

James Ruse Agricultural High School
Year 12 Term1 Mathematics

Question 1

Marks

- (a) Find:
- (i) $\int (4\sin 2x - 6\cos 3x) dx$ 1
- (ii) $\int \frac{4x^3 + 3x^2 - 7}{x^4 + x^3 - 7x + 4} dx$ 1
- (iii) $\int \frac{3x^2 + 4x - 1}{\sqrt{x}} dx$ 2
- (b) Evaluate:
- (i) $\int_{\frac{1}{4}}^1 (4x - 3)^2 dx$ 2
- (ii) $\int_{-1}^1 \sqrt{1 - x^2} dx$ 1
- (c) Find the value of x , giving reasons. 4
- Diagram not to scale*
- (d) By writing $0.\dot{1}\dot{2}$ as a series, express $0.\dot{1}\dot{2}$ as a fraction in simplest terms. 2
- (e) Graph $y = \frac{x+2}{x-1}$ showing all asymptotes and intercepts with the x and y axes. 2

Question 2 (Start A New Page)

Marks

- (a) (i) Graph the region defined by : 2
 $y \geq \sqrt{x}, \quad y \leq x^2 + 1, \quad x \geq 0 \text{ and } x \leq 4.$

(ii) Find the area of the region in (i). 2

(iii) The region in (i) is rotated about the x axis.
Find the Volume of the solid of revolution. 3

(iv) The region in (i) is rotated about the y axis.
Find the Volume of the solid of revolution. 4

- (b) Using the Trapezoidal Rule using 2 strips evaluate (to 4 significant figures) $\int_0^1 e^{-\sin x} dx$ 2

- (c) The gradient function of a curve is given by : 2

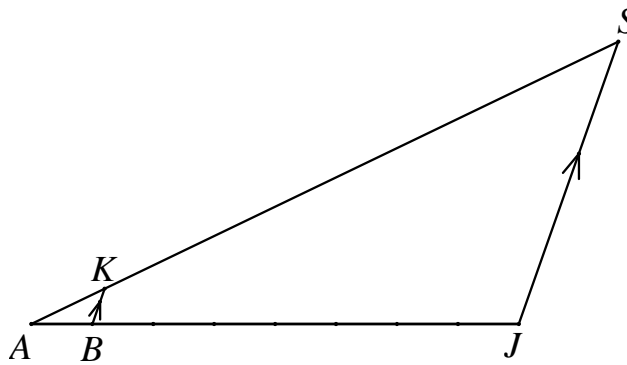
$$\frac{dy}{dx} = e^x(e^x + e^{2x})$$

Find y as a function of x if the point $(\ln 2, 1)$ lies on the curve.

Question 3 (Start a New Page)

Marks

- (a) Find: $\int 3\sec 3x \tan 3x dx$ 1
- (b) On the same axes in the domain $0 \leq x < \pi$ graph $y = \cos \frac{x}{2}$ and $y = \sec \frac{x}{2}$. 4
- (c) (i) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)$ 1
- (ii) Using Simpsons Rule using 5 function values evaluate (to 4 decimal places)
 $\int_0^1 x \operatorname{cosec} x dx.$ 4
- (d) (i) Given $BK \parallel JS$ prove $\triangle ABK \parallel \triangle AJS.$ 2
- (ii) Hence deduce that $BK = \frac{AB \cdot JS}{AJ}$, giving reasons. 1



- (iii) In $\triangle AJS$, AJ is divided into 8 equal intervals such that $AB=BC=CD=DE=EF=FG=GH=HJ.$
 Intervals BK, CL, DM, EN, FP, GQ and HR are drawn parallel to JS in $\triangle AJS$, as shown. 2
 If $JS = 10\text{cm}$ find the value of the length $BK+CL+DM+EN+FP+GQ+HR+JS.$

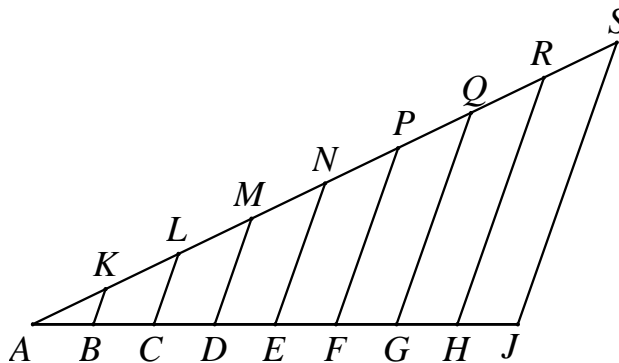


Diagram not to scale

Question 4 (Start A New Page)

Marks

- (a) The sum S of terms in an infinite series is given by :

2

$$S = 1 + 2x + 4x^2 + 8x^3 + \dots$$

Find an expression for the sum to infinity, stating the restrictions on x .

- (b) $ABCD$ is a square with $BC = 145$ cm. Intervals AH , BE , CF and DG are drawn to form square $EFGH$.

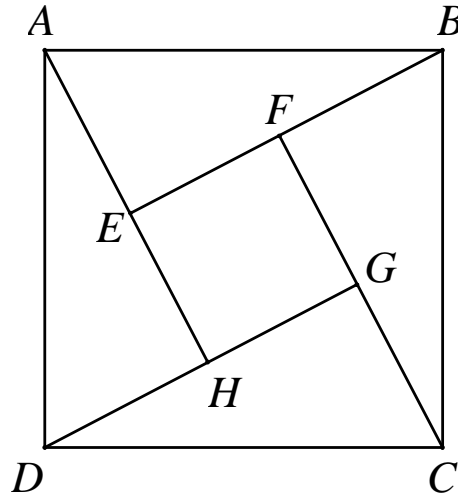


Diagram not to scale

- (i) Prove $\triangle ABE \cong \triangle BCF$. (Hint: Let $\angle ABE = x^\circ$).

4

- (ii) Find the length of CF when $BF = 17$ cm.

1

- (iii) Find the area of square $EFGH$

2

- (c) A man obtains a monthly reducible loan of \$ 200 000 over 25 years, at an interest rate of 8% p.a.

- (i) Show that the amount owing A_n after n monthly repayments of R is :

2

$$A_n = 200000 \left(\frac{151}{150} \right)^n - 150R \left(\left(\frac{151}{150} \right)^n - 1 \right)$$

- (ii) Find the monthly repayment R .

1

- (iii) Find the amount owing after 15 years if $\$R$ is paid each month.

1

- (iv) After 15 years the man changes his repayment to \$1600 each month. Find the number of repayments needed to repay the loan.

2

End of Examination