NEWINGTON COLLEGE

2015
Assessment 2 (HSC mini)

Year 12 Mathematics

General Instructions:

- Date of task - Thursday 19th March (Wk 8B)
- Reading time – 5 mins
- Working time – 120 mins
- Weighting - 30%
- BOSTES-approved calculators may be used.
- A table of standard integrals is provided at the back of the paper.
- Attempt all questions.
- Show all relevant mathematical reasoning and/or calculations.

Total marks – 71

Section I (10 marks)

- Answer questions 1 to 10 on the multiple choice answer sheet provided at the end of this paper.
- Allow about 15 minutes for this section.

Section II (61 marks)

- Answer questions 11 to 14 on the writing paper provided.
- **Start each question in a new writing booklet.**
- Each page must show the candidate's computer number.

Outcomes to be assessed:

H8 Uses techniques of integration to calculate areas and volumes.

H3 Manipulates algebraic expressions involving logarithmic and exponential functions.

H6&H7 Uses the derivative to determine the features of the graph of a function; and uses the features of a graph to deduce information about the derivative.

This page is intentionally blank

Section I **10 Marks**

Attempt Questions 1-10 on the multiple choice answer sheet.
Allow about 15 minutes for this section.

1 If $f'(a)=0$ and $f''(a)<0$ then the function $y=f(x)$ has

(A) A local minimum at $x=a$

(B) A local maximum at $x=a$

(C) A point of inflexion at $x=a$

(D) an x-intercept at $x=a$

2 If $\log_a b=2.5$ and $\log_a c=4$ then $\log_a\left(\sqrt{bc}\right)$ is closest to

(A) 3.16

(B) 6.5

(C) 0.5

(D) 3.25

3 $\dfrac{d}{dx}\left(\dfrac{e^x}{x}\right)=$

(A) e^x

(B) e^{x-1}

(C) xe^x-e^x

(D) $\dfrac{xe^x-e^x}{x^2}$

4 What is the solution to the log equation $\log_2(x+1)=5$?

(A) 4

(B) 11

(C) 24

(D) 31

5 What is the value of $\int_1^3 \frac{4}{x}\,dx$?

(A) $\ln 12$

(B) $\ln 6$

(C) $4\ln 3$

(D) $\frac{1}{4}\ln 3$

6 If $f'(x)=6x^2-2x$ and $f(-1)=5$, then

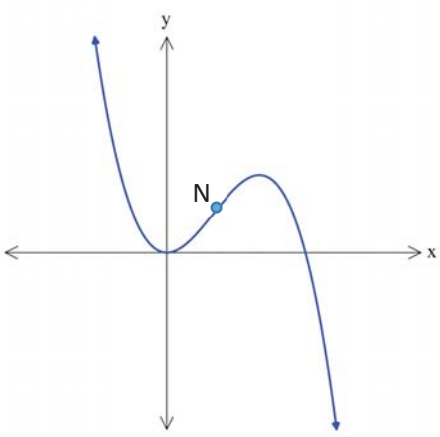
(A) $f(x)=2x^3-x^2+8$

(B) $f(x)=2x^3-x^2+6$

(C) $f(x)=12x+17$

(D) $f(x)=12x-2$

7



The function $f(x)=3x^2-x^3$ is sketched above. The coordinates of the point N are:

(A) (1,1)

(B) (1,2)

(C) (2,4)

(D) (5,4)

8 $\int \frac{1}{3x^2}\,dx =$

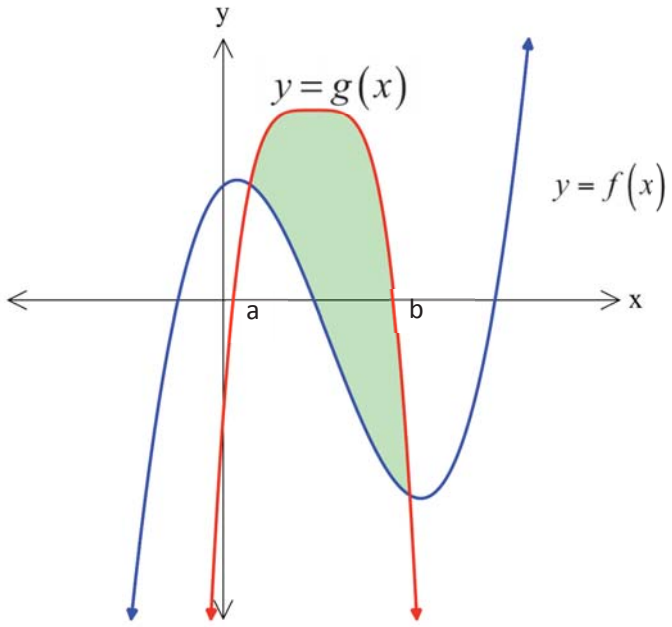
(A) $-\frac{6}{x}+C$

(B) $\frac{1}{x^3}+C$

(C) $-\frac{1}{3x}+C$

(D) $-\frac{1}{9x^3}+C$

9



The shaded area is given by which of the following expressions?

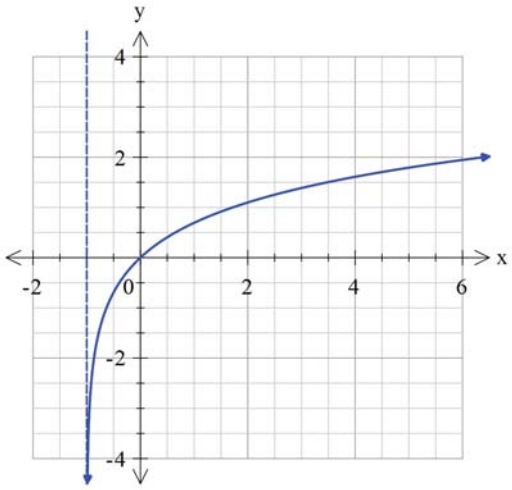
(A) $\int_a^b f(x)-g(x)\,dx$

(B) $\int_a^b g(x)-f(x)\,dx$

(C) $\int_a^b f(x)+g(x)\,dx$

(D) $\pi\int_a^b \left(f(x)-g(x)\right)^2\,dx$

10



The graph above is best described by:

(A) $y = e^{x-1}$

(B) $y = e^{x+1}$

(C) $y = \ln(x-1)$

(D) $y = \ln(x+1)$

End of Section I

Section II
Attempt questions 11-14
Allow about 1 hour and 45 minutes for this section.

Question 11 (15 Marks)- Use a SEPARATE writing booklet.

(a) Evaluate $e^{-0.5}$ to 2 decimal places. 2

(b) Differentiate:

(i) $y = \ln\left(x + e^x\right)$ 2

(ii) $y = \dfrac{\ln x}{x}$ 2

(c) For what values of x is the curve $y = x^3 - 2x^2 + x + 2$ decreasing? 3

(d) Find *a* if $\log_a\left(2\right) = -\dfrac{1}{4}$ 2

(e) (i) State the domain and range of the function $y = e^{-x} + 2$ 2

(ii) Sketch the curve of $y = e^{-x} + 2$, clearly indicating any intercepts or asymptotes. 2

Question 12 (15 Marks)- Use a SEPARATE writing booklet.

(a) (i) Find $\int \sqrt{x} - \dfrac{x^3}{3}\,dx$ 2

(ii) Evaluate $\int_0^1 \dfrac{2x}{1+x^2}\,dx$ 2

(iii) Find $\int 4e^{2x+1}\,dx$ 2

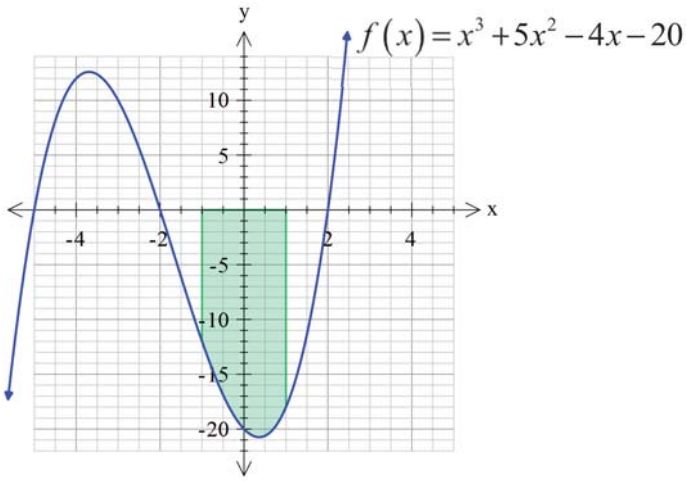
(b) The table below gives three values for the function $f(x) = x^3 + 5x^2 - 4x - 20$

x	-1	0	1
$f(x)$	-12	-20	-18

(i) Using the table above and **using Simpson's Rule** with 3 function values, estimate $I = \int_{-1}^{1} x^3 + 5x^2 - 4x - 20 \, dx$. 2

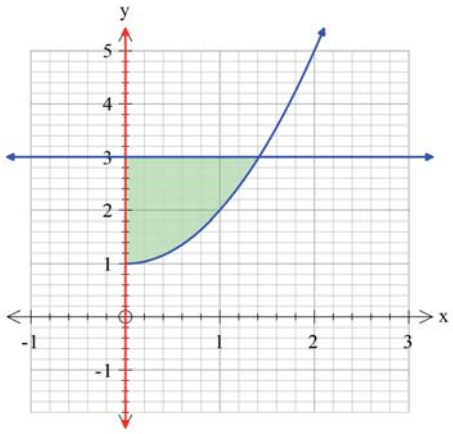
(ii) Using the table above and **using the Trapezoidal Rule** with 3 function values, estimate $I = \int_{-1}^{1} x^3 + 5x^2 - 4x - 20 \, dx$. 2

(iii)



If parts (i) and (ii) above were to be used to estimate the shaded area, which would you use to give an answer closest to the actual area. Briefly explain why. 2

(c)



The area shaded above is bounded by the curve $y = x^2 + 1$ and the line $y = 3$.

Calculate the volume of the solid of revolution when this region is rotated about the ***y*-axis**. 3

Question 13 (16 Marks)- Use a SEPARATE writing booklet.

(a) Consider the curve given by $y = x^3 - 6x^2 + 9x$.

(i) Find the stationary points and determine their nature 3

(ii) Find any points of inflexion. 2

(iii) Sketch the curve in the domain $0 \le x \le 5$. 2

(iv) Find the maximum value of y in this domain. 1

(b)

(i) Differentiate $y = 3^x$ 2

(ii) Find $\int \frac{e^{2x} - 1}{e^x}\, dx$ 2

(c) Find the equation of the **normal** to the curve $y = 3 - 2x^2$ at the point where $x = 1$. 4

Question 14 (15 Marks)- Use a SEPARATE writing booklet.

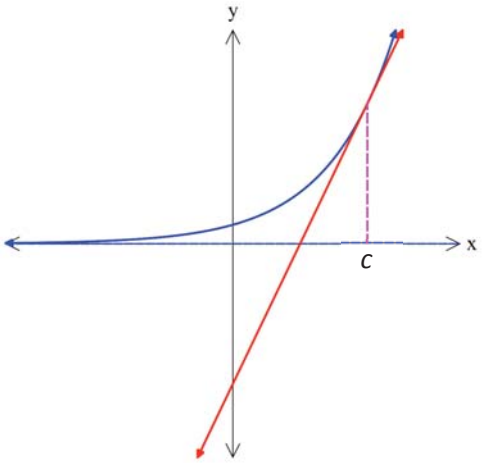
(a) A closed cylindrical can of radius *r* centimetres and height *h* centimetres has a surface area of $120\pi\ cm^2$.

(i) Show that $h = \frac{60 - r^2}{r}$. 1

(ii) Show that the volume of the can *V* can be expressed as $V = 60\pi r - \pi r^3$. 1

(iii) Find the exact value of *r* that gives the maximum volume for the can. 3

(b)



The diagram above shows a graph of the curve $y = e^x$ and the tangent to this curve at the point where $x = c$.

(i) Find the gradient of the tangent at $x = c$. 1

(ii) Find the equation of the tangent at $x = c$. 1

(iii) Find the value of c if the tangent intersects the x-axis at $x = 1$. 2

(c) If the area under the curve $y = \dfrac{1}{2x}$ for $1 \le x \le a$ is 1, find the exact value of a. 3

(d) Solve $2\log x = \log(x+2)$ 3

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$