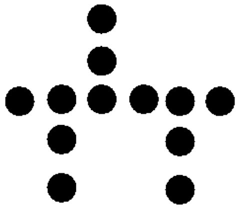
$(\alpha)$ Find a formula for the value $\$B_n$ of the account at the end of the $n^{th}$ month. **2**

$(\beta)$ Find the value of the account at the end of 24 months. Give your answer correct to the nearest dollar. **1**

⬟ THIS IS THE END OF THE EXAMINATION QUESTIONS ⬣

The table below shows the first three stages in a pattern constructed using buttons. 2

1

| Pattern number | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|
| Button Pattern |  |  |  |
| Number of buttons used | 5 | 8 | 11 |

(i) Find the number of buttons needed for the $10^{th}$ pattern ( $P_{10}$ ).

(ii) What is the greatest number of different stages can be illustrated using 1000 buttons?

(i) The area bounded by the curve $y = \log_3 x$, the coordinate axes and the line $y = 2$ is rotated one revolution about the $y$-axis. Find the volume of the solid formed. You may assume that $a^p = e^{p \ln a}$ 4

(i) On the same set of axes, sketch the curves $y = 2\sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$. 2

(ii) Find the area bounded by $y = 2\sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$. 2

(i) Find the volume of the solid formed when the area bounded by the curve $y = (x-2)^2$ and the positive coordinate axes is rotated one revolution about the $y$-axis. 2