100 Years
NORTH SYDNEY BOYS HIGH SCHOOL
1912 - 2012

MATHEMATICS

2014 HSC Course Assessment Task 2

Monday March 10, 2014

General instructions

- Working time – 55 minutes. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer sheet provided (numbered as page 5)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ..... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12M2A – Mr Lin	◯ 12M3A – Mr Zuber	◯ 12M4A – Ms Ziaziaris
	◯ 12M3B – Mr Berry	◯ 12M4B – Mr Lam
◯ 12M2B – Mr Weiss	◯ 12M3C – Mr Lowe	◯ 12M4C – Mr Ireland

Marker's use only.

QUESTION	1-5	6	7	8	9	10	Total	%
MARKS	/5	/9	/8	/9	/10	/8	/44	

Section I: Objective response

Mark your answers on the multiple choice sheet provided. **Marks**

1. What is the value of $e^{-0.2}$ correct to 3 decimal places? **1**

(A) 0.818 (B) 0.819 (C) 1.221 (D) 1.222

2. Which expression is equivalent to $\int \sqrt{5x+1}\,dx$? **1**

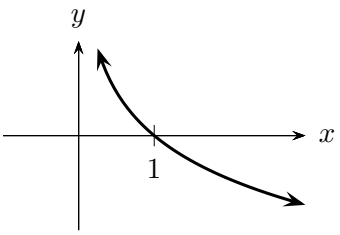
(A) $\frac{1}{10}(5x+1)^{\frac{3}{2}}+C$ (C) $\frac{3}{10}(5x+1)^{\frac{3}{2}}+C$

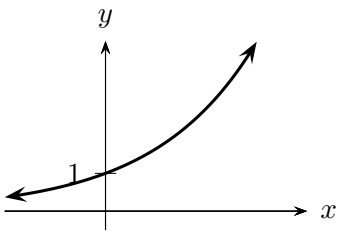
(B) $\frac{2}{15}(5x+1)^{\frac{3}{2}}+C$ (D) $\frac{2}{3}(5x+1)^{\frac{3}{2}}+C$

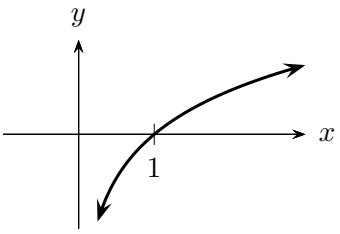
3. If $f(x)=e^{-x}-2e^{-2x}$, what is the value of $f(\ln 2)$? **1**

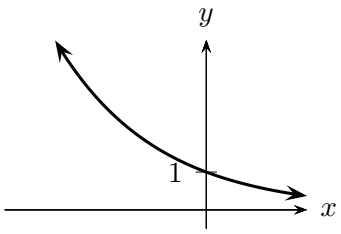
(A) 0 (B) 4 (C) 6 (D) 10

4. Which of the following best represents the shape of the graph $y=\log_e x$? **1**

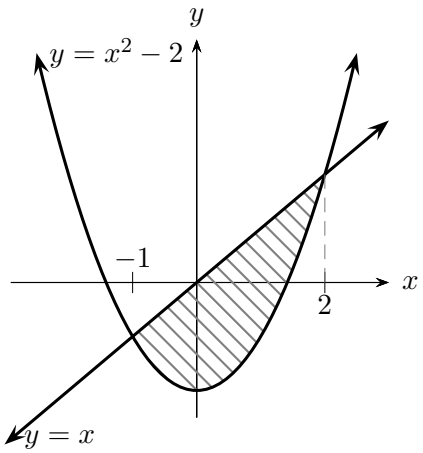
(A)


(C)


(B)


(D)


5. Which of the following integrals is representative of the shaded area? **1**



(A) $\int_{-1}^{2}\left(x-x^2+2\right)\,dx$

(B) $\int_{-1}^{2}\left(x^2+x-2\right)\,dx$

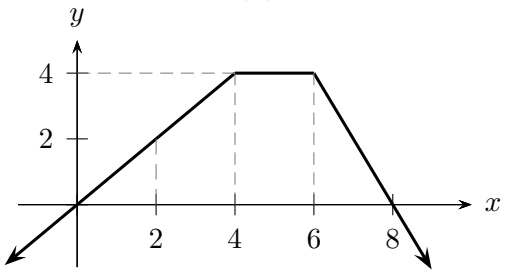
(C) $\int_{-1}^{2}\left(x^2-x-2\right)\,dx$

(D) $\int_{-1}^{2}\left(x-x^2-2\right)\,dx$

End of Section I.
Examination continues overleaf.

Question 6 (9 Marks) Commence a NEW page. **Marks**

(a) The function $y = f(x)$ is shown in the following diagram.



Evaluate:

i. $\int_0^8 f(x)\,dx$. **2**

ii. $\int_{-2}^8 f(x)\,dx$. **1**

(b) Evaluate $\int_1^4 (3x-2)^4\,dx$. **3**

(c) Find the area enclosed by the curves $y = x^2 - x$ and $y = 5x - x^2$. **3**

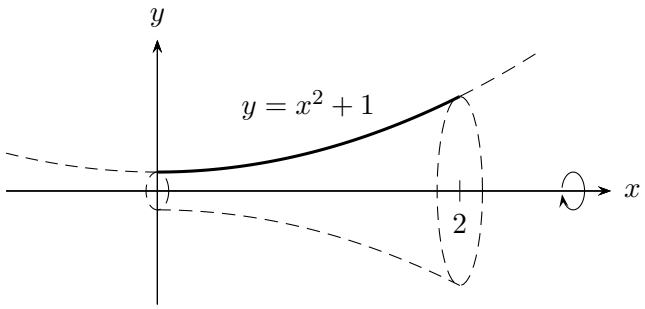
Question 7 (8 Marks) Commence a NEW page. **Marks**

(a) Find the primitives of the following:

i. $\dfrac{x^4 - 6}{x^3}$. **2**

ii. $\sqrt{x}$. **2**

(b) Find the volume of the solid of revolution generated when the curve $y = x^2 + 1$ is rotated about the x axis between $x = 0$ and $x = 2$. **4**



Question 8 (9 Marks) Commence a NEW page. **Marks**

(a) Given the parabola $x^2 = -6y$, find:

i. the coordinates of the vertex. **1**

ii. the coordinates of the focus. **1**

iii. the equation of the directrix. **1**

(b) Find the coordinates of the vertex and focus of the parabola **3**

$$x^2 - 6x - 3y - 12 = 0$$

(c) A parabola has its focus at $(1, -4)$ and the directrix is the line $y = 6$. **3**

Find the equation of the parabola.

Question 9 (10 Marks) Commence a NEW page. **Marks**

(a) Evaluate the following, giving your answer in simplest form.

i. $\dfrac{d}{dx}\left(\dfrac{x}{e^x}\right)$. **2**

ii. $\dfrac{d}{dx}\left(x^2e^{-(x+2)}\right)$. **2**

iii. $\dfrac{d}{dx}\left(\log_e\left(\dfrac{3x+4}{2x-1}\right)\right)$. **2**

(b) For the function $y = \log_e(2x+1)$,

i. Find the domain of the function. **1**

ii. Find the range of the function. **1**

iii. Sketch the function, showing all important information. **2**

Question 10 (8 Marks) Commence a NEW page. **Marks**

(a) Evaluate:

i. $\displaystyle\int_0^2 3e^{1-5x}\,dx$. **2**

ii. $\displaystyle\int xe^{x^2}\,dx$. **2**

(b) Given $y = \log_e x$,

x	1	2	3	4	5
$\log_e x$					

i. Copy and complete the table above, giving the values correct to 3 decimal places. **1**

ii. Use Simpson's Rule with five function values to estimate **3**

$$\int_1^5 \log_e x\,dx$$

End of paper.

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

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1 – Ⓐ Ⓑ Ⓒ Ⓓ
2 – Ⓐ Ⓑ Ⓒ Ⓓ
3 – Ⓐ Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ Ⓓ
5 – Ⓐ Ⓑ Ⓒ Ⓓ

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right) + C$$

NOTE: $\ln x = \log_e x$, $x > 0$

Suggested Solutions

Section I

1. (D) **2.** (B) **3.** (D) **4.** (C) **5.** (A)

Section II

Question 5 (Berry)

(a)

Marking

Q6 Lin

Q7 Lin

Q8 Weiss

Q9 Zuber

Q10 ziaziaris

Q1 B

2 B

3 A

4 B

5 A

Q6) a) i) $2 + 6 + 8 + 4$

$= 20$ 2

ii) $20 - 2$

$= 18$ 1

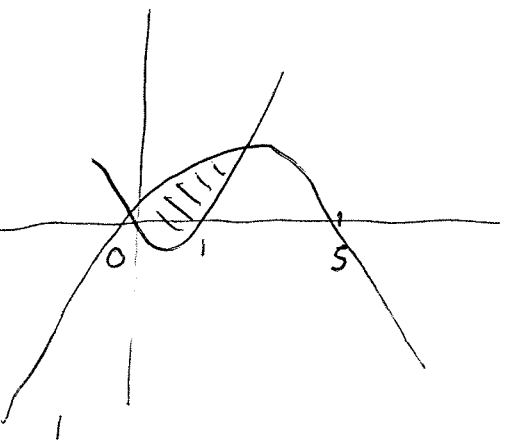
b) $\int_1^4 (3x-2)^4 \, dx$

$= \left[\frac{(3x-2)^5}{15}\right]_1^4$ 1

$= \frac{10^5}{15} - \frac{1}{15}$ 1

$= \frac{10^5 - 1}{15}$ 1

c)



$5x - x^2 = x^2 - x$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$\therefore x = 0$ or 3 1

$\int_0^3 5x - x^2 - (x^2 - x)$ 1

$\int_0^3 6x - 2x^2 \, dx$

$= 3x^2 - \frac{2x^3}{3}\Big]_0^3$

$= 27 - 18 = 9 u^2$ 1

Q7) a) i) $\int x - 6x^{-3}\,dx$ (1)

$= \frac{x^2}{2} + 3x^{-2} + C$ (1)

ii) $\int x^{1/2}\,dx$ (1)

$= \frac{2x^{3/2}}{3} + C$ (1)

b) $V = \pi \int_0^2 y^2\,dx$ (1)

$= \pi \int_0^2 x^4 + 2x^2 + 1\,dx$ (1)

$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^2$ (1)

$= \left(\frac{32}{5} + \frac{16}{3} + 2 \right) \pi\, u^3$

$= 13\frac{11}{15}\,\pi\, u^3$ (1)

Q8)

a) i) $x^2 = 4ay$

$x^2 = -6y$

$a = -\frac{3}{2}$

ii) $V(0,0)$ 1

iii) $S\left(0, -\frac{3}{2}\right)$ 1

iv) $y = \frac{3}{2}$ 1

b) $x^2 - 6x - 3y - 12 = 0$

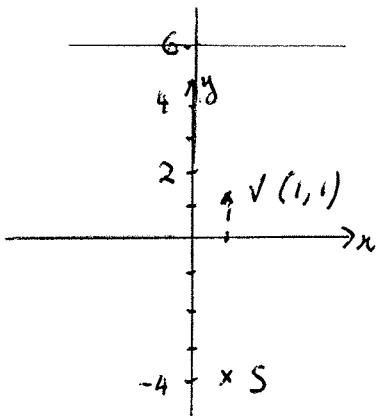
$x^2 - 6x + 9 = 3y + 12 + 9$

$(x-3)^2 = 3(y+7)$ 1

$\therefore$ Vertex $(3, -7)$ 1

Focus $(3, -6\frac{1}{4})$ 1

c)



$(x-1)^2 = -20(y-1)$

1 1

Q9) a) i/ $\frac{d}{dx} \frac{x}{e^x}$

$$= \frac{e^x \cdot 1 - xe^x}{e^{2x}}$$ 1

$$= \frac{e^x(1-x)}{e^{2x}}$$ 1

$$= \frac{1-x}{e^x}$$

ii/ $\frac{d}{dx} x^2 e^{-(x+2)}$

$$= 2x e^{-(x+2)} + -x^2 e^{-(x+2)}$$ 1

$$= x e^{-(x+2)} (2-x)$$ 1

iii/ $\frac{d}{dx} \left(\ln(3x+4) - \ln(2x-1) \right)$ 1

$$= \frac{3}{3x+4} - \frac{2}{2x-1}$$ 1

$$= \frac{6x-3-6x-8}{(3x+4)(2x-1)}$$

$$= \frac{-11}{(3x+4)(2x-1)}$$

Q10)

a) i/ $\int_0^2 3e^{1-5x}\,dx$

$= \frac{-3}{5} e^{1-5x} \Big]_0^2$ 1

$= \frac{-3}{5e^9} + \frac{3}{5e^1}$ 1

ii/ $\int x e^{x^2}\,dx$

$= \frac{1}{2}\int 2x e^{x^2}\,dx$ 1

$= \frac{1}{2} e^{x^2} + C$ 1

b) i/

x	1	2	3	4	5
$\ln x$	0	.693	1.099	1.386	1.609

1

ii/ $\int_1^5 \log_e x\,dx = \frac{1}{3}\left[0 + 1.609 + 4(.693 + 1.386) + 2(1.099)\right]$ 1 1

$= 4.041$ 1