NORTH SYDNEY BOYS HIGH SCHOOL

2012 HSC ASSESSMENT TASK 2

Mathematics

General Instructions

Section I contains 4 multiple choice questions to be answered on the separate answer sheet provided.

Section II is to be answered in the booklet provided, showing ALL necessary working.

- Write using blue or black pen
- Board approved calculators may be used
- Each new question is to be started on a **new page.**
- Attempt all questions

Reading Time – 5 minutes
Working Time – 55 minutes
Total Time – 60 minutes

Class Teacher: Please tick

- O Mr Berry
- O Ms Ziaziaris
- O Mr Lowe
- O Mr Weiss
- O Mr Lam
- O Mr Ireland
- O Mr Fletcher

Student Number:__________________________

(To be used by the exam markers only.)

Question No	1-4	5	6	7	8	9	Total	Total %
Mark	$\frac{}{4}$	$\frac{}{9}$	$\frac{}{11}$	$\frac{}{9}$	$\frac{}{4}$	$\frac{}{6}$	$\frac{}{43}$	$\frac{}{100}$

SECTION I

Question 1

The parabola whose equation is $x^2 = \frac{y}{4}$ has focal length equal to 1

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 4 (D) $\frac{1}{16}$

Question 2

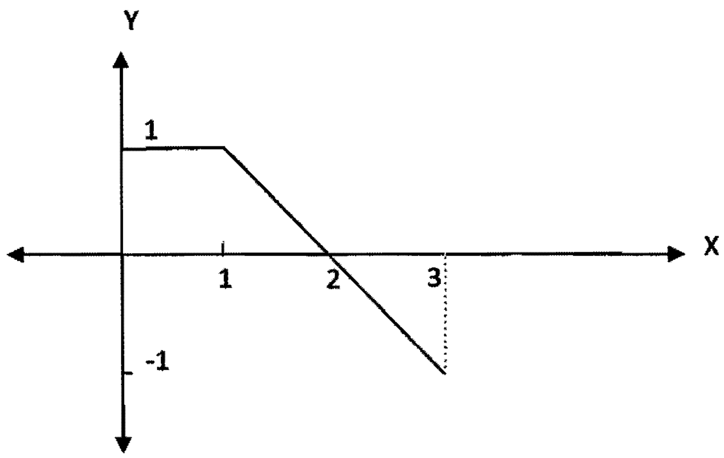
The parabola with equation $(y-3)^2 = 16(x+4)$ has its vertex at 1

(A) (3 ,-4) (B) (-3 , 4) (C) (4 , -3) (D) (-4 , 3)

Question 3

Given the graph of $y = f(x)$ below, the value of $\int_0^3 f(x)dx$ is 1

(A) 0 (B) 1 (C) 2 (D) 3



Question 4

$\int \frac{dx}{2} =$ 1

(A) $2x + C$ (B) $2^0 + C$ (C) $\frac{1}{2}x + C$ (D) $-\frac{1}{2}x + C$

SECTION II

Question 5

Find the indefinite integrals

a) $\int \left(6x - x^4\right)dx$ **1**

b) $\int (3x-5)^4\,dx$ **1**

c) $\int \left(2x^2-1\right)^2 dx$ **2**

Find the definite integrals

d) $\int_0^1 \frac{dx}{(8x+5)^2}$ **2**

e) $\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$ **3**

Question 6

a) Write down the equation of the parabola with focus $(0,-2)$ and directrix $y = 2$. **1**

b) Write down the equation of the parabola with focus $(3,0)$ and vertex $(-1,0)$. **1**

c) Find the equation of the parabola with vertex $(-2,3)$, axis parallel to the Y-axis and passing through the point $(-6,-5)$. **3**

d) For the equation $x = 4y - y^2$, find

i. the vertex **3**

ii. the focal length **1**

iii. the focus **1**

iv. the equation of the directrix **1**

Question 7

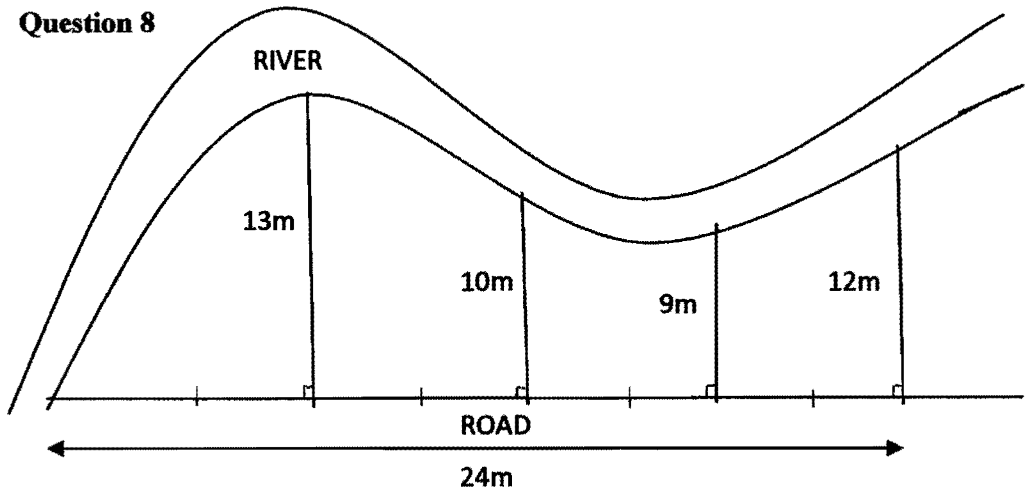
a) Calculate the area under the curve $y = x^2 + 1$ bounded by the X-axis and $x = -1$ and $x = 1$. **2**

b) If the gradient function is $\frac{dy}{dx} = \frac{3x^4 - 1}{x^2}$ and $y = 10$ when $x = 2$, find the primitive function. **3**

c) Find the area between $y = (x-2)(x-1)$ and $y = x - 1$. **4**

Question 8

a)



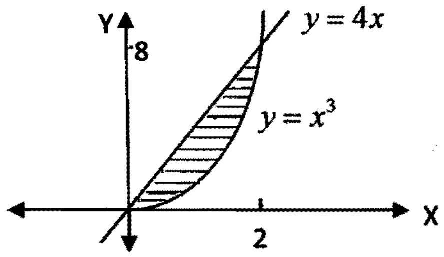
Wasteland bordering a river bank and a straight road was fenced off and used as a recreational park. Perpendicular distances from the road to the river bank are shown on the diagram.

i) Use the Simpson's Rule to approximate the area of the recreational park. **2**

ii) Use the Trapezoidal Rule to approximate the area of the recreational park. **2**

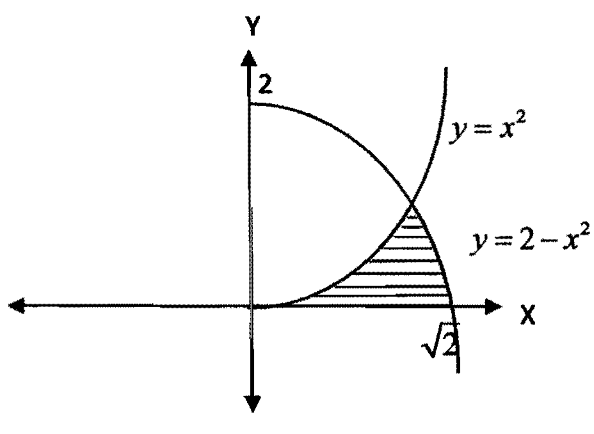
Question 9

a) The shaded region between the curves $y = x^3$ and $y = 4x$ in the first quadrant in the graph below is rotated about the Y-axis. Write down the integral that would calculate the volume of the solid formed. **DO NOT EVALUATE THE INTEGRAL.**



2

b) The shaded area in the diagram below enclosed by $y = x^2$, $y = 2 - x^2$ and the X-axis is rotated about the X-axis. Find the volume of the solid generated correct to 2 decimal places.



4

SOLUTIONS - YR 12 2UNIT ASSESS. TSK 2 2012

1. $4a = \frac{1}{4}$ D. 1

$a = \frac{1}{16}$

2. $(-4,3)$ D. 1

3. $1 + \frac{1}{2} - \frac{1}{2}$ B 1

$= 1$

4. C. 1

5. a) $\frac{6x^2}{2} - \frac{x^5}{5} + C$ 1

$3x^2 - \frac{x^5}{5} + C$

b) $\frac{(3x-5)^5}{15} + C$ 1

c) $\int (4x^4 - 4x^2 + 1)\,dx$ 1

$= \frac{4x^5}{5} - \frac{4x^3}{3} + x + C$ 1

d) $\int_0^1 (8x+5)^{-2}\,dx$

$= \left[\frac{(8x+5)^{-1}}{-8}\right]_0^1$ 1

$= \frac{1}{8}\left[\frac{-1}{8x+5}\right]_0^1$

$= \frac{1}{8}\left[-\frac{1}{13} + \frac{1}{5}\right]$

$= \frac{1}{8} \times \frac{8}{65}$

$= \frac{1}{65}$ 1

e) $\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$= \int_4^9 (x^{1/2} + x^{-1/2})\,dx$

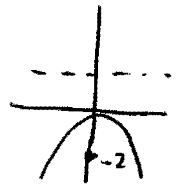
$= \left[\frac{2x^{3/2}}{3} + 2x^{1/2}\right]_4^9$

$= \left[\frac{2}{3} \times 9^{3/2} + 2 \times 9^{1/2} - \left(\frac{2}{3} \times 4^{3/2} + 2 \times 4^{1/2}\right)\right]$

$= 24 - 9\frac{1}{3}$

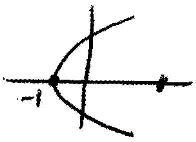
$= 14\frac{2}{3}$

6. a)



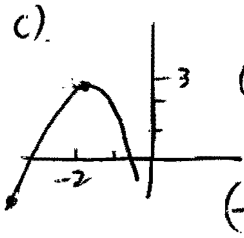
$x^2 = -8y$

b)



$y^2 = 16(x+1)$

c)



$(x+2)^2 = -4a(y-3)$

Sub $(-6,-5)$

$(-6+2)^2 = -4a(-8)$

$16 = 32a$

$a = \frac{1}{2}$

$\therefore$ Eqn: $(x+2)^2 = -2(y-3)$

d) (i) $y^2 - 4y = -x$

$y^2 - 4y + (-2)^2 = -x + 4$

$(y-2)^2 = -(x-4)$

Vertex $(4,2)$

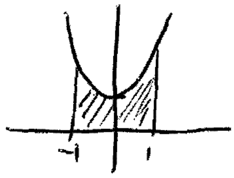
(ii) $4a = 1$

$a = \frac{1}{4}$

(iii) Focus $(3\frac{3}{4}, 2)$

(iv) $x = 4\frac{1}{4}$

7.a)



$$A = 2\int_0^1 (x^2+1)\,dx$$

$$= 2\left[\frac{x^3}{3}+x\right]_0^1$$

$$= 2\left[\frac{1}{3}+1\right]$$

$$= \frac{8}{3} \text{ sq.units}$$

b) $\frac{dy}{dx} = \frac{3x^4-1}{x^2}$

$$= 3x^2 - x^{-2}$$

$$y = \int 3x^2 - x^{-2}\,dx$$

$$y = x^3 - \frac{x^{-1}}{-1} + c$$

$$y = x^3 + \frac{1}{x} + c$$

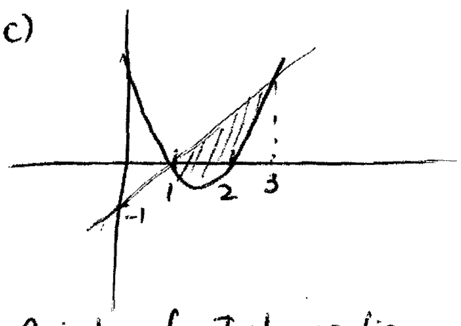
Sub $y=10,\ x=2$

$$10 = 8 + \frac{1}{2} + c$$

$$c = 1\tfrac{1}{2}$$

$$\therefore y = x^3 + \frac{1}{x} + \frac{3}{2}$$

c)



Points of Intersection

$$(x-2)(x-1) = x-1$$

$$(x-1)(x-2-1) = 0$$

$x=1,\ x=3$

$y=0\ \ y=2$

$$A = \int_1^3 (x-1) - (x-2)(x-1)\,dx$$

$$= \int_1^3 x-1-(x^2-3x+2)\,dx$$

$$= \int_1^3 -x^2+4x-3\,dx$$

$$= \left[\frac{-x^3}{3}+2x^2-3x\right]_1^3$$

$$= \left[-9+18-9-\left(\frac{-1}{3}+2-3\right)\right]$$

$$= \left[\frac{4}{3}\right] \text{ sq. units.}$$

8.a) (i)

$$A \doteqdot \frac{6}{3}\{0+4(13+9)+2(10)+12\} \quad (1)$$

$$= 240 \text{ sq. units.} \quad (1)$$

(ii) $A \doteqdot \frac{6}{2}\{0+2(13+10+9)+12\}$ (1)

$$= 228 \text{ sq. units.} \quad (1)$$

9.a) $V = \pi\int_0^8 y^{2/3}$ (1) $- \pi\int_0^8 \frac{y^2}{16}\,dy$ (1)

$$= \pi\left[\frac{3y^{5/3}}{5}\right]_0^8 - \pi\left[\frac{y^3}{48}\right]_0^8$$

b) Point of Intersection

$$x^2 = 2-x^2$$

$$2x^2 = 2 \quad (1)$$

$$x^2 = 1$$

$$x = \pm 1$$

$$V = \pi\int_0^1 x^4\,dx + \pi\int_1^{\sqrt{2}} (4-4x^2+x^4)\,dx \quad (1)$$

$$= \pi\left[\frac{x^5}{5}\right]_0^1 + \pi\left[4x - \frac{4x^3}{3} + \frac{x^5}{5}\right]_1^{\sqrt{2}} \quad (1)$$

$$= \pi\left(\frac{1}{5}\right) + \pi\left(4\sqrt{2} - \frac{8\sqrt{2}}{3} + \frac{4\sqrt{2}}{5} - \left(4 - \frac{4}{3} + \frac{1}{5}\right)\right.$$

$= 1.10u^3$ (1) (0.35π)