QUESTION 1 (15 Marks) (Start a new sheet of paper) MARKS

i) Integrate with respect to x:

a) $\int e^{3x} - \sec^2 2x \ dx$ **2**

b) $\int \pi + \cos 3x \ \ dx$ **2**

c) $\int \ \dfrac{x^2+1}{x\sqrt{x}} \ dx$ **2**

ii) a) Show that $\dfrac{1}{x-1} - \dfrac{1}{x+1} = \dfrac{2}{x^2-1}$. **1**

b) Hence evaluate $\int_3^5 \dfrac{dx}{x^2-1}$ **2**

iii) Differentiate $x\sin x$ and hence find $\int x\cos x \ dx$ **3**

iv) The gradient function of a curve is given by $\dfrac{dy}{dx} = 3x^2 - 8x + 3$ and it is known that the original curve passes through the origin. Find the other points where the original curve crosses the x axis. **3**

QUESTION 2 (15 Marks) (Start a new sheet of paper) MARKS

i) Evaluate $\sum_{r=4}^{7} \dfrac{r^2}{2}$. **1**

ii) The sum of the first n terms of a particular sequence is given by $S_n = n^2 + 2^n$.
Find the first two terms of the sequence (ie Find T_1 and T_2). **2**

iii) In an Arithmetic Sequence, the thirteenth term is 27 and the seventh term is three times the second term. Find the sum of the first ten terms. **4**

iv) The curve $2y = x^2 - 5$ and the line $y = x - 1$ intersect at A and B (see Figure 1).

a) Show that the coordinates of A and B are $(-1,-2)$ and $(3,2)$. **2**

b) Find the area (shown shaded in Figure 1) between the curve and line. **3**

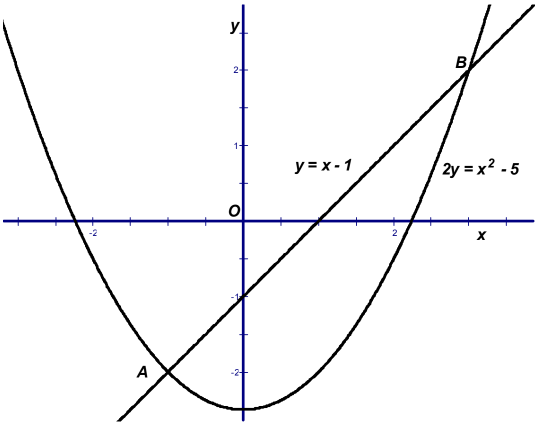

Figure 1

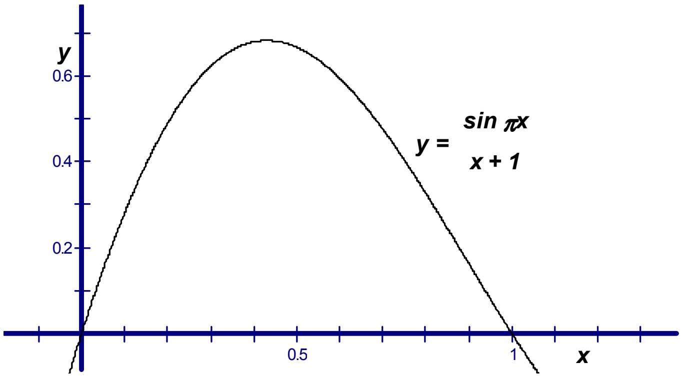

Figure 2

v) The curve $y = \dfrac{\sin \pi x}{x+1}$ is shown in Figure 2 for $0 \le x \le 1$. Use Simpson's Rule with 5 function values to find the area between the curve and the x axis. **3**

QUESTION 3 (15 Marks) (Start a new sheet of paper) MARKS

i) The curve $y = \sqrt{x-1}$ between $x = 1$ and $x = 5$ is rotated about the x axis to form a solid of revolution. Find the exact volume of the solid formed. **3**

ii) a) Sketch the curve $y = \sqrt{9-x^2}$ and, on the same diagram, draw the line $y = x+1$. **3**

b) On your diagram, shade the region or regions where $y \ge x+1$ and $y \le \sqrt{9-x^2}$ simultaneously. **2**

iii) George starts to save \$160 per month and invests it in a savings account which earns 6% pa, compounded monthly.

a) Show that, three months after his first deposit and just before the fourth deposit, his savings are worth \$484.82 (to the nearest cent). **2**

When he has accumulated \$5000, George is able to transfer his money into a higher yielding account which pays 9% pa, compounded monthly.

b) α) At the start of which month will George be able to transfer his money (including that month's \$160) into the new account.

β) Show that the initial investment in the new account will be \$5164.80 (to the nearest cent). **4**

c) Assuming that no further payments are made to the account, how much will George's money be worth 18 months after he transferred to the transferred to the new account ? **1**

QUESTION 4 (15 Marks) (Start a new sheet of paper) MARKS

i) The diagram in Figure 3 (below) shows part of the graph of $y = \ln(x+1)$.

a) Write down the exact coordinates of the point P. **1**

b) Show that the equation, can be expressed in the form $x = e^y - 1$. **1**

c) α) Find the shaded area (in exact form). **3**

β) Hence calculate the exact value of $\int_0^4 \ln(x+1)dx$. **2**

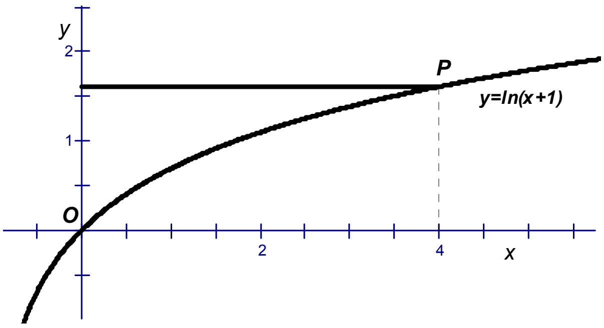

Figure 3

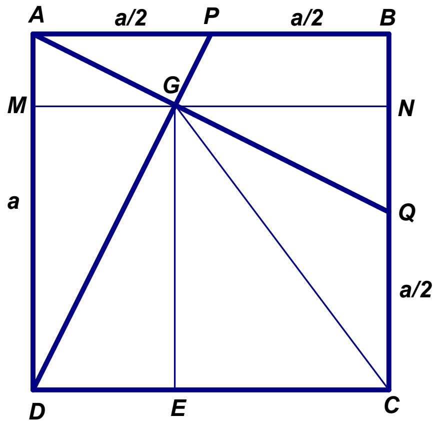

Figure 4

ii) A square $ABCD$, of side length a units, is shown in Figure 4 (above).
The point P is the midpoint of AB and Q is the midpoint of BC.
The lines AQ and DP are drawn and intersect at G.
Construction lines MN, through G parallel to AB and GE, perpendicular from G to DC, have been included.
You may assume, without proof, that the triangles AGP, ABQ, DAP and DMG are similar.

a) Copy the diagram and show that $PD = \dfrac{a\sqrt{5}}{2}$ units. **1**

b) Show that $PG = \dfrac{a\sqrt{5}}{10}$ units. **2**

c) Deduce that $DG = \dfrac{2a\sqrt{5}}{5}$ units. **1**

d) Find the lengths of MD and MG and hence prove that $CG = a$ units. **4**

END OF EXAM