NORTH SYDNEY BOYS
HIGH SCHOOL

# MATHEMATICS

2010 HSC Course Assessment Task 2

**General instructions**

- Working time – 65 minutes.
- Write in the booklet provided. **Each question is to commence on a new page.**
- Write using blue or black pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question (Insufficient/illegible working may cause a deduction of marks).
- Attempt **all** questions.

**Class teacher** (please ✔)

- ◯ 12M21 – Mr Berry
- ◯ 12M22 – Mr Fletcher
- ◯ 12M31 – Mr Rezcallah
- ◯ 12M32 – Mr Lowe
- ◯ 12M33 – Mr Ireland
- ◯ 12M41 – Mr Barrett
- ◯ 12M42 – Mr Trenwith
- ◯ 12M43 – Mr Weiss

**STUDENT NUMBER:** .........................................

Marker's use only.

| **QUESTION** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **Total** | % |
|---|---|---|---|---|---|---|---|---|---|
| **MARKS** | $\overline{10}$ | $\overline{6}$ | $\overline{9}$ | $\overline{10}$ | $\overline{8}$ | $\overline{12}$ | $\overline{12}$ | $\overline{67}$ | |

**Question 1** (10 Marks) Commence a NEW page. **Marks**

(a) Find:

i. $\displaystyle\int (x^3 - 3x^2 + 4)\, dx.$ **2**

ii. $\displaystyle\int (x+2)(2x-5)\, dx.$ **2**

iii. $\displaystyle\int \frac{3x^5 + 2x^3 - 1}{x^2}\, dx.$ **2**

(b) Evaluate:

i. $\displaystyle\int_0^1 x\sqrt{x}\, dx.$ **2**

ii. $\displaystyle\int_2^3 (2x-5)^3\, dx.$ **2**

**Question 2** (6 Marks) Commence a NEW page. **Marks**

(a) The gradient function of a curve is given by $\dfrac{dy}{dx} = 2x + 5$. **3**
If $y = -1$ when $x = 2$, find $y$ as a function of $x$.

(b) Find the values of $k$ for which **3**

$$\int_1^k (x+1)\, dx = 6$$

**Question 3** (9 Marks) Commence a NEW page. **Marks**

(a) A parabola has its vertex at the point $(3, 1)$ and its directrix has equation $y = -1$.

i. What is its focal length? **1**

ii. State the coordinates of the focus. **1**

iii. Find the equation of the parabola. **1**

iv. What is the equation of its axis of symmetry? **1**

(b) A parabola has equation $x^2 + 2x + 8y + 25 = 0$. Write down:

i. the coordinates of its vertex. **2**

ii. the coordinates of its focus. **2**

iii. the equation of its directrix. **1**

**Question 4** (10 Marks) Commence a NEW page. **Marks**

(a) If $\alpha$ and $\beta$ area the roots of $2x^2 + 3x - 4 = 0$, find the value of

i. $\alpha + \beta$. **1**

ii. $\alpha\beta$ **1**

iii. $(\alpha - 3)(\beta - 3)$. **2**

(b) For what values of $p$ will the equation **2**

$$2px^2 - (p+2)x + (p-4) = 0$$

have roots which are reciprocals of each other?

(c) Solve for $x$: **4**

$$\left(x^2 - 2x\right)^2 - \left(x^2 - 2x\right) - 6 = 0$$

**Question 5** (8 Marks) Commence a NEW page. **Marks**

(a) Find the values of $m$ for which **2**

$$5x^2 - 4x + m = 0$$

has real roots.

(b) Find the values of $k$ for which **3**

$$x^2 - (k+2)x + (3k+6)$$

is positive definite.

(c) Find the values of $A$, $B$ and $C$ for which **3**

$$9x^2 + 2x - 5 \equiv Ax(x+1) + B(x+1) + C$$

**Question 6** (12 Marks) Commence a NEW page. **Marks**

(a) Calculate the area of the region between the curve $y = x^2 - 2x$, the $x$ axis and the lines $x = 1$ and $x = 3$. **4**

(b) i. Show that the points of intersection of the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$ are $(5, 15)$ and $(-1, 3)$. **2**

ii. Calculate the area between the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$. **3**

(c) Use Simpson's Rule with five function values to find an approximation to **3**

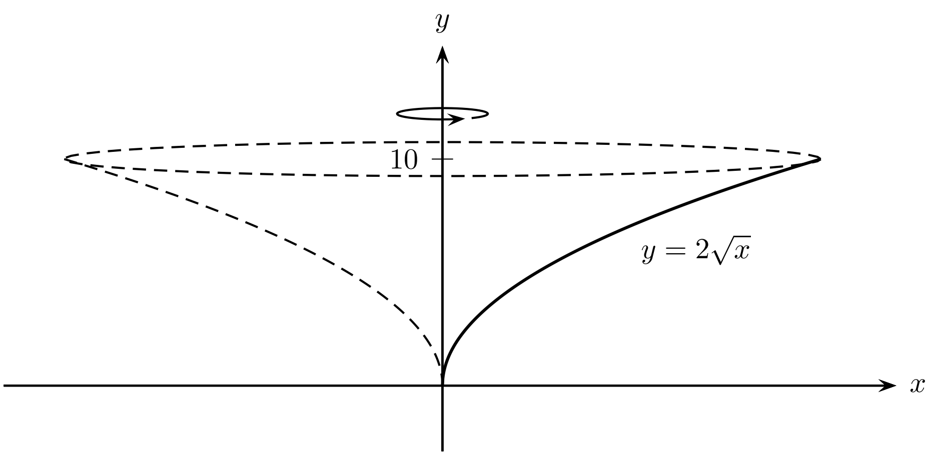
$$\int_1^3 3^{x-1}\,dx$$

**Question 7** (12 Marks) Commence a NEW page. **Marks**

(a) i. Find the locus of the point $P(x, y)$ which moves such that its distance from the point $A(1, -2)$ is always twice the distance from the point $B(7, -8)$. **3**

ii. Describe this locus geometrically, clearly stating its features. **2**

(b) A glass is obtained by rotating part of the parabola $y = 2\sqrt{x}$ about the $y$ axis as shown. **3**




The glass is 10 cm deep.
Find the volume of liquid that the glass will hold.

(c) i. Draw a neat sketch of the parabola $x^2 = -4ay$. **1**

ii. The area between the parabola and its latus rectum is rotated about the $x$ axis. Calculate the volume of the solid which is generated. **3**

**End of paper.**

MATHEMATICS YEAR 12 2UNIT ASSESSMENT 2 — SOLUTIONS

## QUESTION 1

(a) (i) $\int (x^3 - 3x^2 + 4)\,dx = \dfrac{x^4}{4} - x^3 + 4x + c$    2

no c $-1$

(ii) $\int (x+2)(2x-5)\,dx = \int (2x^2 - x - 10)\,dx$    1

$= \dfrac{2}{3}x^3 - \dfrac{x^2}{2} - 10x + c$    1

(iii) $\int \dfrac{3x^5 + 2x^3 - 1}{x^2}\,dx = \int (3x^3 + 2x - x^{-2})\,dx$    1

$= \dfrac{3}{4}x^4 + x^2 + x^{-1} + c$    1

(b) (i) $\int_0^1 x\sqrt{x}\,dx = \int_0^1 x^{3/2}\,dx$

$= \left[\dfrac{2}{5}x^{5/2}\right]_0^1$    1

$= \dfrac{2}{5} - 0 = \dfrac{2}{5}$    1

(ii) $\int_2^3 (2x-5)^3\,dx = \left[\dfrac{(2x-5)^4}{8}\right]_2^3$    1

$= \left(\dfrac{1}{8} - \dfrac{(-1)^4}{8}\right) = \dfrac{1}{8} - \dfrac{1}{8} = 0$    1

## QUESTION 2

(a) $\dfrac{dy}{dx} = 2x + 5$

$y = x^2 + 5x + c$    1

When $x = 2$, $y = -1$ $\Rightarrow -1 = 2^2 + 5 \times 2 + c$    1

$c = -15$    1

$\therefore y = x^2 + 5x - 15$

(b) $\int_1^k (x+1)\,dx = \left[\dfrac{x^2}{2} + x\right]_1^k$    1

$= \left(\dfrac{k^2}{2} + k\right) - \left(\dfrac{1}{2} + 1\right)$

$= \dfrac{k^2}{2} + k - \dfrac{3}{2} = 6$    1

i.e. $k^2 + 2k - 3 = 12$

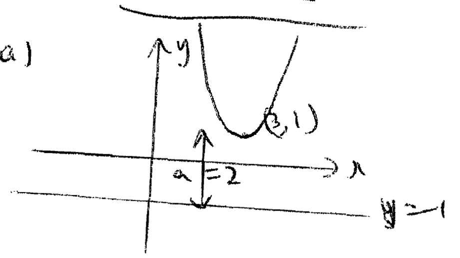
$k^2 + 2k - 15 = 0$

$(k+5)(k-3) = 0$    1

$k = -5$ or $k = 3$

QUESTION 3

(a)




(i) $a = 1 - -1 = 2$ 1

(ii) focus is at $(3,3)$ 1

(iii) equation of parabola is

$(x-3)^2 = 8(y-1)$ 1

(iv) axis of symmetry has equation $x = 3$ 1

(b) (i) $x^2 + 2x + 8y + 25 = 0$

$x^2 + 2x + 1 = -8y - 24$

$(x+1)^2 = -8(y+3)$ 1

$\therefore$ vertex is at $(-1,-3)$ 1

(ii) $a = 2$ 1

$\therefore$ focus is at $(-1,-5)$ 1

(iii) directrix has equation $y = -1$ 1

QUESTION 4

(a) (i) $\alpha + \beta = -\frac{b}{a} = -\frac{3}{2}$ 1

(ii) $\alpha\beta = \frac{c}{a} = -\frac{4}{2} = -2$ 1

(iii) $(\alpha-3)(\beta-3) = \alpha\beta - 3\alpha - 3\beta + 9$

$= \alpha\beta - 3(\alpha+\beta) + 9$ 1

$= -2 - 3 \times -3/2 + 9$

$= \frac{23}{2}$ 1

(b) $\alpha\beta = 1$

i.e. $\frac{c}{a} = 1$

i.e. $\frac{p-4}{2p} = 1$ 1

$p - 4 = 2p$

$p = -4$ 1

(c) Let $u = x^2 - 2x$

then equation becomes $u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3$ or $u = -2$ 1

i.e. $x^2 - 2x = 3$ or $x^2 - 2x = -2$

$x^2 - 2x - 3 = 0$ $\qquad$ $x^2 - 2x + 2 = 0$

$(x-3)(x+1) = 0$ $\qquad$ $\downarrow$

$x = 3, x = -1$ $\quad$ 2 $\quad$ no solutions 1

$\therefore$ valid solutions are $x = 3$ or $x = -1$

QUESTION 5

(a) real roots $\Rightarrow \Delta \geq 0$

i.e. $(-4)^2 - 4\times 5\times m \geq 0$ 1

$16 - 20m \geq 0$

$20m \leq 16$

$m \leq \frac{4}{5}$ 1

(b) positive definite $\Rightarrow a > 0, \Delta < 0$ 1

$a = 1 > 0$

$\Delta = (k+2)^2 - 4(3k+6)$

$= k^2 + 4k + 4 - 12k - 24$

$= k^2 - 8k - 20$

$= (k-10)(k+2)$ 1

we want $(k-10)(k+2) < 0$

i.e. $-2 < k < 10$ 1

(c) $9x^2 + 2x - 5 \equiv Ax(x+1) + B(x+1) + C$

sub. $x = -1 \Rightarrow C = 9 - 2 - 5 = 2$ 1

Sub $x = 0 \Rightarrow B + C = -5 \Rightarrow B = -7$ 1

equate coefficients of $x^2 \Rightarrow A = 9$ 1

OR $Ax(x+1) + B(x+1) + C = Ax^2 + Ax + Bx + B + C$

$= Ax^2 + (A+B)x + (B+C)$

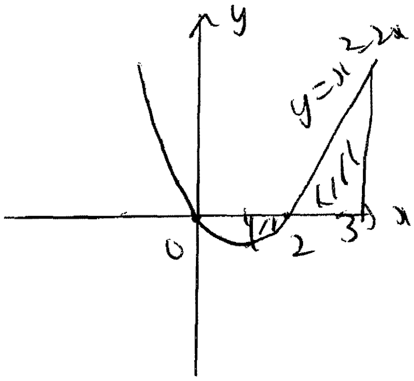
equate coefficients of $x^2 \Rightarrow A = 9$ 1

equate coefficients of $x \Rightarrow A + B = 2 \Rightarrow B = -7$ 1

equate constants $\Rightarrow B + C = -5 \Rightarrow C = 2$ 1

QUESTION 6

(a)




Required area $= \int_2^3 (x^2-2x)\,dx + \left|\int_1^2 (x^2-2x)\,dx\right|$ 1

$= \left[\frac{x^3}{3}-x^2\right]_2^3 + \left|\left[\frac{x^3}{3}-x^2\right]_1^2\right|$ 1

$= \left(9-9-\frac{8}{3}+4\right) + \left|\left(\frac{8}{3}-4-\frac{1}{3}+1\right)\right|$ 1

$= \frac{4}{3} + \left|-\frac{2}{3}\right|$ 1

$= \frac{4}{3}+\frac{2}{3} = 2$ units$^2$ 1

(b) (i) $x^2-2x = 2x+5$

$x^2-4x-5=0$

$(x-5)(x+1)=0$ 1

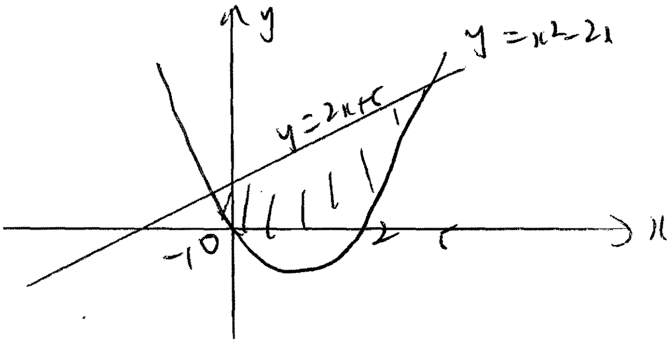
$x=5$ or $x=-1$

when $x=5$, $y=2\times5+5=15$ 1

when $x=-1$, $y=2\times-1+5=3$

i.e. points of intersection are $(5,15)$ and $(-1,3)$

(ii)




required area

$= \int_{-1}^{5} \left(2x+5-(x^2-2x)\right)dx$

$= \int_{-1}^{5} (4x+5-x^2)\,dx$ 1

$= \left[2x^2+5x-\frac{x^3}{3}\right]_{-1}^{5}$ 1

$= \left(50+25-\frac{125}{3}\right)-\left(2-5+\frac{1}{3}\right)$ 1

$= \frac{100}{3}+\frac{8}{3} = 36$ units$^2$

(c) $\int_1^3 3^{x-1}\,dx \doteqdot \frac{3-1}{12}\left(3^0+4\times3^{\frac12}+2\times3^1+4\times3^{1\frac12}+3^2\right)$ 1

$= \frac{1}{6}\left(1+4\sqrt3+6+12\sqrt3+9\right)$

$= \frac{~~16+16\sqrt3~~\ 8+8\sqrt3}{3}$

$\doteqdot$ ~~[illegible]~~ 7.285 1

wrong rule - no marks

QUESTION 7

(a) (i) $PA^2 = (x-1)^2 + (y+2)^2$
$PB^2 = (x-7)^2 + (y+8)^2$ 1

$PA = 2PB$

$\therefore PA^2 = 4PB^2$

i.e. $x^2 - 2x + 1 + y^2 + 4y + 4 = 4x^2 - 56x + 196 + 4y^2 + 64y + 256$ 1

$3x^2 - 54x + 3y^2 + 60y + 447 = 0$

$x^2 - 18x + y^2 + 20y + 149 = 0$ 1

$(x-9)^2 + (y+10)^2 = -149 + 81 + 100$
$= 32$

(ii) Locus is a circle centre $(9, -10)$ radius $4\sqrt{2}$ 2

−1 for each mistake

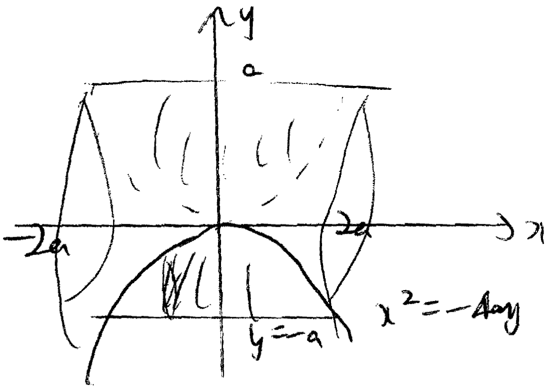
(b) (i) $V = \pi \int_0^{10} x^2\, dy$ $y = 2\sqrt{x} \rightarrow y^2 = 4x$
$x = \frac{y^2}{4}$
$x^2 = \frac{y^4}{16}$ 1

$= \pi \int_0^{10} \frac{y^4}{16}\, dy$

$= \left[\frac{\pi y^5}{80}\right]_0^{10}$ 1

$= \frac{\pi}{80} \times 10^5 = \frac{2500\pi}{2}$ units$^3$ 1

(c) (i)




1

(ii) required volume = volume of cylinder $- \pi \int_{-2a}^{2a} y^2\, dx$

$= \pi \times a^2 \times 4a - 2\pi \int_0^{2a} \frac{x^4}{16a^2}\, dx$

$= 4\pi a^3 - 2\pi \left[\frac{x^5}{80a^2}\right]_0^{2a}$

$= 4\pi a^3 - \frac{4\pi a^3}{5} = \frac{16\pi a^3}{5}$ units$^3$