

MLC SCHOOL



YEAR 12 MATHEMATICS

Half Yearly Examination

Tuesday 18th March 2008

Student Number:20546387.....

Teacher:Mrs. Montensen =.....

General Instructions:

- Time Allowed: 3 hours plus 5 minutes reading time
- Write using blue or black pen
- Board approved calculators may be used
- Attempt all questions
- Questions are of equal value
- Start each question in a new booklet
- All necessary working should be shown in every question
- Write your student number on the top right hand corner of each booklet
- A table of standard integrals is provided at the back of this paper

Assessment Criteria:

- Clear, logical working with appropriate mathematical notation
- Neat diagrams of the appropriate size
- Correct solutions
- Reasoning ability

Question 1 (12 Marks) Use a SEPARATE writing booklet

Marks

~~(a)~~ Evaluate $\sqrt{4^2 - \pi}$ correct to two decimal places. 2

~~(b)~~ Rationalise the denominator of $\frac{1}{3 - \sqrt{2}}$ 2

~~(c)~~ Solve $x + 7 \geq 5$ and graph the solution on the number line. 2

~~(d)~~ Solve $x^2 + 2x - 8 = 0$ 2

~~(e)~~ Simplify $\left(\frac{5x^2y^3}{x^4y}\right)^3$ 2

~~(f)~~ A new racing bike is reduced by 25%. The sale price is \$720.
Find the original price of the racing bike. 2

Question 2 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) Express $0.2\dot{7}$ as a fraction in its simplest form

2

(b) (i) Sketch the function $y = \sqrt{25 - x^2}$, clearly labelling any intercepts with the axes.

2

(ii) Hence state the domain and range of the function $y = \sqrt{25 - x^2}$.

2

(c) The definition of an odd function $f(x)$ is given by $f(-x) = -f(x)$. Show that the function $f(x) = x^5 + x$ is an odd function.

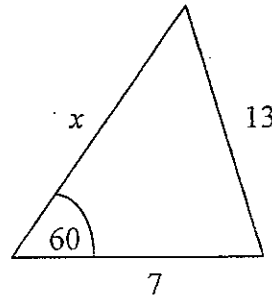
2

(d) The diagram shows a triangle with sides 7cm, 13cm and x cm, and an angle of 60° as marked.

4

(i) Use the cosine rule to show that $x^2 - 7x = 120$.

(ii) Hence find the exact value of x .



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Question 3 (12 Marks) Use a SEPARATE writing booklet

Marks

~~(a)~~ The points A and B have coordinates $(0, 1)$ and $(2, 3)$ respectively. 6

~~(i)~~ Find the coordinates of the midpoint AB .

~~(ii)~~ Find the gradient of the line AB .

~~(iii)~~ Find the equation of the perpendicular bisector of AB .

~~(iv)~~ The point P lies on the line $y = 2x - 9$ and is equidistant from A and B . Find the coordinates of P .

~~(b)~~ Solve $\cos\theta = -\frac{1}{2}$, for $0 \leq \theta \leq 360^\circ$ 2

~~(c)~~ Show that $\sec\theta - \cos\theta = \sin\theta \tan\theta$ 2

~~(d)~~ Solve for x : $8^x = \frac{1}{16}$ 2

Question 4 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) Consider the parabola with equation $x^2 - 4y + 4 = 0$.

3

(i) Find the coordinates of the vertex of the parabola.

(ii) Find the coordinates of the focus of the parabola.

(b) For what values of k will the equation $x^2 + (k+2)x + 4 = 0$ have real roots?

3

(c) Let A and B be the points $(-1, 0)$ and $(2, 0)$ respectively and let P be the variable point (x, y) .

6

(i) Write down expressions for PA and PB in terms of x and y .

(ii) Suppose that P moves such that $PA = 2PB$. Deduce that P moves on a circle.

(iii) Hence find the centre and radius of this circle.

Question 5 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 1$ at the point $(-2, -1)$. Find m and b . 3

(b) Consider the curve given by $f(x) = -\frac{1}{3}x^3 - x^2 + 3x + 1$.

(i) Find any turning points and determine their nature. 3

(ii) Find any points of inflexion. 2

(iii) Sketch the curve for the domain $-6 \leq x \leq 3$. 2

(iv) For what values of x is $\frac{dy}{dx} > 0$? 1

(v) What is the maximum value of $f(x)$ in the given domain? 1

Question 6 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) Evaluate $\sum_{n=4}^6 (3n + 2)$.

1

(b) An infinite geometric series has a first term of 8 and a limiting sum of 12. Calculate the common ratio.

2

(c) Mrs. Babbage plants cabbages in her garden in rows. The first row has 35 cabbages, the second row has 39 cabbages and each succeeding row has 4 more cabbages than the previous row.

5

(i) Calculate the number of cabbages in the 12th row.

(ii) Which row would be the first to contain more than 200 cabbages?

(iii) Mrs. Babbage runs out of room in her garden after planting 945 cabbages. How many rows has she planted?

(d) The sum of the first 3 terms of an arithmetic series is 27 while the sum of the next 4 terms is 120. Find the sum of the first 10 terms.

4

Question 7 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) Find $\int 6x^2 dx$.

2

(b) Evaluate $\int_1^2 \frac{1}{x^2} dx$.

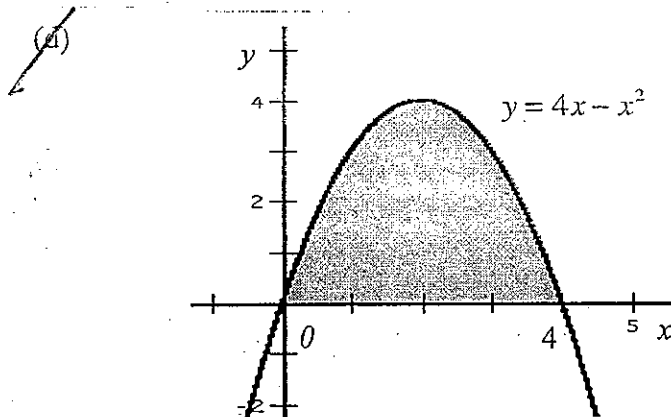
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(c) (i) The trapezoidal rule is used to approximate the integral $\int_1^3 (16 - x^2) dx$ using four subintervals. Find this approximation giving your answer correct to 3 significant figures.

3

(ii) Explain whether this approximation is larger or smaller than the true answer, without calculating the true answer. You may sketch the curve to help.

1



4

The diagram shows the graph of the function $y = 4x - x^2$.

(i) Find the area of the shaded region enclosed by the curve $y = 4x - x^2$ and the x-axis.

(ii) Write down the pair of inequations that specify the shaded region.

Question 8 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) The table shows the values of a function $f(x)$ for five values of x .

3

x	1	1.5	2	2.5	3
y	5	1	-2	3	7

Use Simpson's Rule with these five values to estimate $\int_1^3 f(x) dx$.

(b) At the beginning of each year, Sarah will deposit \$1000 in a fund paying 10% each year compound interest.

(i) What will be the value of Sarah's first deposit when it has been invested for n years?

1

(ii) Show that after n years the total value of all her deposits will be

2

$$\$ \frac{1000(1.1)(1.1^n - 1)}{0.1}$$

(iii) Sarah decides to withdraw all her funds at the end of 20 years. How much money will she have?

1

(c) Samantha and Peter borrow \$280 000 to buy a house. They take the loan over a period of 25 years at an interest rate of 6% per annum, compounded monthly. They repay an amount of \$ M per month. Let A_n be the balance owing after the n^{th} repayment.

(i) Show that $A_2 = 280000 \times 1.005^2 - M(1 + 1.005)$

2

(ii) Find an expression for A_n and use it to calculate their monthly repayments, rounded to the nearest dollar.

2

(iii) Find how much Samantha and Peter pay back altogether after 25 years.

1

Question 9 (12 Marks) Use a SEPARATE writing booklet

Marks

(a) The function $f(x)$ is given by $f(x) = \begin{cases} x+3 & \text{for } x \geq 4 \\ x^2 - 9 & \text{for } x < 4 \end{cases}$

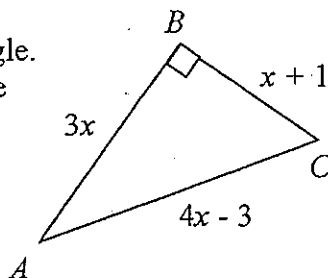
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(i) Find $f(6) + f(2)$

(ii) Find the value of x that produces $f(x) = -9$

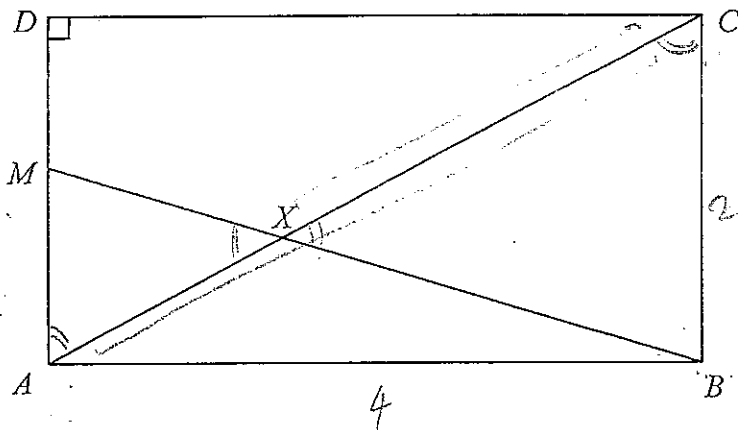
(b) In the diagram $\triangle ABC$ is a right-angle. Find the value(s) of x and hence the length of the hypotenuse.

3



(c) In the diagram, $ABCD$ is a rectangle and $AB = 2AD$.

6



The point M is the midpoint of AD . The line BM meets AC at X .

(i) Show that the triangles AXM and BXC are similar.

(ii) Show that $\frac{AC}{CX} = \frac{3}{2}$.

(iii) Show that $(AB)^2 = \frac{9}{5}(CX)^2$.

END OF PAPER