MLC SCHOOL

YEAR 12

HALF YEARLY EXAMINATION

APRIL 2001

MATHEMATICS

Time allowed - 2 Hours

DIRECTIONS

- ALL questions may be attempted.
- Questions are NOT of equal value.
- All necessary working should be shown.
- Marks may be deducted for careless or badly arranged work.
- Approved calculators may be used.
- Start each question in a new booklet.
- Each booklet is to be handed in separately.
- Write your student number on each booklet.
- For any question not attempted hand in a blank booklet with the question number and your student number written on it.
- A table of standard integrals is provided on the last page.

QUESTION 1. (Start a separate booklet) (16 marks)

Marks

a) Find the value of $\dfrac{13 \cdot 81^2}{8 \cdot 09 + \sqrt{4.62}}$

correct to (i) 2 decimal places

(ii) 1 significant figure (2)

b) Simplify $\dfrac{4y}{3} - \dfrac{y-7}{2}$ (2)

c) Fully factorise $8n^3 - 64$ (2)

d) Solve for m:

i) $m^2 - 4m + 1 = 0$ (leave answer in simplified surd form) (3)

ii) $|3m-1| = m$ (2)

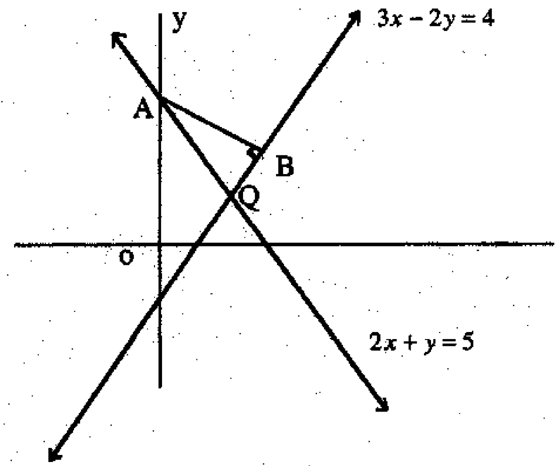
iii) $2m^2 - 9m - 5 < 0$ (2)

e) If $\dfrac{6-\sqrt{3}}{2-\sqrt{3}} = a + \sqrt{b}$, find the values of a and b (2)

f) After a discount of 13% was given, I paid $27840 for my new car.
What was the original marked price? (1)

QUESTION 2 (Start a separate booklet) (9 marks) *Marks*

Diagram is not to scale



The diagram shows the graphs of the two lines $3x - 2y = 4$ and $2x + y = 5$.
The point Q is the point of intersection of the two lines. A is the y intercept of $2x + y = 5$.
The line AB is perpendicular to the line $3x - 2y = 4$.

(a)	Find the co-ordinates of a Q.	*(2)*
(b)	State the co-ordinates of A.	*(1)*
(c)	Find the gradient of AB and hence the equation of AB.	*(2)*
(d)	Write down the three inequations which would define the region enclosed by ΔABQ	*(1)*
(e)	Calculate the perpendicular distance of A to the line $3x - 2y = 4$. (Leave your answer in exact form).	*(2)*
(f)	If the length of QB = a units, write down an expression for the area of ΔABQ	*(1)*

QUESTION 3. (Start a separate booklet) (15 marks) *Marks*

a) Differentiate the following with respect to x

(i) $6x^2 + \frac{1}{x^2}$ *(2)*

(ii) $\frac{1}{(2x+7)^6}$ *(2)*

(iii) $\frac{4x+1}{x-1}$ *(2)*

b) Integrate $\left(t^2 + \sqrt{t}\right)$ with respect to t. *(1)*

c) (i) Show that the derivative of $y = x(4-x)^5$ is $2(4-x)^4(2-3x)$ *(2)*

(ii) Hence find $\int (4-x)^4 (2-3x)\,dx$ *(1)*

d) Evaluate $\int_{-1}^{4} \frac{x+1}{3}\,dx$ *(3)*

e) Consider the following solution to find the $\int (4x+1)^{1/3}\,dx$ *(2)*

$$\int (4x+1)^{1/3}\,dx = \frac{[4x+1]^{4/3}}{4/3} + c$$

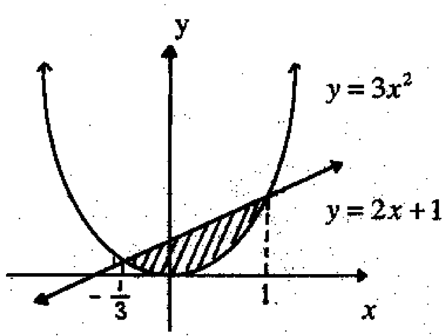
$$= \frac{3(4x+1)^{4/3}}{4} + c$$

This solution is ***incorrect***. Correctly find $\int (4x+1)^{1/3}\,dx$.

QUESTION 4. (Start a separate booklet) (16 marks) *Marks*

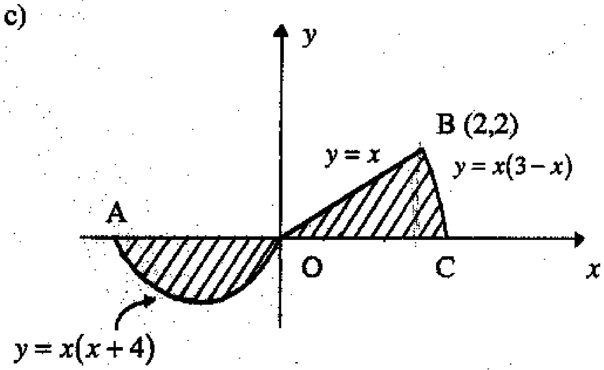
a) Find the volume of the solid formed when the curve $y = \sqrt[3]{x}$ is rotated about the $x-axis$ from $x = 1$ to $x = 8$. *(3)*

b)



Calculate the area of the region shown, bounded by the graph of $y = 3x^2$ and $y = 2x + 1$ given that the graphs intersect at $\left(\frac{-1}{3}, \frac{1}{3}\right)$ and $(1,3)$ *(3)*

c)

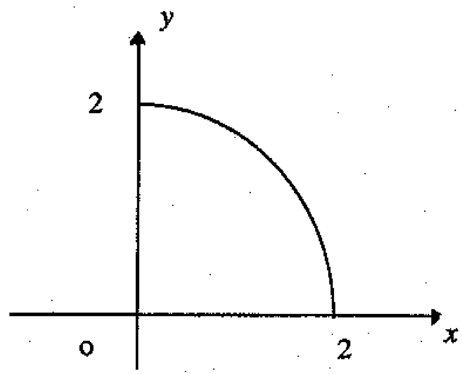


The shaded area shown is bounded by parts of the graphs of $y = x(x+4)$, $y = x$ $y = x(3-x)$ and the $x-axis$. The co-ordinates of B are (2,2)

i) Find the co-ordinates of the points A and C. *(1)*

ii) Calculate the area of the shaded region shown. *(4)*

d) Consider the quadrant of a circle of radius 2 units (as shown below). By using 5 ordinates, the following table of ordinates resulted:



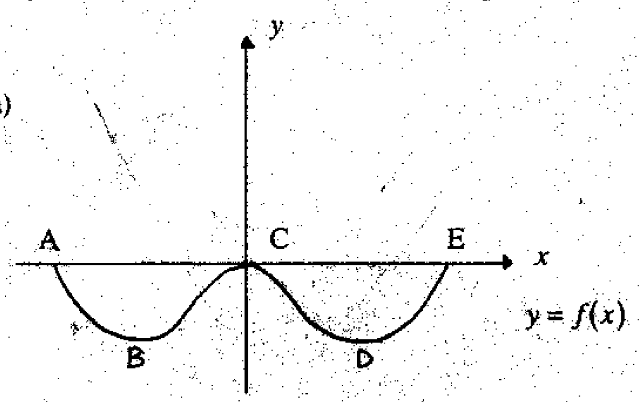
x	0	0.5	1	1.5	2
y	2	1.93	1.73	1.32	0

i) Use these values and Simpsons rule to find the approximate area of the quadrant and hence the circle. *(3)*

ii) Compare this answer with the area given by using the formula $A = \pi r^2$. What is the percentage error using Simpson's Rule? (Answer to 1 decimal place) *(2)*

QUESTION 5. (Start a separate booklet) (15 marks) *Marks*

a)



The following questions refer to the graph of $y = f(x)$ above.
Answer using the letters A, B, C, D and E.

(i) Which points show stationary points? *(1)*

(ii) Between which pairs of adjacent points must there be points of inflexion? *(1)*

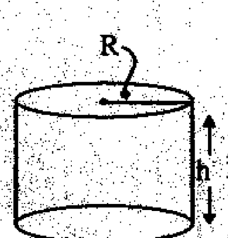
(iii) Between which pairs of adjacent points is the curve increasing? *(1)*

(iv) Where is $\frac{dy}{dx} < 0$? (Give the pairs of adjacent points). *(1)*

b) (i) Show that the graph of $y = x^3 - 8$ has one stationary point. Verify that it is in fact a point of inflexion. *(3)*

(ii) Sketch the graph of $y = x^3 - 8$, showing clearly the intercepts. *(2)*

c)



The sum of the radius R and height h of a cylinder is 60cm.

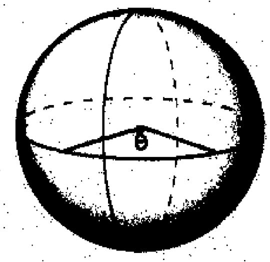
(i) Express h in terms of R. *(1)*

(ii) Show that the volume of the cylinder is given by $V = \pi(60R^2 - R^3)$ *(1)*

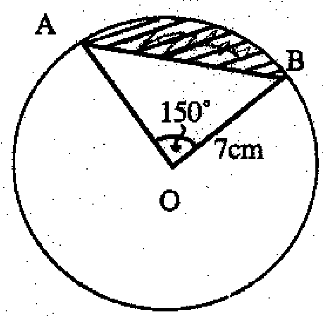
(iii) Hence find the maximum volume possible. *(4)*

QUESTION 6. (Start a separate booklet) (9 marks) *Marks*

a) Taking the radius of the Earth as 6341 km, what is the size of angle θ that would be subtended at the Earth's centre by an arc on the equator of length 9000 km? (Answer to the nearest minute). *(2)*



b)



The diagram shows the minor segment of a circle subtending an angle at the centre of 150° and a radius of 7 cm.

(i) Show that the exact area of the minor segment is $\dfrac{49(5\pi-3)}{12}$ units^2 *(2)*

(ii) Find the area of the major segment. (Answer to 3 significant figures) *(2)*

c) The length of an arc 11.2 cm and the area of a sector is 28.7 cm^2 when an angle of θ is subtended at the centre of circle, radius r. Find r and θ. (Answer θ to nearest radian) *(3)*

END OF PAPER