Name:	
Class:	

YEAR 12

ASSESSMENT TEST 2
TERM 1, 2013

MATHEMATICS

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

General Instructions:

- *All* questions may be attempted
- *All* questions are of equal value
- Standard Integral Tables will be supplied
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each question must show your Candidate Number.

QUESTION 1 **15 Marks** **Marks**

(a) Find the following indefinite integrals:

(i) $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$ **2**

(ii) $\int\frac{x^2+2x-1}{x^2}\,dx$ **2**

(iii) $\int xe^{3x^2}\,dx$ **1**

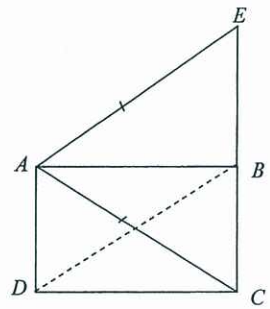
(iv) $\int\cos\pi\,dx$ **1**

(b) A rectangle measures 64 cm by 2 cm. The rectangle is changed in size, so that the length is decreased by $33\frac{1}{3}\%$ and the width is increased by $33\frac{1}{3}\%$.

(i) Find the ratio of the new area of the rectangle to the original area. **2**

(ii) How many times must this process be performed so that the new shape is similar in shape to the original one? **2**

(c) $ABCD$ is a rectangle. CB is produced to E, so that $AE = AC$.



Not to Scale

(i) Prove that $\Delta ABE \equiv \Delta ABC$, giving reasons. **3**

(ii) Prove that $AEBD$ is a parallelogram, giving reasons. **2**

QUESTION 2 15 Marks START A NEW PAGE

Marks

(a) Sketch, on a number plane, the graph of $y=\frac{2-x}{x+1}$. **3**
Label all intercepts and asymptotes.

(b) Three **different** positive numbers, 2, x, y are the first, second and twelfth terms of an arithmetic sequence. They are also consecutive terms of a geometric sequence.

(i) Show that $y=11x-20$. **2**

(ii) Find the values of x and y. **3**

(c) The line $y=\frac{1}{2}$ intersects the curve $y=\sin x$, in the domain $0\le x\le 2\pi$,

at two points $A\left(\frac{\pi}{6},\frac{1}{2}\right)$ and $B\left(\frac{5\pi}{6},\frac{1}{2}\right)$.

(i) Calculate the area bounded by the curve $y=\sin x$ and the line $y=\frac{1}{2}$, **3**
between points A and B.

(ii) This area is rotated about the x-axis. Use the trapezoidal rule with 5 function values to find an approximation to the volume of the solid of revolution formed. **4**
Give your answer to 2 decimal places.

QUESTION 3 15 Marks START A NEW PAGE

(a) The first four terms of a sequence are:

$$1,\ \frac{1+2}{1+3},\ \frac{1+2+3}{1+3+5},\ \frac{1+2+3+4}{1+3+5+7}.$$

(i) Find a formula for T_n, the nth term of the sequence. **2**

(ii) Show that T_n is always greater than $\frac{1}{2}$. **1**

(b) The area bounded by the x and y axes, the line $y=1$ and the graph $y=\ln x$ is rotated about the y axis. Calculate the volume of the solid of revolution formed. **3**

(c) Use Simpson's rule with 5 function values to estimate the area bounded by the the graph $y=-\sqrt{25-x^2}$, the x and y axes and the line $x=4$. **4**
Give your answer to 2 decimal places.

Question 3 continued over page

(d) Anna opens a bank account to save for the deposit to buy a house. At the beginning of each month she makes a deposit into the account. Her first deposit is \$100. Each month she increases her deposit by 1%. The interest rate on the account is 6% p.a., paid monthly.

(i) Show that the amount in her account, A_n at the end of n months, is given by the formula: **3**

$$A_n=20100\left(1\cdot01^n-1\cdot005^n\right).$$

(ii) Calculate the total amount of interest, to the nearest dollar, that Anna received on her investment at the end of 10 years. **2**

QUESTION 4 15 Marks START A NEW PAGE

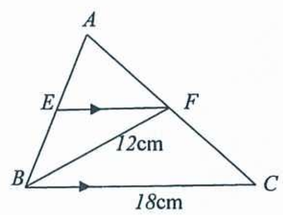
(a) Show on a number plane the region representing the set: **3**

$$\{(x,y): y\le -3x+6\}\cap\{(x,y):(x-1)^2+(y+2)^2<16\}$$

(b) The first term of an infinite geometric series is 6. Each term is double the sum of all the terms that follow.

Find the values of the common ratio and the limiting sum of the series. **3**

(c) In ΔABC, $\angle ABC=2\angle ACB$. BF bisects $\angle ABC$ and EF is parallel to BC.
$BC=18$ cm and $BF=12$ cm.



Not to Scale

(i) Prove that ΔEBF and ΔFBC are similar isosceles triangles, giving reasons. **3**

(ii) Show that the length of EB is 8 cm, giving reasons. **2**

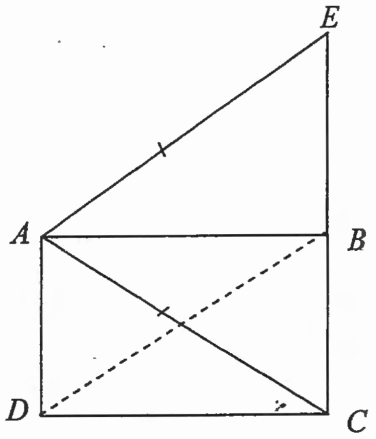
(iii) Calculate the length of AF, giving reasons. **4**

END OF EXAMINATION

MATHEMATICS: Question 1

Suggested Solutions	Marks	Marker's Comments
a) i) $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx = \int\left(x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)dx$		
$= \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$	1	Convert to index form
$= \frac{2}{3}x\sqrt{x} - 2\sqrt{x} + c$	1	Integrate
ii) $\int \frac{x^2+2x-1}{x^2}dx = \int\left(1+\frac{2}{x}-\frac{1}{x^2}\right)dx$	1	Divide through
$= x + 2\ln x + \frac{1}{x} + c$	1	Integrate
iii) $\int xe^{3x^2}dx = \frac{1}{6}e^{3x^2}+c$	1	Integrate
iv) $\int \cos\pi\, dx = \int -1\, dx$	$\frac{1}{2}$	Evaluate $\cos\pi$
$= -x + c$	$\frac{1}{2}$	Integrate
b) Let L_n, W_n & A_n be the length, width & area (respectively) after the nth adjustment of dimensions.		
i) $\therefore L_0 = 64,\ W_0 = 2,\ A_0 = 128$	$\frac{1}{2}$	Original area
$L_1 = 64 \times \frac{2}{3} = \frac{128}{3}$, $W_1 = 2 \times \frac{4}{3} = \frac{8}{3}$		
$\therefore A_1 = \frac{1024}{9}$	$\frac{1}{2}$	New area
$\therefore$ Ratio of areas $= \frac{\left(\frac{1024}{9}\right)}{128}$		
$= \frac{8}{9}$ (or $8:9$)	1	Ratio
ii) For similar shape, proportion of width to length must reverse		
ie $\frac{W_n}{L_n} = \frac{L_0}{W_0}$ ⋮ $\left[\frac{\left(\frac{4}{3}\right)}{\left(\frac{2}{3}\right)}\right]^n = 1024$	1	Produce correct equation to be solved
$\frac{2\left(\frac{4}{3}\right)^n}{64\left(\frac{2}{3}\right)^n} = \frac{64}{2}$ ⋮ $2^n = 1024$		
$\therefore n = 10$	1	Number of times

MATHEMATICS: Question 1

Suggested Solutions	Marks	Marker's Comments
c) i) In $\triangle ABE$ & $\triangle ABC$: AB is common $AE = AC$ (given) 	1	Corresponding sides
$\angle ABC = 90^\circ$ (angle of rectangle ABCD is 90°)	1	Corresponding angles
$\angle ABE + \angle ABC = 180^\circ$ (angle sum of straight angle is 180°)	$\frac{1}{2}$	Setting out to show equivalence of angles
$\therefore \angle ABE = 90^\circ$ $\therefore \triangle ABE \equiv \triangle ABC$ (RHS)	$\frac{1}{2}$	Correct reason for congruence
ii) $AD \parallel BC$ (opposite sides of rectangle ABCD are parallel) $\therefore AD \parallel BE$ (BE is extended from BC) $BC = BE$ (corresponding sides of congruent $\triangle ABE$ & $\triangle ABC$ equal) But $BC = AD$ (opposite sides of rectangle ABCD are equal) $\therefore AD = BE$	$1\frac{1}{2}$	Sound reasoning that corresponds to relevant geometric features & properties
$\therefore$ ADBE is a parallelogram (one pair of sides both equal & parallel)	$\frac{1}{2}$	Valid sufficiency condition chosen

1/2

MATHEMATICS: Question 2

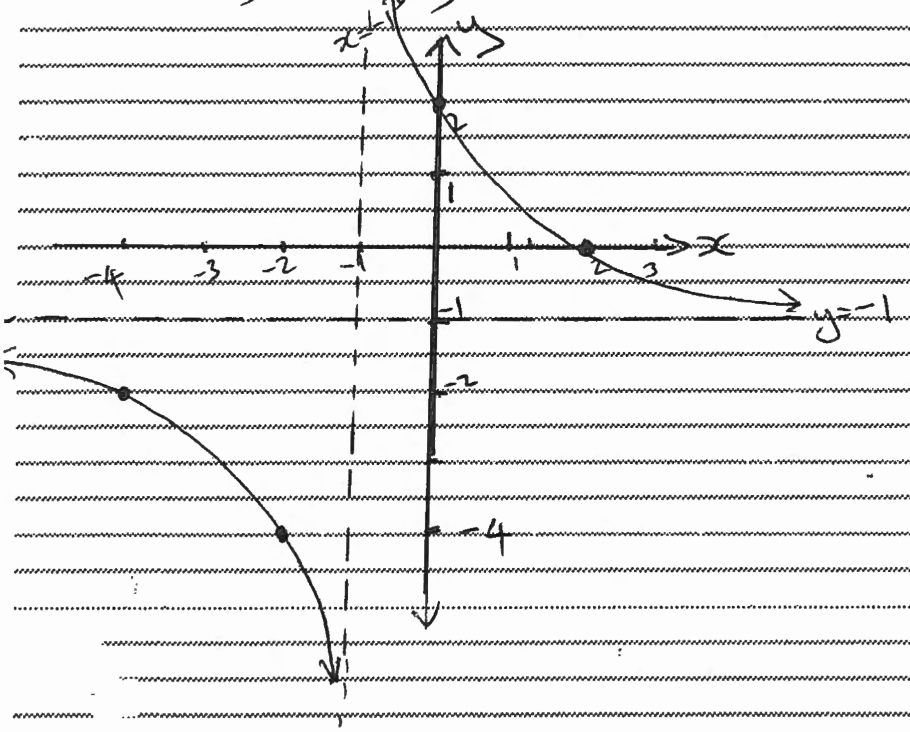
Suggested Solutions	Marks	Marker's Comments

(a) $y = \dfrac{2-x}{x+1}$

vertical asymptote at $x=-1$

horizontal asymptote at $y=-1$

$(2,0)$ $(0,2)$



problems

* inconsistent scale on axes.
* didn't lock a point in on 2nd branch.
* forgot horizontal asymptote.

$-\tfrac{1}{2}$ off for each error.

b)(i) $T_1 = 2$ $T_2 = x$ $T_{12} = y$

$d = T_2 - T_1$ $T_1 = 2 = a$

$= x - 2$

$d = x - 2 = \tfrac{1}{2}$ mk

$T_{12} = a + 11d$ — $\tfrac{1}{2}$

$= 2 + 11(x-2)$ — $\tfrac{1}{2}$

$\therefore y = 11x - 20$ — $\tfrac{1}{2}$

(ii) G.P. $T_1 = 2$ $T_2 = x$ $T_3 = y$

$\therefore r = \dfrac{x}{2} = \dfrac{y}{x}$ $\therefore x^2 = 2y$ — $\tfrac{1}{2}$

sub into $y = 11x - 20$ — $\tfrac{1}{2}$

$\dfrac{x^2}{2} = 11x - 20$

$x^2 = 22x - 40$ — $\tfrac{1}{2}$

$x^2 - 22x + 40 = 0$

$(x-20)(x-2) = 0$ — $\tfrac{1}{2}$

a lot forgot to make the 40 positive

MATHEMATICS: Question 2 (cont)

Suggested Solutions	Marks	Marker's Comments
$\therefore x = 20$ or $x = 2$	1/2	1/2 off for each error
when $x = 20$, $y = 200$		
when $x = 2$, $y = 2$ but $x \neq y$		
$\therefore x = 20, y = 200$ only	1/2	
(c) (i) $A = \int_{\pi/6}^{5\pi/6} (\sin x - \tfrac{1}{2})\,dx$		
$= \left[-\cos x - \tfrac{1}{2}x\right]_{\pi/6}^{5\pi/6}$		
$= \left(-\cos\frac{5\pi}{6} - \frac{5\pi}{12}\right) - \left(-\cos\frac{\pi}{6} - \frac{\pi}{12}\right)$		* maximum 2 mks if
$= \frac{\sqrt{3}}{2} - \frac{5\pi}{12} + \frac{\sqrt{3}}{2} + \frac{\pi}{12}$		$\sqrt{3}$ or 0.684853 (no working)
$= \left(\sqrt{3} - \frac{\pi}{3}\right)$ units^2		is the answer?
(ii) $f(x) = \sin x - \frac{1}{2}$		
$V = \pi\left[f(x)\right]^2$		* majority of students didn't write out
$= \pi \sin^2 x - \pi\left(\frac{1}{2}\right)^2$		$f(x)$, so I had no idea what they were
5 fn values = 4 applications		substituting into!!!

x	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$
$f(x)$	$\frac{\pi}{4}-\frac{\pi}{4}$ 0	$\frac{3\pi}{4}-\frac{\pi}{4}$ $=\frac{2\pi}{4}$	$\pi-\frac{\pi}{4}$ $\frac{3\pi}{4}$	$\frac{2\pi}{4}$	0

* A lot of students forgot to multiply by π.

$V = \frac{\frac{\pi}{6}}{2}\left[f\left(\frac{\pi}{6}\right) + 2f\left(\frac{2\pi}{6}\right) + 2f\left(\frac{3\pi}{6}\right) + 2f\left(\frac{4\pi}{6}\right) + f\left(\frac{5\pi}{6}\right)\right]$

$= \frac{\pi}{12}\left[0 + 2\left(\frac{2\pi}{4}\right) + 2\left(\frac{3\pi}{4}\right) + 2\left(\frac{2\pi}{4}\right) + 0\right]$

* Some students found the area, lost 1 mk.

$= \frac{\pi}{12}\left(\frac{4\pi + 6\pi + 4\pi}{4}\right)$

* 1/2 mk off if you squared it together instead of individually

$= \frac{\pi}{12} \cdot \frac{7\pi}{2}$

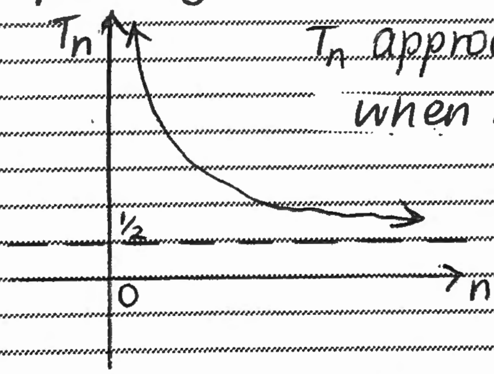
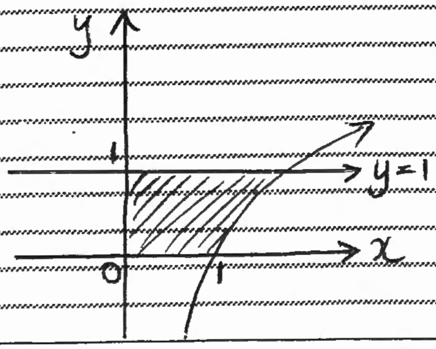
$= \frac{7\pi^2}{24}$

* −1/2 if not to 2dp.

$= 2.878634617 \quad \therefore V = 2.88$ units^3

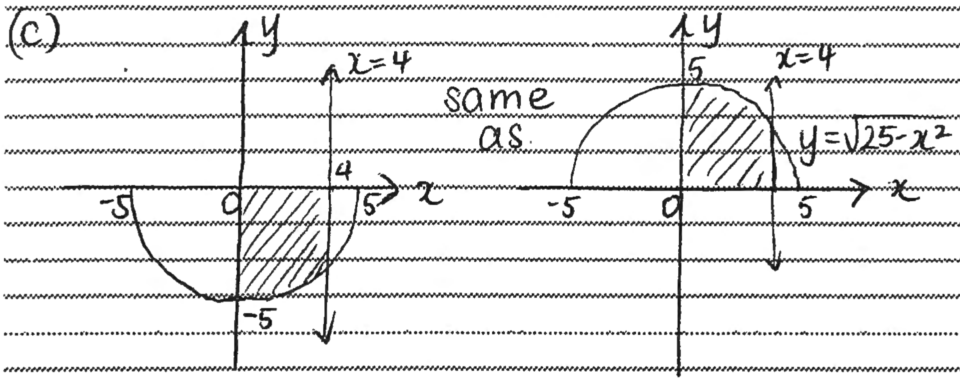
* 1/2 off for each error

. . YR 12 HY 2013 **MATHEMATICS** **Question** ..3....

Suggested Solutions	Marks	Marker's Comments
3(a)(i) $T_n = \dfrac{1+2+3+4+\cdots+n}{1+3+5+7+\quad+2n-1}$		
Let $S_n = 1+2+3+4+\cdots+n$ AP		
$a=1,\ d=1,\ S_n = \frac{n}{2}(1+n)$		
Let $R_n = 1+3+5+7+\cdots+2n-1$ AP		
$a=1,\ d=2,\ R_n = \frac{n}{2}(1+2n-1)$		
$= n^2$		
$\therefore\ T_n = \dfrac{\frac{n}{2}(1+n)}{n^2}$	1 1	numerator sum denominator sum
$= \dfrac{(1+n)}{2n}$		
(ii) $T_n = \frac{1}{2n} + \frac{1}{2}$	½	Splitting T_n
Since $n>0$, $\frac{1}{2n} > 0$	½	reason
$\therefore\ T_n > \frac{1}{2}$		
OR graphically:		
 T_n approaches $T_n=\frac{1}{2}$ when $n>0$ $\therefore T_n > \frac{1}{2}$	1	drawing graph with explanation
(b)  $y = \ln x$ $x = e^y$ $V = \pi\int_a^b x^2\,dy$		

Suggested Solutions	Marks	Marker's Comments
$V = \pi \int_0^1 e^{2y}\,dy$	1	Integral
$= \pi \left[\frac{1}{2} e^{2y}\right]_0^1$		
$= \pi \left[\frac{1}{2}e^2 - \frac{1}{2}e^0\right]$	1	integration and substitution
$= \pi \left[\frac{e^2}{2} - \frac{1}{2}\right]$		
$= \frac{\pi}{2}(e^2 - 1)$ units^3	1	answer with working.

(c)

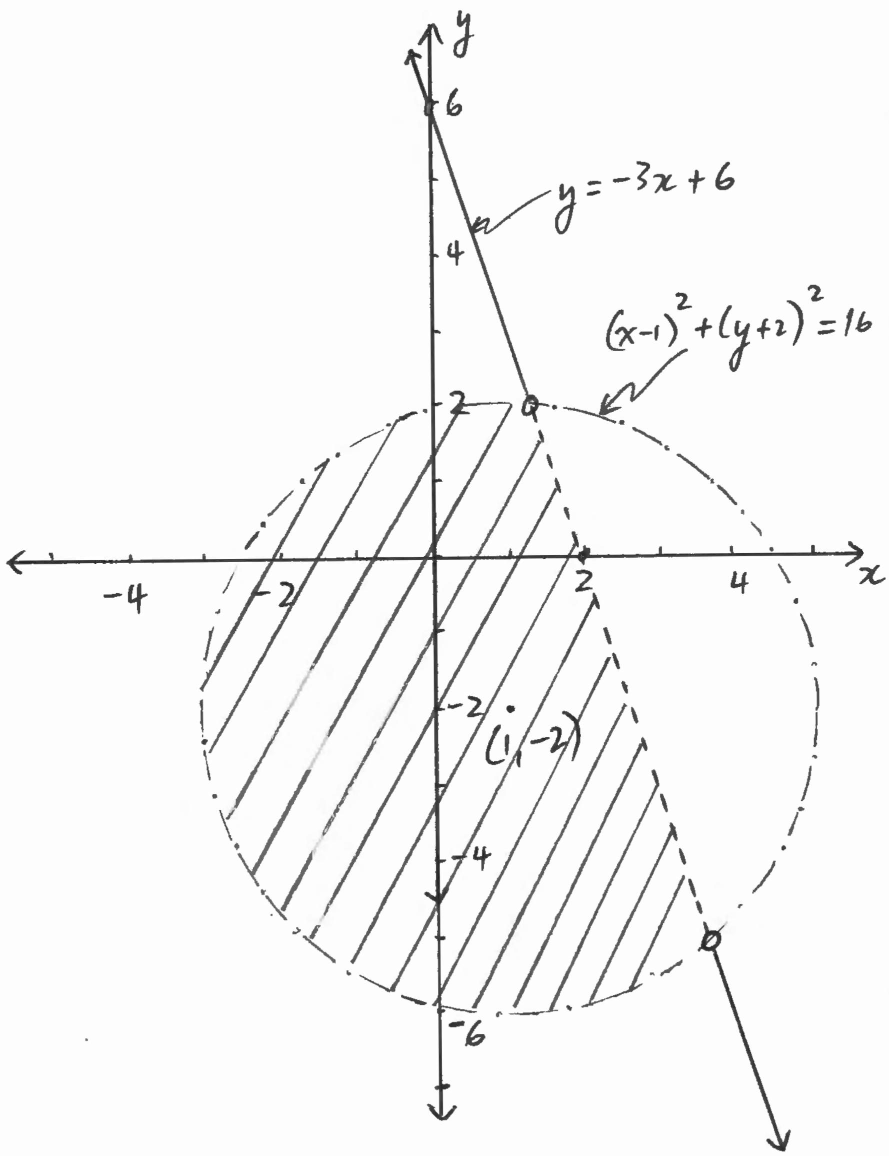


x	0	1	2	3	4
y	5	$\sqrt{24}$	$\sqrt{21}$	4	3

Suggested Solutions	Marks	Marker's Comments
(table above)	$1\frac{1}{2}$	Table values
$A = \frac{1}{3}\left[5 + 4(\sqrt{24} + 4) + 2(\sqrt{21}) + 3\right]$	2	substitution into correct formula
$\doteqdot 17.58702311$		
$\doteqdot 17.59$ units^2 (to 2 dec. pl.)	$\frac{1}{2}$	answer
OR/		
$A_1 = \frac{2-0}{6}\left[5 + 4\sqrt{24} + \sqrt{21}\right]$	1	substitution into correct formula
$A_2 = \frac{4-2}{6}\left[\sqrt{21} + 4 \times 4 + 3\right]$	1	
Total area $\doteqdot 17.58702311$		
$\doteqdot 17.59\ u^2$	$\frac{1}{2}$	answer

MATHEMATICS **Question……..**

Suggested Solutions	Marks	Marker's Comments
(d) (i) $A_1 = 100(1.005)$		
$A_2 = 100(1.005)^2 + 100(1.01)(1.005)$		
$A_3 = 100(1.005)^3 + 100(1.01)(1.005)^2 + 100(1.01)^2(1.005)$		
$\therefore A_n = 100(1.005)^n + \cdots + 100(1.01)^{n-1}(1.005)$	1	Series
$A_n = 100(1.005)^n \dfrac{\left[\left(\frac{1.01}{1.005}\right)^n - 1\right]}{\left[\frac{1.01}{1.005} - 1\right]}$	1	Sum
$= \dfrac{100\left[1.01^n - 1.005^n\right]}{\left[\frac{1.01 - 1.005}{1.005}\right]}$		
$= \dfrac{100(1.005)}{0.005}\left[1.01^n - 1.005^n\right]$		
$= 20100\left[1.01^n - 1.005^n\right]$	1	Simplifying
(ii) $A_{120} = 20100(1.01^{120} - 1.005^{120})$		
$= \$29767.90$	$\frac{1}{2}$	A_{120}
Total invested : $a = 100, r = 1.01, n = 120$		
$S_{120} = \dfrac{100(1.01^{120} - 1)}{1.01 - 1}$		
$= \$23003.87$	1	S_{120}
Interest $= A_{120} - S_{120}$		
$= \$6764$ to nearest dollar	$\frac{1}{2}$	Interest

MATHEMATICS: Question 4		– PAGE 1 –
Suggested Solutions	**Marks**	**Marker's Comments**
a) 	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	for st. line for dotty line for circle centre (1,-2) and r = 4 for dotty line. for open ring at points of intersection shaded region $-\frac{1}{2}$ for not labelling graphs $-\frac{1}{2}$ for not using compass $-\frac{1}{2}$ for incorrect/ poor scale $-\frac{1}{2}$ for no centre

MATHEMATICS: Question 4 PAGE 2

Suggested Solutions	Marks	Marker's Comments
b) $T_n = 2(S_\infty - T_n)$	1	$S_\infty = \frac{a}{1-r}$ (formula)
$\therefore 6 = 2(S_\infty - 6)$	1	$S_\infty = 9$
$\Rightarrow S_\infty = 9$	1	$r = \frac{1}{3}$
i.e $\frac{a}{1-r} = 9$		
$\therefore 1-r = 6/9$		
$\therefore r = \frac{1}{3}$		
Hence common ratio is $\frac{1}{3}$ and the limiting sum is 9.		
c) (i) In Δ's EBF and FBC		
• let $\angle EBF = x°$		
Now $\underline{\angle EBF = \angle FBC} = x$ … (FB bisects $\angle ABC$)	$\frac{1}{2}$	for $\angle EBF = \angle FBC$
• $\angle EFB = \angle FBC$ … (alternate angles on parallel lines EF and BC, are equal)	$\frac{1}{2}$	
But $\angle FCB = \frac{1}{2}\angle ABC$ … (given) $= x$	$\frac{1}{2}$	
$\therefore \underline{\angle FCB = \angle EBF}$ … (from •)		
$\therefore \Delta EBF \parallel\!\mid \Delta FBC$ … (equiangular)	$\frac{1}{2}$	
But $\angle FBC = \angle FCB$ … (proved above)		
Hence ΔFBC is isosceles … (base angles are equal)	$\frac{1}{2}$	for isosceles ΔFBC

Suggested Solutions	Marks	Marker's Comments
But ΔFBC is similar to ΔEBF and since ΔFBC is isosceles, ΔEBF must be isosceles.	$\frac{1}{2}$	isosceles ΔEBF
(ii) From $\Delta EBF \parallel\!\mid \Delta FBC$,		
$\frac{EB}{FB} = \frac{BF}{BC}$ … (corresponding sides in similar Δ's are in proportion)	$\frac{1}{2}$ $\frac{1}{2}$	ratio for correct reason
ie. $\frac{EB}{12} = \frac{12}{18}$	$\frac{1}{2}$	for sub.
$\therefore EB = 12^2/18$		
$= 8$ cm.	$\frac{1}{2}$	final correct answer
(iii) $EB = EF = 8$ cm … (ΔEBF isosceles)	$\frac{1}{2}$	for $EF = 8$ cm
$FC = 12$ cm … (ΔFBC isosceles)	$\frac{1}{2}$	$FC = 12$ cm with reason
In ΔAEF & ΔABC, • $\angle EAF$ is common • $\angle ABC = \angle AEF$ … (corresponding angles on parallel lines EF and BC are equal)		
$\therefore \Delta AEF \parallel\!\mid \Delta ABC$ … (equiangular)	1	Similar Δ's
Hence $\frac{AF}{AC} = \frac{FE}{BC}$ … (corresponding ~~sides~~ on parallel sides EF and BC are equal)	1	correct ratio
But $AC = AF + FC$ $= AF + 12$		

MATHEMATICS: Question 4		PAGE 4
Suggested Solutions	**Marks**	**Marker's Comments**
Hence $\dfrac{AF}{AF+12} = \dfrac{8}{18}$ Solving for AF gives 9.6 cm i.e $AF = 9.6$ cm	1	for correct answer