Moriah College
בית ספר הר המוריה

YEAR 12

MATHEMATICS

PRETRIAL
TERM 1 2015

Time Allowed: 3 hours + 5mins reading time

Examiners: CO BT BR BO

Instructions:

- **USE A BLACK PEN.**
- Answer every QUESTION on a NEW PAGE.
- SHOW all working.
- Draw clear, well labelled, BIG diagrams.
- Marks may be deducted for careless or untidy work.
- Board approved calculators may be used.

Question 1 – (10 marks)

1. What is 4.09784 correct to three significant figures.

   A) 4.09 B) 4.10 C) 4.097 D) 4.098

2. Find $\int \pi \, dx$

   A) $\pi x + c$ B) $\frac{\pi x^2}{2}$ C) $\pi + c$ D) $\frac{\pi^2}{2} + c$

3. Solve : $(2^x)^2 = 2^8$

   A) $x = 6$ B) $x = 4$ C) $x = 2\sqrt{2}$ D) $x = 8$

4. Factorise: $3a^2 + 10a - 8$

   A) $(3a-2)(a+4)$ B) $(3a+2)(a-4)$
   C) $(3a+4)(a-2)$ D) $(3a+4)(a+2)$

5. Simplify: $\frac{x^2 - 4x}{2x - 8}$

   A) $x-2$ B) $\frac{x-2}{4}$ C) $\frac{x}{2}$ D) $\frac{x^2-2}{-8}$

6. Solve the simultaneous equations $2x + y = 3$ and $x - 2y = 4$.

   A) $x = -1,\ y = 2$ B) $x = 1,\ y = 1$
   C) $x = 2,\ y = -1$ D) $x = -2,\ y = 1$

7. Solve for θ:
$\sqrt{3}\tan\theta+3=0$ for $0^0 \le \theta \le 360^0$

A) $\theta=150^0, 330^0$ B) $\theta=60^0, 240^0$
C) $\theta=120^0, 300^0$ D) $\theta=30^0, 210^0$

8. $\int (2x+1)^{\frac{1}{3}}\,dx$

A) $\frac{3}{4}(2x+1)^{\frac{4}{3}}+c$ B) $-\frac{2}{3}(2x+1)^{\frac{-1}{3}}+c$

C) $\frac{8}{3}(2x+1)^{\frac{4}{3}}+c$ D) $\frac{3}{8}(2x+1)^{\frac{4}{3}}+c$

9. Find the derivative of x^2e^{2x} with respect to x

A) $2x^2e^{2x}$ B) $4x^2e^{2x}$ C) $(2x+x^2)e^{2x}$ D) $(1+x)2xe^{2x}$

10. Find the values of a and b if $(\sqrt{a}+\sqrt{2})^2=5+2\sqrt{b}$

A) $a=25,\ b=2$ B) $a=25,\ b=6$
C) $a=3,\ b=6$ D) $a=3,\ b=2$

Question 11 (start each question on a new page) (15 marks)

a) Solve for x:

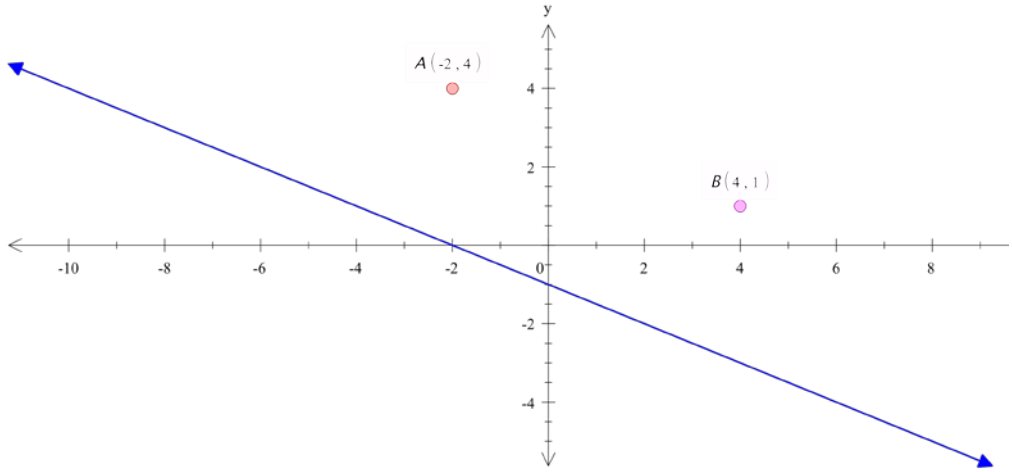
$$\frac{x-5}{3}-\frac{x+1}{4}=5$$

3

b) Find the integers a and b such that $\frac{7}{5+3\sqrt{2}}=a-b\sqrt{2}$

3

c) In the quadrilateral ABCD the coordinates of the points A and B are $(-2,4)$ and $(4,1)$ respectively. The equation of the line DC is $x+2y+2=0$.



i) Find the gradients of AB and DC. Hence, explain why he quadrilateral is a trapezium. **2**

ii) Find the length of AB **1**

iii) The line BC is parallel to the y-axis. Find the coordinates of C. **1**

iv) The line AD is parallel to the x-axis. Find the coordinates of D. **1**

v) Find the perpendicular distance from B to DC **2**

vi) Hence, find the area of the trapezium ABCD **2**

Question 12 (start each question on a new page) (15 marks)

a) Differentiate: $\left(2e^{3x}-4\right)^7$ **2**

b) The first three terms of a sequence are $20, 15, 11\frac{1}{4}$

i) Give a reason why the sequence is geometric? **1**

ii) Find the 8th term of this sequence.(give answer in index form) **1**

iii) Write an expression for the sum of n terms of this sequence. (give answer in simplified index form) **1**

iv) Find the limiting sum of this sequence. **1**

b) Find the equation of the normal to the curve $y=x+e^{2x}$ at the point where $x=0$ **4**

c) Consider the parabola $y^2=8(x+2)$

i) Find the coordinates of the vertex. **1**

ii) Find the coordinates of the focus. **1**

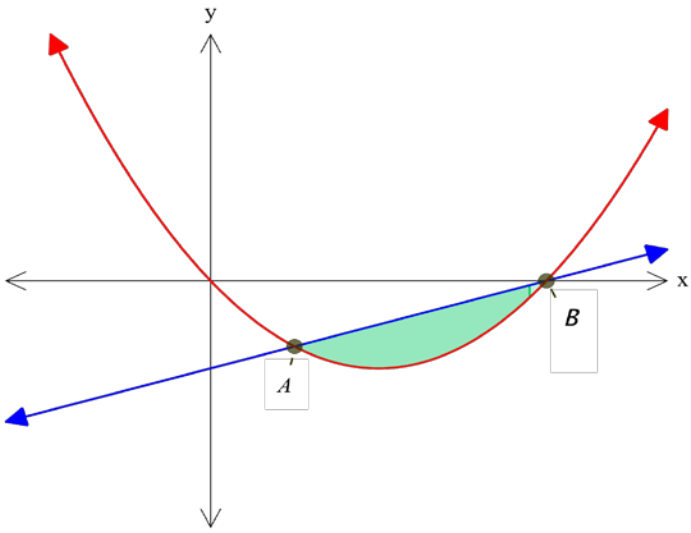
iii) Find the equation of the directrix. **1**

iv) Find the end points of the latus rectum **1**

d) Evaluate: $\int_1^2 e^{3x}\ dx$ (give answer to 2 decimal places) **2**

Question 13 (start each question on a new page) **(15 marks)**

a) The graphs of $y=x-4$ and $y=x^2-4x$ intersect at A and B.



i) Find the x-co-ordinates of the points of intersection of the 2 graphs. **2**

ii) Find the area of the region bounded by $y=x-4$ and $y=x^2-4x$. **3**

b) The quadratic equation $2x^2+8x+k=0$ has roots α and β. Use this information to evaluate:

i) $\alpha+\beta$ **1**

ii) Given that $\alpha^2\beta+\alpha\beta^2=6$, find the value of k. **2**

c) For what values of m does the equation $2x^2+mx+8=0$ have 2 positive, unequal real roots? **3**

d) A tourist drives $25\ km$ from town P on a bearing of 150^0T to town R. He then drives $45\ km$ on a bearing of $022^0\ T$ to town Q.

i) Draw the diagram into your examination booklets and show that $\angle PRQ=52^0$. (give reasons) **2**

ii) Find the distance from P to Q. (to 2 decimal places) **2**

Question 14 (start each question on a new page) (15 marks)

a) A function $f(x)$ is defined by $f(x)=x^3-3x^2$ for $-3\le x\le 4$.

i) Find the x and y intercepts **2**

ii) Find the stationary points and their nature **3**

iii) Sketch the curve $y=f(x)$, clearly showing the intercepts and the point of inflexion. **2**

iv) Find the range of $f(x)$. **2**

b) Use Simpson's rule with 5 function values to find an approximation to

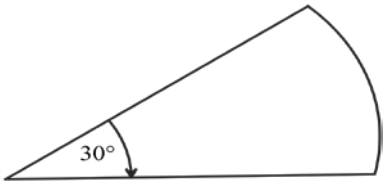
$$\int_0^4 xe^x\ dx$$

(give answer to 2 decimal places) **3**

c) If $\sin\theta=x$, express $\dfrac{1-\cos^2\theta}{\sec^2\theta}$ in terms of x **3**

Question 15 (start each question on a new page) **(15 marks)**

a) Sami calculated that the area of the sector below is $4\pi \ cm^2$.

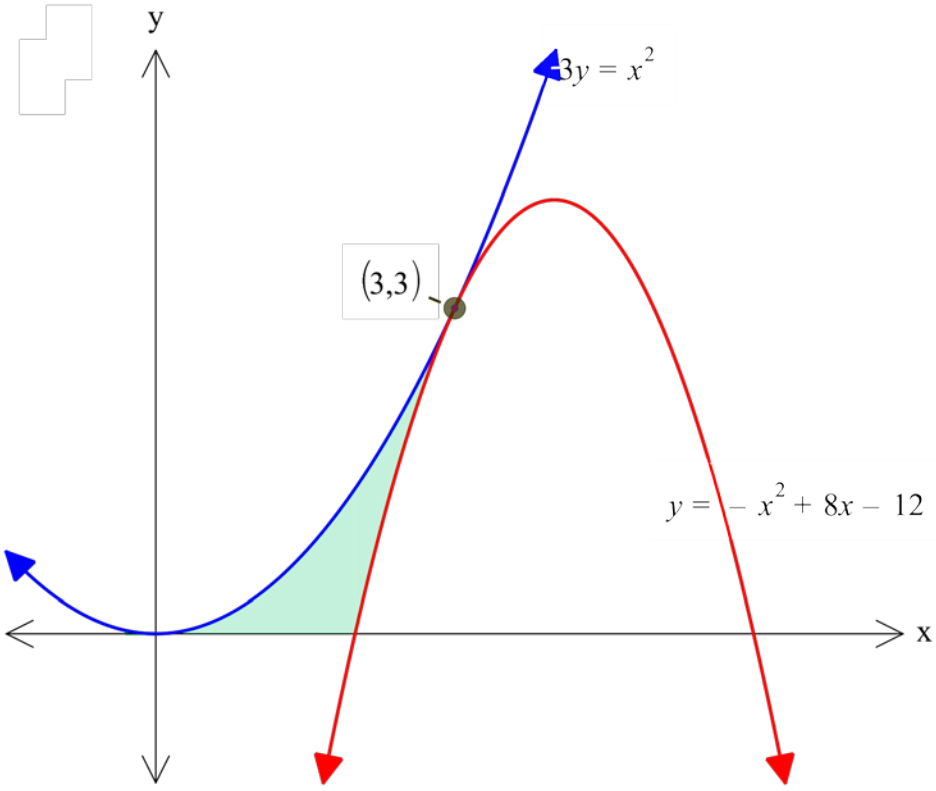


i) Find the radius of the sector. **1**

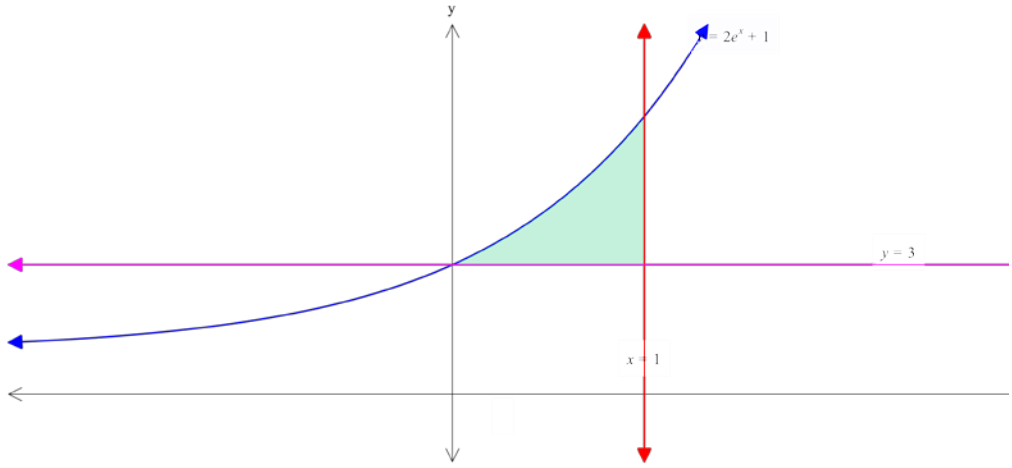
ii) Find the perimeter of the sector. **2**

b) The graphs of $3y=x^2$ and $y=-x^2+8x-12$ are shown on the same system of axes. These curves meet at $(3,3)$ as shown.

Calculate the area enclosed by the curves $3y=x^2$, $y=-x^2+8x-12$ and the x-axis. **4**



d) The area enclosed by the curve $y=2e^x+1$ and the lines $x=1$ and $y=3$ is shaded as shown in the diagram.



i) Show that the volume of the solid formed when this shaded region is rotated about the x-axis can be expressed as

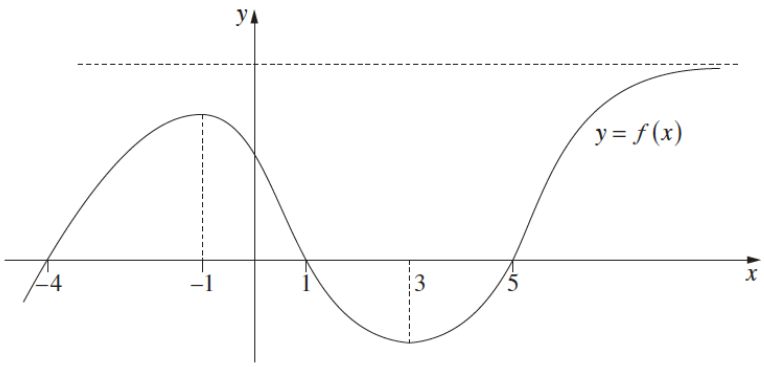
$$V=4\pi\int_0^1\left(e^{2x}+e^x-2\right)dx.$$

2

ii) Calculate the exact volume of the solid formed.

2

e) The diagram shows the graph of $y=f(x)$



(i) For which values of x is the derivative, $y=f'(x)$, negative?

1

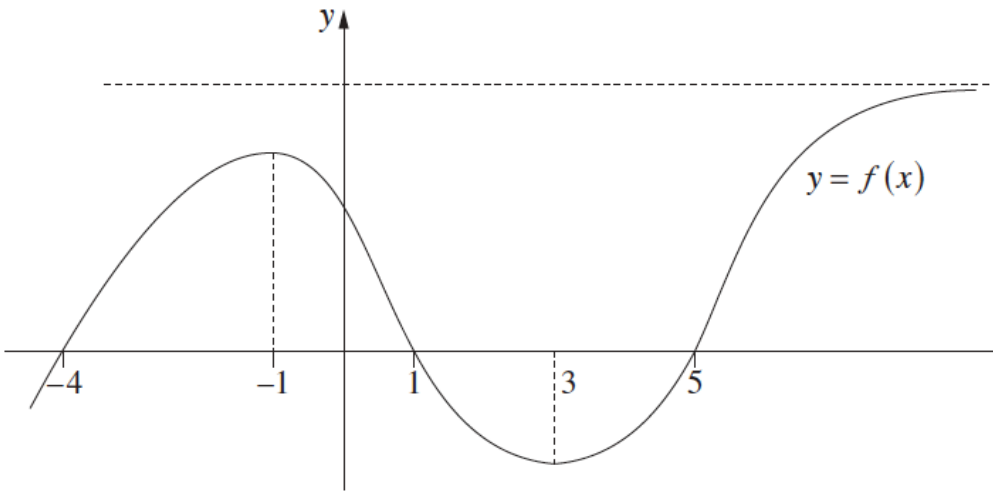
(ii) What happens to $f'(x)$ for large values of x?

(iii) Sketch the graph of $y=f'(x)$
on the attached tear-off sheet

1

2

Name …… Teacher ……..



Question 16 (start each question on a new page) (15 marks)

a) Differentiate $(x^4+8)^5$ hence find $\int_0^1 x^3\left(x^4+8\right)^4\ dx$. **3**

b) If $\tan^2\theta+2\sec^2\theta=5$, find the value of $\sin^2\theta$. **2**

c) $K(k,k-e^{-k})$, $L(-4,-3)$ and $M(5,9)$

Show that the area of ΔKLM is $A=\frac{3}{2}\left(3e^{-k}+k+7\right)$ **3**

d) A farmer is fencing a paddock using P metres of fencing. The paddock is to be in the shape of a sector of a circle with radius r and sector angle θ.

i) Show that the length of the fencing required to fence the perimeter of the paddock is $P=r(\theta+2)$. **1**

ii) Show that the area of the sector is $A=\frac{1}{2}\text{Pr}-r^2$. **1**

iii) Find the radius of the sector, in terms of P, that will maximize the area of the paddock. **2**

iv) Find the angle θ,that gives the maximum area of the paddock. **1**

v) Explain why it is only possible to construct a paddock in the shape of a sector if $\frac{P}{2(\pi+1)}<r<\frac{P}{2}$. **2**

END OF TEST

Name ……..

MULTIPLE CHOICE GRID

10 marks

1.	**A**	**B**	**C**	**D**
2.	**A**	**B**	**C**	**D**
3.	**A**	**B**	**C**	**D**
4.	**A**	**B**	**C**	**D**
5.	**A**	**B**	**C**	**D**
6.	**A**	**B**	**C**	**D**
7.	**A**	**B**	**C**	**D**
8.	**A**	**B**	**C**	**D**
9.	**A**	**B**	**C**	**D**
10.	**A**	**B**	**C**	**D**

STANDARD INTEGRALS

$\int x^n dx$ = $\frac{1}{n+1} x^{n+1}$, $n \neq -1$; $x \neq 0$, if $n < 0$.

$\int \frac{1}{x} dx$ = $\ln x$, $x > 0$.

$\int e^{ax} dx$ = $\frac{1}{a} e^{ax}$, $a \neq 0$.

$\int \cos ax dx$ = $\frac{1}{a} \sin ax$, $a \neq 0$

$\int \sin ax dx$ = $-\frac{1}{a} \cos ax$, $a \neq 0$

$\int \sec^2 ax dx$ = $\frac{1}{a} \tan ax$, $a \neq 0$

$\int \sec ax \tan ax dx$ = $\frac{1}{a} \sec ax$, $a \neq 0$

$\int \frac{1}{a^2 + x^2} dx$ = $\frac{1}{a} \tan^{-1} \frac{x}{a}$, $a \neq 0$

$\int \frac{1}{\sqrt{a^2 - x^2}} dx$ = $\sin^{-1} \frac{x}{a}$, $a > 0, -a < x < a$

$\int \frac{1}{\sqrt{x^2 - a^2}} dx$ = $\ln \left\{ x + \sqrt{(x^2 - a^2)} \right\}$, $|x| > |a|$

$\int \frac{1}{\sqrt{x^2 + a^2}} dx$ = $\ln \left\{ x + \sqrt{(x^2 + a^2)} \right\}$

NOTE: $\ln x = \log_e x$, $x > 0$.