# NEWINGTON COLLEGE

# 2014
# Assessment 2 (HSC mini)

# Year 12 Mathematics

## General Instructions:

- Date of task - Monday 31st March (Wk 10B)
- Reading time – 5 mins
- Working time – 120 mins
- Weighting - 30%
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of the paper.
- Attempt all questions.
- Show all relevant mathematical reasoning and/or calculations.

## Total marks – 70

### Section I (10 marks)

- Answer questions 1 to 10 on the multiple choice answer sheet provided at the end of this paper.
- Allow about 15 minutes for this section.

### Section II (60 marks)

- Answer questions 11 to 14 on the writing paper provided.
- **Start each question on a new page.**
- Each page must show the candidate's computer number.

## Outcomes to be assessed:

**H8** Uses techniques of integration to calculate areas and volumes.

**H3** Manipulates algebraic expressions involving logarithmic and exponential functions.

**H6&7** Uses the derivative to determine the features of the graph of a function; and uses the features of a graph to deduce information about the derivative.

## Section I 10 marks

**Attempt Questions 1-10**
**Allow about 15 minutes for this section.**

1 $\int e^{3x}dx =$

(A) $3e^{3x}+C$

(B) $e^{3x}+C$

(C) $\frac{1}{3}e^{3x}+C$

(D) $\frac{1}{3}e^{x}+C$

2 Convert $\frac{\pi}{7}$ radians into degrees to the nearest minute.

(A) $25^{o}43'$

(B) $51^{o}26'$

(C) $80^{o}47'$

(D) $0^{o}27'$

3 If $\log_2 x = 3$ , then:

(A) $x=9$

(B) $x=\frac{3}{2}$

(C) $x=6$

(D) $x=8$

**4** The exact value of $\sin\left(\frac{5\pi}{4}\right)$ is:

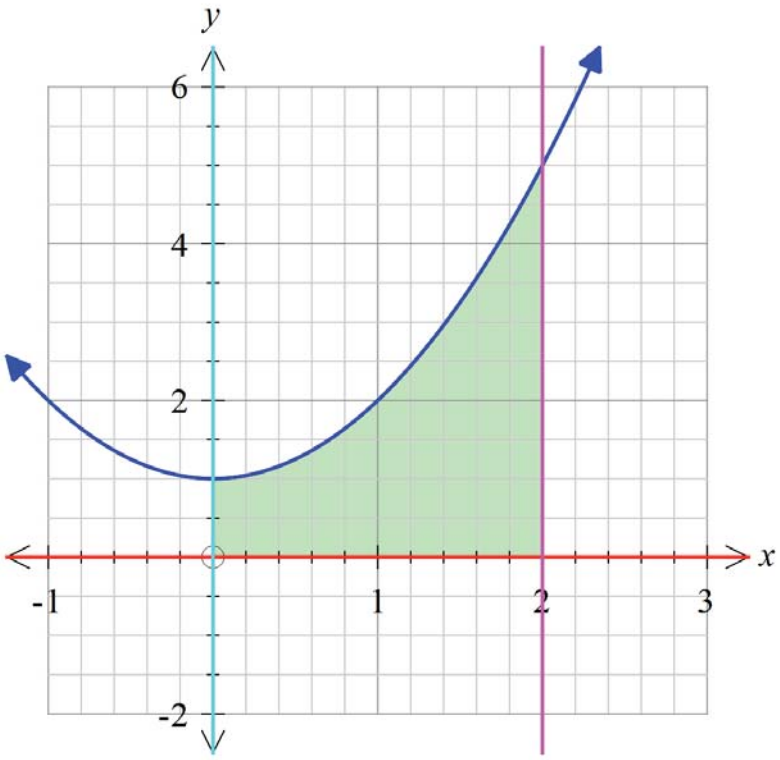
(A) $\frac{1}{\sqrt{2}}$

(B) $-\frac{1}{\sqrt{2}}$

(C) $1$

(D) $-1$

**5** Which of the following will find the shaded area on the diagram below? Note: the curve shown is $y = x^2 + 1$.




(A) $\int_0^5 2 - \sqrt{y-1}\, dy$

(B) $\int_1^5 2 - \sqrt{y-1}\, dy$

(C) $\int_0^2 x^2 + 1\, dx$

(D) $\int_0^2 5 - (x^2 + 1)\, dx$

**6** The gradient of the curve $y = \ln x$ at the point where $x = \frac{1}{2}$ is

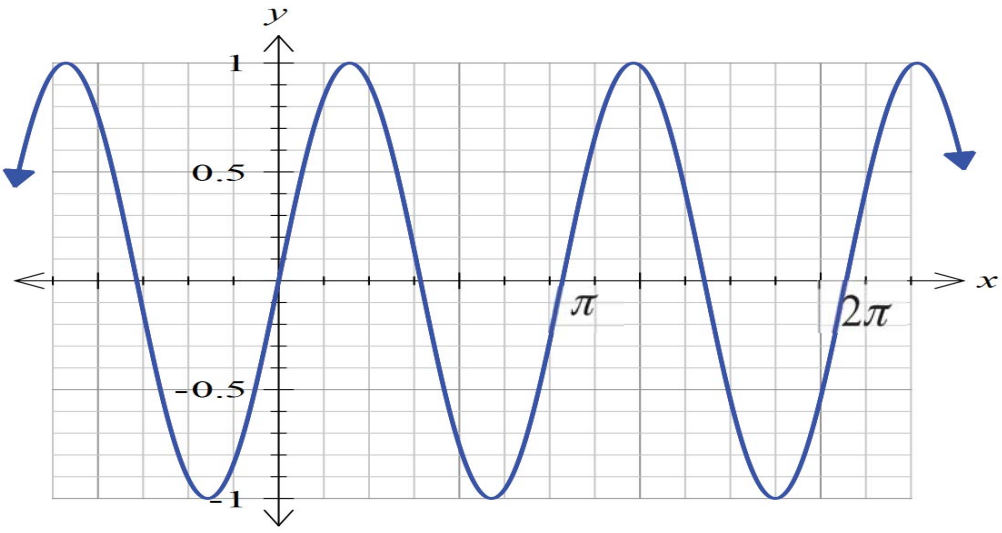
(A) $e^{\frac{1}{2}}$

(B) $\ln \frac{1}{2}$

(C) $\frac{1}{2}$

(D) $2$

**7** The graph below would be best described by which equation?




(A) $y = \sin x$

(B) $y = 2\sin x$

(C) $y = \sin 2x$

(D) $y = \sin \frac{x}{2}$

**8** The derivative of $y = x^2e^x$ is given by

(A) $\frac{dy}{dx} = 2xe^x$

(B) $\frac{dy}{dx} = x^2e^{x-1} + 2xe^x$

(C) $\frac{dy}{dx} = 2xe^{x-1}$

(D) $\frac{dy}{dx} = x^2e^x + 2xe^x$

**9** If $2\log_a x = \log_a 9$, then

(A) $x = 3 \; only$

(B) $x = \pm 3$

(C) $x = 4\frac{1}{2}$

(D) $x = 81$

**10** $\int_1^2 \frac{dx}{x^2} =$

(A) $\ln 4$

(B) $\frac{1}{2}$

(C) $1\frac{3}{4}$

(D) $\frac{7}{24}$

**End of Section I**

## Section II 60 marks

**Attempt Questions 11-14**
**Allow about 1 hour and 45 minutes for this section.**

**Question 11** (15 Marks) – Use a SEPARATE writing booklet.

(a) Evaluate $\log_2 7$ to 2 decimal places. **2**

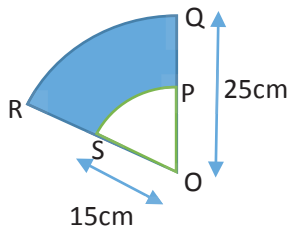
(b) Differentiate:

(i) $y = \dfrac{e^x}{x^2}$ **2**

(ii) $y = (\ln x)^5$ **2**

(c) Find $\displaystyle\int \frac{\sqrt{x}+1}{x}\,dx$ **2**

(d) PS and QR are arcs of concentric circles with radii 15cm and 25cm respectively and O as the centre. The angle at the centre $\angle ROQ$ is $60^o$.
Diagram is not to scale.




Calculate **in terms of** $\pi$ :

(i) The area of the shaded region PQRS **3**

(ii) The perimeter of the shaded region PQRS **2**

(e) Sketch the curve $y = e^x + 2$ showing any asymptotes or intercepts clearly. **2**

**End of Question 11**

**Question 12** (15 Marks) – Use a SEPARATE writing booklet.

(a) (i) Copy and complete the table of values for $y = \sqrt{4x^2 - 1}$ in your writing booklet. **1**

| $x$ | 5 | 5.5 | 6 | 6.5 | 7 |
|---|---|---|---|---|---|
| $y$ | $\sqrt{99}$ | $\sqrt{120}$ | | | |

(ii) Use Simpson's rule with 5 function values to approximate $\int_5^7 \sqrt{4x^2 - 1}\, dx$ to 3 decimal places. **3**

(b) (i) Differentiate $e^{3x^2-4x}$. **1**

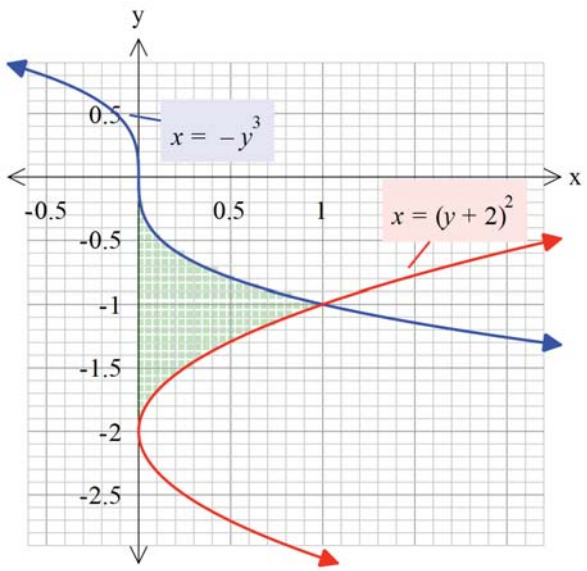
(ii) Hence, find $\int (6x-4)e^{3x^2-4x}\, dx$. **1**

(c) Find the equation of the tangent to the curve $y = e^{-2x}$ at the point where $x = -1$. **4**

(d) The diagram below shows the graphs of $x = -y^3$ and $x = (y+2)^2$

(i) Show algebraically that the point of intersection of the graphs $x = -y^3$ and $x = (y+2)^2$ is $(1, -1)$. **1**

(ii) Find the shaded area on the graph below enclosed by the curves $x = -y^3$, $x = (y+2)^2$ and the y-axis. **4**




**End of Question 12**

**Question 13** (15 Marks) – Use a SEPARATE writing booklet.

(a) Differentiate:

(i) $y = x\tan x$ **2**

(ii) $y = \ln(\sin x)$ leaving your answer in simplest form. **2**

(b) Evaluate $\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx$ **3**

(c) (i) Evaluate $\int_{-2}^{2} x^5\,dx$ **2**

(ii) Using a graph, or otherwise, explain why this integral does not give the area enclosed by this curve and the x-axis between $x = -2$ and $x = 2$ . **2**

(d) Consider the function $y = 2\cos\left(x + \frac{\pi}{2}\right)$ for $0 \le x \le 2\pi$ .

(i) State the amplitude and the period **2**

(ii) Make a neat sketch of the graph in the given domain. **2**

**End of Question 13**

**Question 14** (15 Marks) – Use a SEPARATE writing booklet.

(a) By writing $y = \sec x$ as $y = (\cos x)^{-1}$, show that $\frac{d}{dx}(\sec x) = \sec x \tan x$ **3**

(b) The region enclosed by the curve $y = 2\sqrt{x}$ between $x = 1$ and $x = 4$ is rotated about the x-axis. Find the volume of the solid of revolution formed. **3**

(c) Consider the curve $f(x) = x - \log_e x$

(i) State the domain of $f(x)$. **1**

(ii) Show that $f'(x) = \frac{x-1}{x}$. **2**

(iii) By examining $f''(x)$ show that the curve is concave up for all values of $x$ in the domain. **2**

(iv) Find the coordinates and nature of the turning point. **2**

(v) Sketch the curve showing clearly what happens as $x \to 0^+$. **2**

**End of Paper**

## STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section 1

Solutions Y12 Maths Menai 2014

Multiple Choice

| | |
|---|---|
| Q1 | C |
| 2 | A |
| 3 | D |
| 4 | B |
| 5 | C |
| 6 | D |
| 7 | C |
| 8 | D |
| 9 | A |
| 10 | B |

Section II. Question 11

(a) $\log_2 7 = \frac{\log_e 7}{\log_e 2}$ ✓

$= 2.81$ (2 dp) ✓

(b)(i) $\frac{dy}{dx} = \frac{x^2e^x - 2xe^x}{x^4}$ ✓

$u = e^x \quad u' = e^x$
$v = x^2 \quad v' = 2x$ } ✓

(ii) $\frac{dy}{dx} = 5(\ln x)^4 \times \frac{1}{x}$ ✓

(c) $\int \frac{x^{\frac{1}{2}}+1}{x}\,dx = \int x^{-\frac{1}{2}} + \frac{1}{x}\,dx$ ✓

$= 2x^{\frac{1}{2}} + \ln x + c.$ ✓

(d) $60° = \frac{\pi}{3}^c$ ✓

(i) $A = \frac{1}{2}r_2^2\theta - \frac{1}{2}r_1^2\theta$

$= \frac{1}{2} \times \frac{\pi}{3}(25^2 - 15^2)$ ✓

$= \frac{200\pi}{3}$ cm$^2$ ✓

(ii) $SP = r_1\theta$ $\quad RQ = r_2\theta$

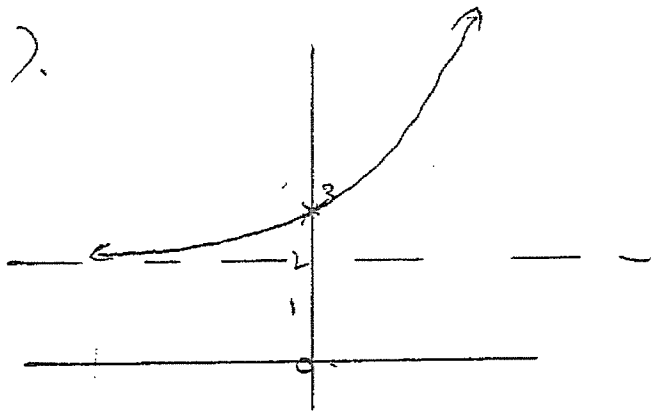
$= \frac{15\pi}{3}$ $\quad = \frac{25\pi}{3}$ } ✓

$= 5\pi$

$PQRS = 2 \times 10 + 5\pi + \frac{25\pi}{3}.$

$= 20 + \frac{40\pi}{3}$ ✓

(e).




asymptote + intercept ✓
shape ✓

Question 12

(a)

| $x$ | 5 | 5.5 | 6 | 6.5 | 7 |
|---|---|---|---|---|---|
| $y$ | $\sqrt{99}$ | $\sqrt{120}$ | $\sqrt{143}$ | $\sqrt{168}$ | $\sqrt{195}$ |

$$\int_5^7 \sqrt{4x^2-1}\,dx \approx \frac{\frac{1}{2}}{3}\left\{\sqrt{99}+\sqrt{195}+4\left\{\sqrt{120}+\sqrt{168}\right\}+2\times\sqrt{143}\right\}$$

$$= 23.9657\ldots$$

$$= 23.916 \text{ (3dp)}$$

(b)(i) $\frac{d}{dx} e^{3x^2-4x} = (6x-4)e^{3x^2-4x}$

(ii) $\int (6x-4)e^{3x^2-4x}\,dx = e^{3x^2-4x} + C$

(c) $y = e^{-2x}$

$\frac{dy}{dx} = -2e^{-2x}$

At $x=-1$ $\frac{dy}{dx} = -2e^2$

and $y = e^2$

Equation of tangent is $y - e^2 = -2e^2(x+1)$

(d) Sub $x=-1$ to satisfy both equations

$x = -y^3$ $\quad$ $x = (y+2)^3$

LHS $= 1$ $\quad$ RHS $= -(-1)^3 = 1$ $\quad$ LHS $= 1$ $\quad$ RHS $= (-1+2)^3 = 1$

LHS $=$ RHS $\quad$ LHS $=$ RHS

(ii) $A = \int_{-1}^{0} -y^3\,dy + \int_{-2}^{-1} (y+2)^2\,dy$ ✓✓ one each integral

$$= -\left[\frac{y^4}{4}\right]_{-1}^{0} + \left[\frac{(y+2)^3}{3}\right]_{-2}^{-1}$$

$$= -\left(0 - \frac{1}{4}\right) + \left(\frac{1}{3} - 0\right)$$

$$= \frac{1}{4} + \frac{1}{3}$$

$$= \frac{7}{12} u^2$$

# Question 13.

(a) (i) $\frac{dy}{dx} = x\sec^2 x + \tan x$ ✓✓

(ii) $\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$ ✓

$= \cot x$ ✓

(b) $\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx = \left[\ln(1+e^x)\right]_0^{\ln 2}$ ✓

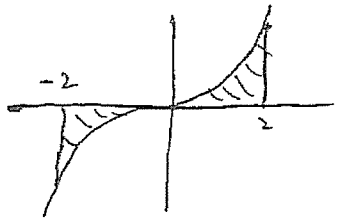
$= \ln(1+e^{\ln 2}) - \ln(1+e^0)$ ✓

$= \ln 3 - \ln 2$ ✓

$= \ln \frac{3}{2}$. ✓

(c) (i) $\int_{-2}^{2} x^5\,dx = 0$ ✓

(d) (ii)

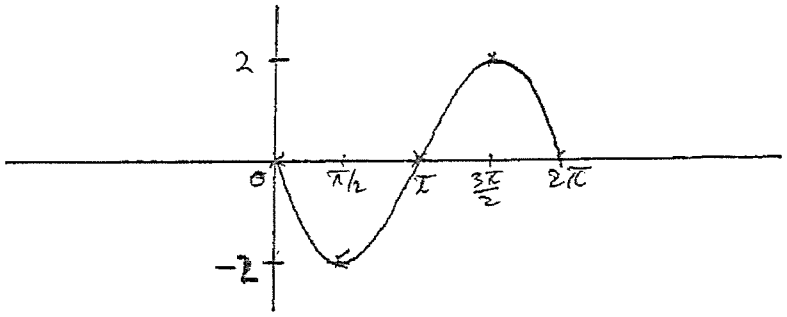



since $x^5$ is odd function shaded areas are equal. Since area to LHS of y-axis is below x-axis it is negative when calculated by definite integral Hence 'areas cancel'. ✓✓

(d) (i) $y = 2\cos\left(x + \frac{\pi}{2}\right)$ amplitude = 2 ✓

period = $2\pi$. ✓

(ii) graph is cosine.. amp 2 curve shifted $\frac{\pi}{2}$ to left.




✓✓

## Question 14

(a) $y = (\cos x)^{-1}$ ✓

$\frac{dy}{dx} = -(\cos x)^{-2} \times -\sin x$. ✓

$= + \frac{1}{\cos^2 x} \times \sin x$.

$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ ✓

$= \sec x \tan x$.

(b) $V = \pi \int_1^4 (2\sqrt{x})^2 dx$ ✓

$= \pi \int_1^4 4x^1 dx$.

$= \pi \left[2x^2\right]_1^4$ ✓

$= \pi (32 - 2)$

$= 30\pi \ u^3$ ✓

(c) (i) $x > 0$

(ii) $f'(x) = 1 - \frac{1}{x}$ ✓

$= \frac{x-1}{x}$

(iii) $f''(x) = \frac{1}{x^2} > 0$ ✓ for all $x$ in domain since $x^2 > 0$.

$\therefore$ concave up.

(iv) turning point when $f'(x) = 0$

$\therefore x = 1 \quad y = 1 - \ln 1$

$= 1$

nature - since concave up, must be local min at $(1,1)$.

(v) As $x \to 0^+$ $y \to \infty$.