Moriah College
MATHEMATICS DEPARTMENT

Mathematics
Year 12

2 Unit Pre-Trial 2013

Examiners: P. Brown, G. Busuttil

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided on page 12.
- All necessary working should be shown in every question

Total Marks (100)

- Attempt Questions 1 -16
- Section I: 10 multiple choice
- Section II: Questions 11 – 16
- Use a **SEPARATE** answer book for each question

STUDENT NUMBER:_____ **CLASS TEACHER:**_____

Section I
(10 Marks): Answer on the Multiple Choice Answer Sheet provided.

1 What is 5.076 31 correct to three significant figures?

(A) 5.07

(B) 5.08

(C) 5.076

(D) 5.077

2 Which of the following is equal to $\frac{1}{3\sqrt{2}-\sqrt{5}}$?

(A) $\frac{3\sqrt{2}-\sqrt{5}}{13}$

(B) $\frac{3\sqrt{2}+\sqrt{5}}{13}$

(C) $\frac{3\sqrt{2}-\sqrt{5}}{23}$

(D) $\frac{3\sqrt{2}-\sqrt{5}}{23}$

3 The quadratic equation $x^2+8x+20=0$ has roots α and β.
What is the value of αβ + (α + β)?

(A) -28

(B) -12

(C) 12

(D) 28

4 What is the perpendicular distance of the point (3, 2) from the line $y=2x+3$?

(A) $\frac{7}{\sqrt{3}}$

(B) $\frac{11}{\sqrt{3}}$

(C) $\frac{7}{\sqrt{5}}$

(D) $\frac{11}{\sqrt{5}}$

5 Let $y = e^x$

Which expression is equal to $\log_e(y^3)$?

(A) e^{3x}

(B) e^{x^3}

(C) $3x$

(D) x^3

6 What is the value of $\int_1^4 \frac{1}{5x} dx$?

(A) $\frac{1}{5}\ln 5$

(B) $\frac{2}{5}\ln 2$

(C) $\ln 20$

(D) $\ln 25$

7 What are the solutions of $-2\cos\theta^o = \sqrt{3}$ for $0^o \le \theta^o \le 360^o$?

(A) 30^o *and* 330^o

(B) 30^o *and* 210^o

(C) 150^o *and* 330^o

(D) 150^o *and* 210^o

8 If $\log_a 6 = 3.1$ and $\log_a 5 = 2.7$, which of the following is true?

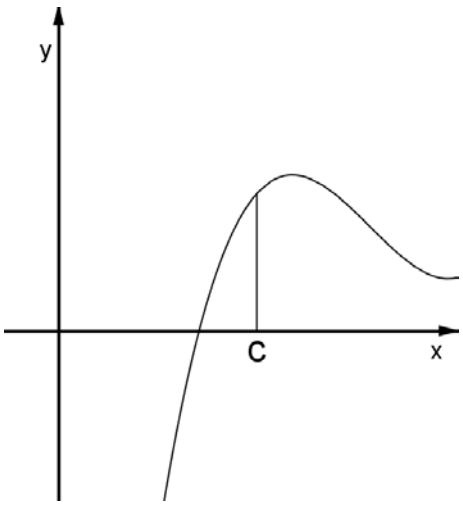
(A) $\log_a 30 = 8.37$

(B) $\log_a 30 = 5.8$

(C) $\log_a 11 = 5.8$

(D) $\log_a 11 = 8.37$

9



Which of the following statements is true?

(A) $f'(c) > 0 \text{ and } f''(c) < 0$

(B) $f'(c) > 0 \text{ and } f''(c) > 0$

(C) $f'(c) < 0 \text{ and } f''(c) < 0$

(D) $f'(c) < 0 \text{ and } f''(c) > 0$

10 The solutions to $|2x+3| < 9$ are:

(A) $-6 < x < 3$

(B) $-3 < x < 6$

(C) $x < -3, x > 6$

(D) $x < -6, x > 3$

END OF SECTION 1

Student number: _____

MULTIPLE CHOICE ANSWER SHEET

Circle the letter of the correct answer

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D

Section II

Question 11 (15 Marks): (Start a new booklet) **Marks**

(a) Evaluate $\frac{2+\sqrt{2}}{7(e^2-4)}$ correct to 3 decimal places. **2**

(b) If $\frac{2}{3+2\sqrt{2}} = a+b\sqrt{2}$, find a and b. **2**

(c) Evaluate $\int_0^3 2dx$ **2**

(d) Solve $2^{2x}-7(2^x)-8=0$ **2**

(e) If $g(x)=7x^3-3x+1$, find $g'(1)$ **2**

(f) Simplify $\frac{a^2+a-2}{a+2}\times\frac{a^2-3a}{a^2-4a+3}$ **2**

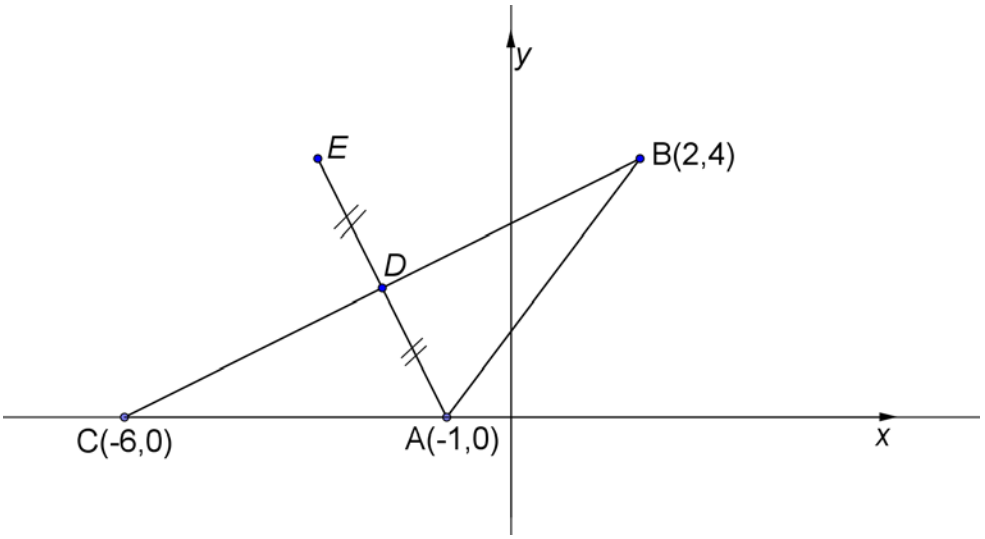
(g) Differentiate

(i) $e^{x^2}+2$ **1**

(ii) $5x\log_e(x^3+1)$ **2**

Question 12 (15 Marks): (Start a new booklet) **Marks**

(a)



In the diagram A, B and C are the points (-1, 0), (2, 4) and (-6, 0) respectively. D has coordinates (-2, 2) and is the midpoint of AE.

(i) Find the length of the interval AB. **1**

(ii) Find the equation of the circle with centre at B which passes through the point A. **2**

(iii) Find the size of $\angle CAB$ (to the nearest degree). **2**

(iv) Find the midpoint of BC. **1**

(v) Show that the equation of the line BC is $x-2y+6=0$ **2**

(vi) Find the perpendicular distance of A from the line BC in simplest exact form. **2**

(vii) What type of quadrilateral is ABEC? Give reasons for your answer. **2**

(b) Find A, B and C if
$2x^2-5x+7\equiv 2A(x+1)^2+B(x+1)+C$ **3**

Question 13 (15 Marks): (Start a new booklet) **Marks**

(a) Consider the curve given by $y = 2x^3 - 9x^2 + 12x$

(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Show that a point inflexion occurs at $x = \frac{3}{2}$ **1**

(iii) Sketch the graph $y = f(x)$, indicating clearly any important features. **2**

(iv) For what values of x is the curve concave up? **1**

(b) The first four terms of a sequence are 3, 6, 9, 12

(i) Show that 102 is a term of this sequence **1**

(ii) Hence, or otherwise, find the sum of the terms of this sequence between 100 and 200 **2**

(c) Find $\lim_{x \to 3} \frac{x-3}{x^2-9}$ **2**

(d) Evaluate $\int_0^4 \frac{9x^2}{3+x^3} dx$ correct to 2 decimal places **3**

Question 14 (15 Marks): (Start a new booklet) **Marks**

(a) Find:

(i) $\int (3x-4)^{10}\,dx$ **1**

(ii) $\int \frac{x^5+x}{x}\,dx$ **2**

(iii) $\int e^{7x}\,dx$ **1**

(iv) $\int \frac{3}{6x-2}\,dx$ **2**

(b) The third term of a geometric series is 2.5 and the seventh term of the same series is 40.

(i) Find the common ratio **2**

(ii) If $r > 0$, find the first term to exceed 1 000 000 **2**

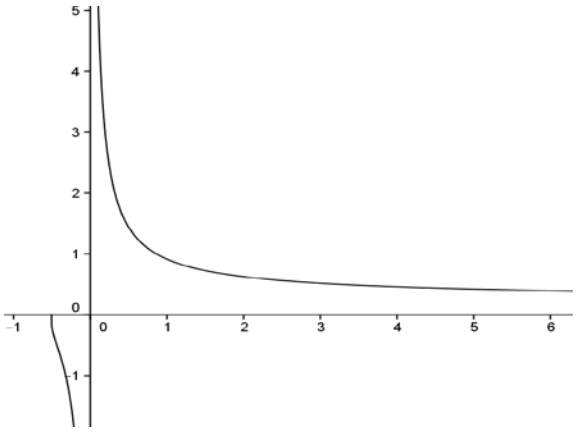
(c) (i) Express $\sin\theta\cos\theta + \frac{\cos^3\theta}{\sin\theta}$ as a single trigonometric ratio. **3**

(ii) Hence, solve $\sin\theta\cos\theta + \frac{\cos^3\theta}{\sin\theta} = 1$ for $0^\circ \le \theta \le 360^\circ$. **2**

Question 15 (15 Marks): (Start a new booklet) **Marks**

(a) Given that $f'(x) = 2x + 2$ and $f(2) = 13$, find $f(x)$. **2**

(b) The diagram below shows part of the graph of $y = \dfrac{1}{\log_e(2x+1)}$



(i) Copy and complete the table of values for $y = \dfrac{1}{\log_e(2x+1)}$ with each value correct to 3 decimal places. **1**

x	0.5	1.0	1.5	2.0
y				

(ii) Use the trapezoidal rule with four function values to approximate $\int_{0.5}^{2} \dfrac{1}{\log_e(2x+1)}\,dx$, correct to three decimal places. **2**

(c) The area bounded by the curve $y = x^2 + 2$ and the line $y = 6$ is rotated about the y-axis. Find the volume of the solid formed. Leave your answer in exact form. **3**

(d) Q is the point (0,2) and R(4, -2). The point P(x,y) moves such that the angle QPR is a right angle.

(i) Draw a diagram to represent the above information. **1**

(ii) Show that the equation of the locus of P is a circle, by finding its centre and radius. **3**

(e) Find the equation of the tangent to the curve $y = xe^x$ where x=1. Give your answer in general form. **3**

Question 16 (15 Marks): (Start a new booklet) **Marks**

(a) Given the parabolic equation:

$$x^2 - 4x - 36 = -8y$$

Find:

(i) The coordinates of the vertex **2**

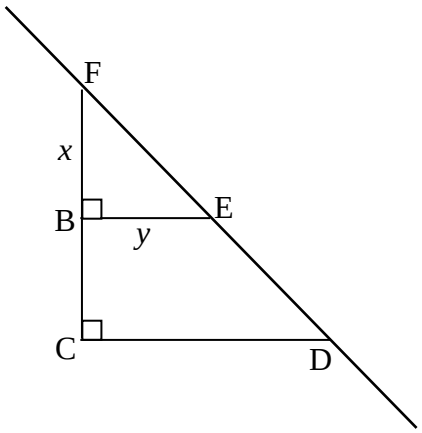
(ii) The equation of the directrix **1**

(iii) The coordinates of the focus **1**

(iv) Find the co-ordinates of the endpoints of the latus rectum. **2**

(b) Using the logarithm laws, find the value of $\log_6 4 - 2\log_6 12$ **3**

(c) A farmer wishes to fence some of her land as shown in the diagram below. Fences are to be erected at FC, CD and BE. The side FD is a river and no fence is needed there. CD is twice the length of BE and FB = BC. $\angle FBE = \angle FCD = 90^\circ$. Let FB=*x metres* and BE=*y*



(i) Copy the diagram into your writing booklet.

(ii) Write an expression in terms of *x* and *y* for:

a. the area of $\Delta FCD, and$ **1**

b. the length, L, of fencing the farmer would need. **1**

(iii) If the total area of the land to be enclosed is $1200\,m^2$, show that the length of fencing L is given by $L = 2x + \frac{1800}{x}$ **1**

(iv) Hence, find the values of *x* and *y* for which the length of fencing required will be a minimum. **3**

END OF EXAMINATION

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$