NORTH SYDNEY BOYS HIGH SCHOOL

2009 YEAR 12 HSC
ASSESSMENT TASK 2

Mathematics

General Instructions

- Working time – 65 minutes
- Write in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

Total Marks (54)

- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Weiss
- O Mr Fletcher
- O Mr Lowe
- O Mr Ireland
- O Mr Trenwith
- O Mr Rezcallah
- O Mr Barrett

Student Number: .....

Question	1	2	3	4	5	6	7	Total	Total
Mark	/8	/6	/8	/8	/8	/7	/9	/54	/100

Question 1 (8 marks)

(a) Find (i) $\int (3x^2 + 1)\,dx$ 2

(ii) $\int \left(\frac{1}{x^2} - \sqrt{x} \right) dx$ 2

(b) Evaluate $\int_0^1 (2x + 1)^4\,dx$ 2

(c) If $f'(x) = 4x - 1$, and $f(-1) = 6$, find $f(x)$. 2

Question 2 (6 marks)

Solve for x, giving exact answers:

(a) $(x - 2)^2 = 5$ 2

(b) $x^4 = 8x^2 + 9$ 2

(c) $x^2 > 9x$ 2

Question 3 (8 marks)

(a) Sketch the parabola $x^2 = -12y$, showing all important features. 2

(b) A parabola has its focus at $S(1, 3)$, and its directrix has equation $x = -5$. Write down the equation of this parabola. 3

(c) Find the coordinates of the focus, and the equation of the directrix for the parabola $x^2 + 4x - 6y + 10 = 0$. 3

Question 4 (8 marks)

(a) The expression $2x^2 - x + 4$ has zeros α and β. Find the values of

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

(iv) $\alpha^2 + \beta^2$ 2

(b) Form a quadratic equation whose roots are $1 \pm \sqrt{3}$. 2
Write your answer in the form $ax^2 + bx + c = 0$

Question 5 (8 marks)

(a) Write down the equation of the circle centred on $C(-3, 1)$, with a radius of 4 units. **2**

(b) The points A and B have coordinates $(-1, 2)$ and $(0, 4)$ respectively.
Derive the equation of the locus of a point P which satisfies

(i) P is twice as far from A as from B. **3**

(ii) $AP \perp BP$. **3**

Question 6 (7 marks)

(a) Use the discriminant to show that the roots of $3x^2 + 4x - 1 = 0$ are real. **3**
Then write down two more properties of these roots.

(b) (i) Show that the equation $ax^2 - (a+1)x + 1 = 0$ has real roots for all values of a. **2**

(ii) It is given that the vertex of $y = ax^2 - (a+1)x + 1$ lies on the x-axis. **2**
Find the coordinates of this vertex.

Question 7 (9 marks)

(a) Express x^2 in the form $a(x+1)^2 + b(x+1) + c$ **3**

(b) Solve $2^{2x+1} + 2^x = 1$ [working needed to get **any** marks] **3**

(c) A straight line through the origin has a gradient of m. **3**
A parabola has an equation of the form $y = ax^2 + m$.
For what value of m (other than $m = 0$) is the line a tangent to the parabola?

[Show all working]

Question 1

(a) (i) $\int (3x^2+1)\,dx = \underline{x^3 + x + c}$

answer ... 1
'c' ... 1
[2]

(ii) $\int \left(\frac{1}{x^2} - \sqrt{x}\right)dx = \int (x^{-2} - x^{1/2})\,dx$

$= \frac{x^{-1}}{-1} - \frac{x^{3/2}}{3/2}\,dx$

$= -\frac{1}{x} - \frac{2}{3}\sqrt{x^3} + c$

$-\frac{1}{x}$ ... 1
$-\frac{2}{3}x^{3/2}$ ... 1
[2]

(b) $\int_0^1 (2x+1)^4\,dx = \frac{1}{10}\left[(2x+1)^5\right]_0^1$ ... 1

$= \frac{1}{10}(243 - 1)$

$= \frac{121}{5}$ ... 1

[2]

(c) $f'(x) = 4x - 1$

$f(x) = 2x^2 - x + c$ ... 1 (with or without c)

$(-1, 6)$: $6 = 2 + 1 + c$

$c = 3$ ... 1

$f(x) = 2x^2 - x + 3$

[2]

Question 2

(a) $(x-2)^2 = 5$

$x - 2 = \pm\sqrt{5}$

$x = 2 \pm \sqrt{5}$

... ½

... ½

2

(b) $x^4 = 8x^2 + 9$

$x^4 - 8x^2 - 9 = 0$

$(x^2 - 9)(x^2 + 1) = 0$

$x = \pm 3$

... ½ for $(x-9)(x+1)$

... ½

2

(c) $x^2 > 9x$

$x^2 - 9x > 0$

$x(x-9) > 0$

$x < 0$ or $x > 9$

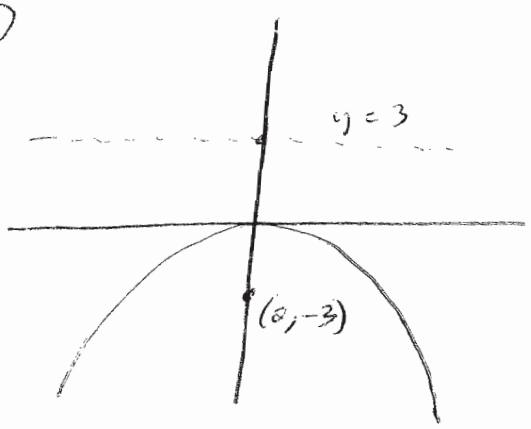
roots ... ½

inequality ... ½

2

Question 3

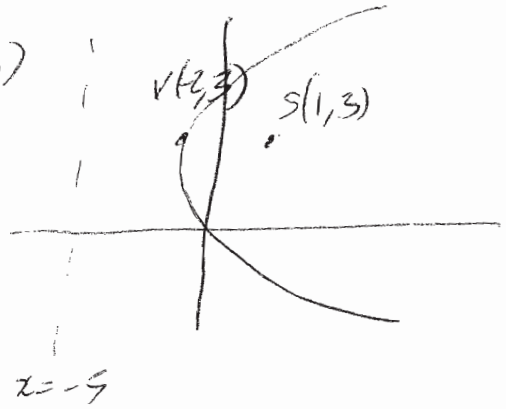
(a)



Focus
Position
Shape/Orientation
} 1 off for each error

2

(b)



$(y-3)^2 = 12(x+2)$

Orientation ... 1
12 ... 1
$y-3$, $x+2$ ... 1

3

(c) $x^2 + 4x - 6y + 10 = 0$

$x^2 + 4x = 6y - 10$
$+4 \qquad +4$

$(x+2)^2 = 6(y-1)$ ... 1

~~V~~ $(-2, 1)$

$a = 1\frac{1}{2}$

$\therefore S\left(-2, 2\frac{1}{2}\right)$ ... 1

dir: $y = -\frac{1}{2}$ ... 1

3

Question 4

(a) (i) $\alpha + \beta = \frac{1}{2}$ ... 1

(ii) $\alpha\beta = 2$ ... 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ ... 1

$= \frac{1/2}{2}$

$= 1/4$ ... 1

2

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ... 1

$= \left(\frac{1}{2}\right)^2 - 2(2)$

$= -\frac{15}{4}$ ... 1

2

(b) sum of roots $= 2$

prod. of roots $= (1+\sqrt{3})(1-\sqrt{3})$

$= -2$

any one of these ... 1

$\therefore x^2 - 2x - 2 = 0$ ... 1

2

Question 5

(a) $(x+3)^2 + (y-1)^2 = 16$ ... ½ ... ½ [2]

(b) $PA = 2PB$ ... ½

$PA^2 = 4PB^2$

$(x+1)^2 + (y-2)^2 = 4x^2 + 4(y-4)^2$ ... ½

$x^2 + 2x + 1 + y^2 - 4y + 4 = 4x^2 + 4y^2 - 32y + 64$ [3]

$3x^2 + 3y^2 - 2x - 28y + 59 = 0$ ... ½

(c) $m_{AP} \cdot m_{BO} = -1$

$\frac{y-2}{x+1} \cdot \frac{y-4}{x} = -1$

$(y-2)(y-4) = -x(x+1)$

$y^2 - 6y + 8 = -x^2 - x$

$x^2 + y^2 + x - 6y + 8 = 0$

Question 6

(a) $\Delta = b^2 - 4(3)(-1)$

$= 28 > 0$

$\therefore$ roots are real ... 1

also distinct ... 1

and irrational --- 1

3

(b) (i) $\Delta = (a+1)^2 - 4 \cdot a \cdot 1$ ... 1

$= a^2 + 2a + 1 - 4a$

$= a^2 - 2a + 1$

$= (a-1)^2$

$\geq 0 \quad \forall a$ ... 1

$\therefore$ real roots $\forall a$

2

(ii) equal roots $\Rightarrow a = 1$ ... 1

$\therefore x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ ---------- 1

$\therefore V(1, 0)$

2

Question 7

(a) $x^2 \equiv a(x+1)^2 + b(x+1) + c$

$x = -1 \Rightarrow 1 = c$

$x = 0 \Rightarrow 0 = a + b + 1$ $\qquad a + b = -1$

$x = -2 \Rightarrow 4 = a - b + 1$ $\qquad a - b = 3$

$(+)$ $\quad 2a = 2$

$a = 1$

$b = -2$

$\therefore x^2 \equiv (x+1)^2 - 2(x+1) + 1$

OR

$x^2 \equiv a(x+1)^2 + b(x+1) + c$

$= ax^2 + 2ax + a + bx + b + c$

$= ax^2 + (2a+b)x + (a+b+c)$

$\therefore$ $\underline{a = 1}$ $\qquad 2a + b = 0$ $\qquad a + b + c = 0$

$2 + b = 0$ $\qquad 1 - 2 + c = 0$

$\underline{b = -2}$ $\qquad \underline{c = 1}$

$\therefore x^2 \equiv (x+1)^2 - 2(x+1) + 1$

(b) $2^{2x+1} + 2^x = 1$

$2(2^x)^2 + 2^x = 1$ ... 1

let $m = 2^x$

$2m^2 + m - 1 = 0$

$(2m-1)(m+1) = 0$ ... 1

$m = \frac{1}{2}$ $\quad m = -1$

$2^x = \frac{1}{2}$ $\quad 2^x = -1$

$x = -1$ $\quad$ no solution ... 1

3

(No working = No marks)

(c) $ax^2 + m = mx$ ... 1

$ax^2 - mx + m = 0$

$\Delta = 0$ for tangent ... 1

$m^2 - 4am = 0$

$m(m - 4a) = 0$

$m = 0$ $\quad m = 4a$ ... 1

stated already

3