

QUESTION 1

Start on a New Sheet of paper

MARKS

- a) Evaluate exactly :
- i) $\int_0^1 (3-2x)^3 dx$ 2
- ii) $\int_0^1 e^{\pi x} dx$ 1
- iii) $\int_1^4 (\sqrt{x}-2)^2 dx$ 3
- b) Find the centre and radius of the circle $4x^2 + 4y^2 - 12x + 16y = 11$. 3
- c) The gradient function of a curve is given by $\frac{dy}{dx} = x^2 - 4x$. If the curve passes through the point (2,-3), find the equation of the curve. 2
- d) i) Write down the primitive of $\sec^2 2x$. 1
- ii) Hence evaluate $\int_0^{\frac{\pi}{8}} (\tan^2 2x) dx$ exactly. 3

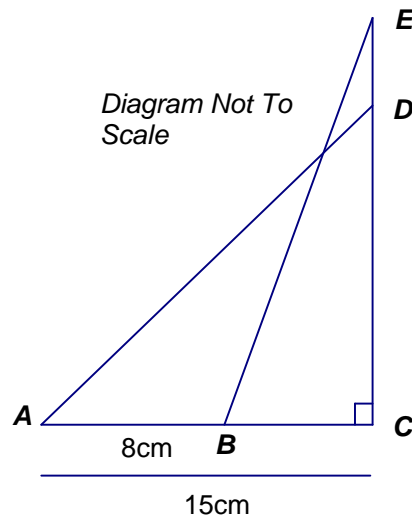
QUESTION 2

Start on a New Sheet of paper

MARKS

- a) 3

In the diagram, $AD = BE = 25\text{cm}$.
 Also, $\angle C$ is a right angle, $AB = 8\text{cm}$
 and $AC = 15\text{cm}$.
 Find the length of DE .



Question 2 is continued on the next page

QUESTION 2 (Continued)

MARKS

- b) The first and last terms of an Arithmetic series are 10 and 60 respectively and the sum of the series is 3535. Find:
- i) the number of terms in the series 2
 - ii) the common difference 2
- c) i) Use Simpson's Rule to estimate $\int_0^2 4^x dx$ using 5 function values. 2
- ii) Use the Trapezium Rule with 5 function values to approximate the same integral. 2
- iii) Given that $\frac{d}{dx}(4^x) = (4^x) \ln 4$, find the exact value of $\int_0^2 4^x dx$. 2
- iv) Hence, find the percentage error caused by using Simpson's Rule with 5 function values. (Answer to 2 Decimal Places) 2

QUESTION 3 Start on a New Sheet of paper

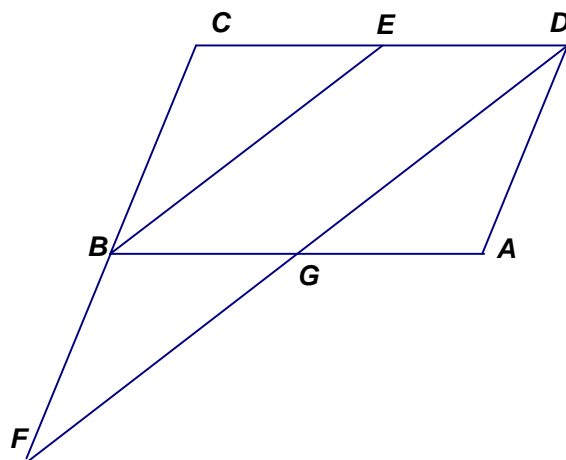
MARKS

- a) Sally invests \$6000 at the start of every year into a superannuation fund which pays 9% per annum, compounded annually.
- i) How much will Sally's investment be worth at the end of the first year? 1
 - ii) How much will Sally's investment be worth at the end of the 20th year? (Answer to the nearest dollar) 3
 - iii) If, instead of paying yearly, Sally pays \$500 monthly and the interest is accumulated monthly at 9% p.a., what will Sally's investment now be, after 20 years, to the nearest dollar? 3

b)

In the diagram alongside, $ABCD$ is a parallelogram.
 E is the midpoint of DC .
 DF is parallel to EB .

Copy the diagram and prove that $FB = AD$.



4

Question 3 is continued on the next page

QUESTION 3 (Continued)**MARKS**

- c) The area enclosed between the curve $y = \ln x$, the two coordinate axes and the line $y = 1$ is rotated about the y -axis. Find the volume so generated.

4**QUESTION 4 Start on a New Sheet of paper****MARKS**

- a) i) Prove that $\sqrt{2} - 1$, 1 and $\sqrt{2} + 1$ are in geometric sequence. **1**
- ii) If these are the first three terms of a geometric sequence, find, in exact form, the 5th term in the sequence. **2**
- iii) If the sequence is reversed to form the start of a geometric series, starting $\sqrt{2} + 1$, 1 , $\sqrt{2} - 1$, find the sum to infinity of that series, simplifying your answer if possible. **2**
- b) Shade the region where $y \geq -\sqrt{9 - x^2}$. **3**
- c) i) Show that the curves $y = \cos \frac{\pi x}{2}$ and $y = 2(e^{x-1} - 1)$ intersect at $(1, 0)$. **2**
- ii) By considering the gradient of the exponential curve, or otherwise, explain why these curve will not cross anywhere else. **2**
- iii) Find the area enclosed between these curves and the y -axis ($x \geq 0$), leaving your answer in an exact form. **3**

END OF EXAM