100 Years
NORTH SYDNEY BOYS HIGH SCHOOL
1912 – 2012

MATHEMATICS

2015 HSC Course Assessment Task 2

March 2, 2015

General Instructions

- Working time –55 minutes (plus 5 minutes reading time).
- Write using blue or black pen. Diagrams may be sketched in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt all questions.

Section I - 4 marks

- Mark your answers on the answer sheet provided.

Section II – 42 marks

- Commence each new question on a new page.
- Show all necessary working in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: ……………………………………….. **# BOOKLETS USED:** ……….

Class (please ✓)

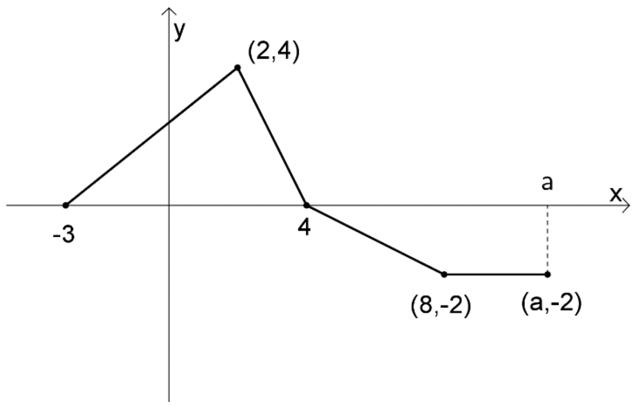
☐ Ms Lee ☐ Mr Berry ☐ Mr Ireland

☐ Ms Ziaziaris ☐ Mr Lam

☐ Mr Zuber ☐ Mr Lin

Question	1-4	6	7	8	9	**Total**
Marks	$\overline{4}$	$\overline{10}$	$\overline{12}$	$\overline{10}$	$\overline{10}$	$\overline{46}$

Q1. Using the graph of $y = f(x)$ below,



determine the value of a which satisfies the condition:

$$\int_{-3}^{a} f(x)dx = 0$$

(A) 5

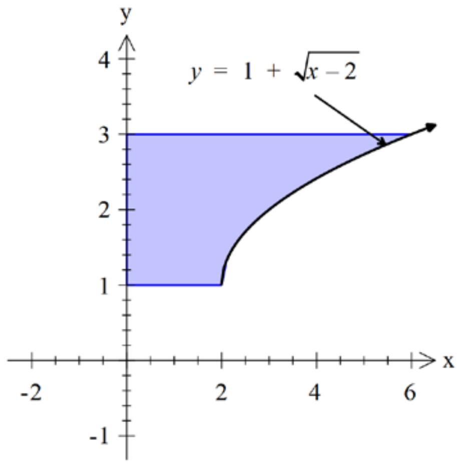
(B) 9.5

(C) 13

(D) 18

Question 2 on the next page

Q2. A section of the curve $y = 1 + \sqrt{x-2}$, ranging from the values $(2,1)$ to $(6,3)$ is rotated about the y-axis:



The volume of the resulting solid of revolution can be calculated using the expression:

(A) $\pi \int_{2}^{6} ((y-1)^2 + 2)^2 \, dy$

(B) $\pi \int_{2}^{6} ((y-1)^2 + 2) \, dy$

(C) $\pi \int_{1}^{3} ((y-1)^2 + 2) \, dy$

(D) $\pi \int_{1}^{3} ((y-1)^2 + 2)^2 \, dy$

Q3. A sequence is defined:

$$T_1 = 1, \quad T_{n+1} = (T_n)^2 - 4$$

What is the sum of the first four terms?

(A) 12

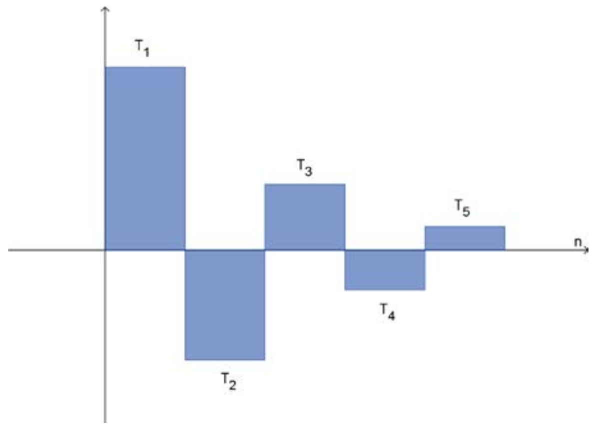
(B) 14

(C) 21

(D) 24

Question 4 on the next page

Q4. The values for a geometric sequence $T_n = a\, r^{n-1}$ are plotted on a bar graph:



Which statement must be true?

(A) $a > 0,\ 0 < r < 1.$

(B) $a > 0, -1 < r < 0.$

(C) $a < 0, 0 < r < 1.$

(D) $a < 0,\ -1 < r < 0.$

End of Section I

Section II – 42 marks

Question 6 (10 marks) Commence on a NEW page Marks

(a) Evaluate $\sum_{k=4}^{7}(-1)^k\, k^2$ 2

(b) Consider the arithmetic sequence $213, 207, 201, 195, \ldots$,.

i) What is the value of the 16th term? 2

ii) What is the sum of the first 16 terms? 2

(c) In a geometric series, the third term is 18 and the sixth term is -486.

i) Find the common ratio. 2

ii) Hence find the sum of the first 12 terms in the series. 2

Question 7 (12 marks) Commence on a NEW page. Marks

(a) Find the indefinite integrals:

i) $\int (5x - x^4)\, dx$ 1

ii) $\int (2x-1)^5\, dx$ 1

iii) $\int \frac{3+t}{t^3}\, dt$ 2

(b) If $f'(x) = 6x^2 - 2x + 3$ and the curve $y = f(x)$ passes through $(1,2)$, find $f(x)$. 3

(c) Evaluate the following:

i) $\int_1^8 \frac{dx}{\sqrt{x}}$ 2

ii) $\int_{-2}^{2} (5x^2 + x)^2\, dx$ 3

Question 8 (10 marks) Commence on a NEW page. Marks

(a) By considering the recurring decimal $0.5\overline{32}$ as the <u>sum of an infinite geometric series</u>, express $0.5\overline{32}$ as a fraction of two integers. 3

Note: You must use a geometric sequence to gain these marks.

(b) The sum of a series S_n is given as:

$$S_n = 5n^2 - 9n$$ 3

Find an expression for the nth term T_n

(c) A miner is digging a horizontal tunnel.

The first 100 metres of the tunnel has already been dug, so the miner starts at the 100 metre mark.

The miner is equipped with a pickaxe which can excavate 40 metres of the tunnel before it breaks. When the pickaxe breaks, the miner returns to the start of the tunnel to collect a new pickaxe, then returns to the end of the tunnel and continues excavating.

i) Write an algebraic expression for the length of the tunnel the moment the nth pickaxe breaks. 1

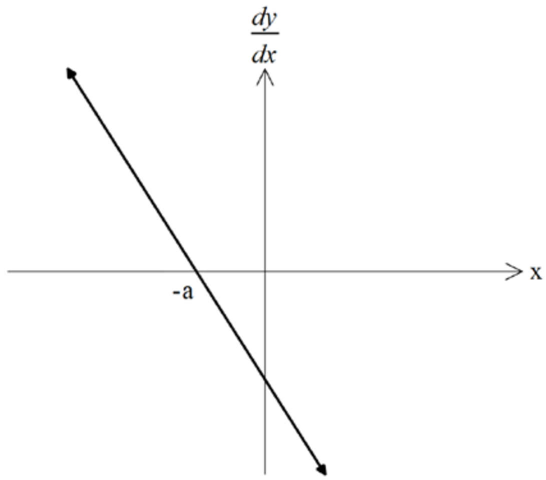
ii) What is the total distance the miner has travelled when he has returned to the start of the tunnel after using the nth pickaxe? 1

iii) How many pickaxes are need if the miner is contracted to travel a maximum distance of 3 km during his shift? 2

Examination continues on the next page.

Question 9 (10 marks) Commence on a NEW page. Marks

(a) Copy the following curve into your exam booklet: 2



<u>On the same set of axes</u>, draw a possible primitive function for the curve.

(b) A recent study in the journal *Science* estimates that approximately 8.75 million metric tons (MT) of plastic waste entered the world's oceans during the Year 2010.

Using this research, the following data is suggested as an estimate for the amount of plastic waste entering the ocean in specific years:

Year	1935	1950	1965	1980	1995	2010
Plastic waste entering the ocean *(Million MT per year)*	0.10	0.24	0.60	1.46	3.58	8.75

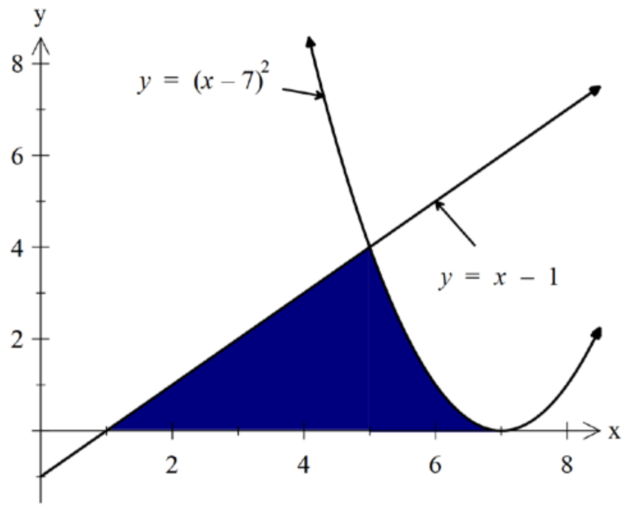
i) Use the trapezoidal rule with all six values to approximate the area under a curve based on this data. 2

ii) Explain why the area under this curve gives an approximation for the total amount of plastic waste in the ocean at the end of 2010. 2

Write a maximum of three sentences. You may annotate a diagram if you wish.

Question 9 continues on the next page.

(c) The graphs $y = x - 1$ and $y = (x-7)^2$ are shown in the diagram below.



i) Show the curves intersect at (5, 4). 1

ii) Find the area of the region defined by the intersection of the two curves and the x-axis. 3

End of the Examination

2Unit HSC Task 2 2015 Solutions

Q1. Area $= (7)(4)(\frac{1}{2}) - (4)(2)(\frac{1}{2}) - (a-8)2 = 0$

$10 + 16 - 2a = 0$

$a = 13$

C

| C | A | D | B |

Q2: $y = 1 + \sqrt{x-2}$

$\sqrt{x-2} = y - 1$

$x = (y-1)^2 + 2$

$V = \pi \int_1^3 x^2\,dy = \pi \int_1^3 \left((y-1)^2+2\right)^2 dy$ A

Q3: $T_1 = 1$

$T_2 = 1^2 - 4 = -3$

$T_3 = (-3)^2 - 4 = 5$

$T_4 = (5)^2 - 4 = 21$

$T_1 + T_2 + T_3 + T_4 = 24$ D

Q4. $a > 0$, $-1 < r < 0$ B

Q6.

(a) $\sum_{k=4}^{7} (-1)^k k^2 = 4^2 - 5^2 + 6^2 - 7^2 = \boxed{-22}$

✓ Correct use of $k=4$ to $k=7$

✓ answer

(b) 213, 207, 201, 195, ...

$a = 213$

$d = -6$

(i) $T_n = 213 + (-6)(n-1)$ ✓

$T_{16} = 213 - 6(15)$ ✓

$\boxed{= 123}$

(ii) $S_{16} = \frac{(T_1 + T_{16})}{2} \times 16$ ✓

$= \frac{(213 + 123)}{2} \times 16$

$\boxed{= 2{,}688}$ ✓

(c) (i)

$T_3 = ar^2 = 18$ ✓

$T_6 = ar^5 = -486$

$\therefore \frac{ar^5}{ar^2} = \frac{-486}{18}$

$r^3 = -27$

$\therefore \boxed{r = -3}$ ✓

$a = 2$

(ii) $S_{12} = \frac{a(r^{12}-1)}{r-1}$ ✓

$= \frac{2((-3)^{12}-1)}{-4}$

$= -265{,}720$ ✓

Q7

(a) (i) $\int (5x - x^4)\,dx = \frac{5}{2}x^2 - \frac{1}{5}x^5 + C$ ✓

(ii) $\int (2x-1)^5\,dx = \frac{1}{12}(2x-1)^6 + C$ ✓

(iii) $\int \frac{3+t}{t^3}\,dt = \int \left(\frac{3}{t^3} + \frac{1}{t^2}\right)dt$

$= -\frac{3}{2t^2} - \frac{1}{t} + C$

or $-\frac{3}{2}t^{-2} - t^{-1} + C$

or $-\left(\frac{2t+3}{2t^2}\right) + C$

Any form acceptable

✓2 correct answer

✓ progress.

must have +C for 2 marks

(b) $f'(x) = 6x^2 - 2x + 3$

$f(x) = \int (6x^2 - 2x + 3)\,dx$

$= 2x^3 - x^2 + 3x + C$ ✓

But $f(1) = 2$

$\therefore\ 2 = 2(1) - (1) + 3(1) + C$ ✓

$C = -2$

$\boxed{\therefore\ f(x) = 2x^3 - x^2 + 3x - 2}$ ✓

Must write final solution $f(x) = \ldots$

Q7 (c)

i) $\int_1^8 \frac{dx}{\sqrt{x}} = \left[+2\sqrt{x}\right]_1^8$

$= 2(\sqrt{8} - 1)$

(ii) $\int_{-2}^{2} (5x^2 + x)^2\,dx = \int_{-2}^{2} (25x^4 + 10x^3 + x^2)\,dx$ ✓

$= \underbrace{\int_{-2}^{2} (25x^4 + x^2)\,dx}_{\text{even func}} + \underbrace{\int_{-2}^{2} 10x^3\,dx}_{\text{odd func}}$

$= 2\int_0^2 (25x^4 + x^2)\,dx + 0$

$= 2\left[\frac{25}{5}x^5 + \frac{1}{3}x^3\right]_0^2$ ✓

$= 2\left(5(2)^5 + \frac{8}{3}\right)$

$= \frac{976}{3}$ ✓

Q6

(a) $0.5\overline{32} = 0.5 + 0.032 + 0.00032 + \dots$

$$= \frac{1}{2} + \frac{32}{1000}\underbrace{\left(1 + \frac{1}{(100)} + \frac{1}{(100)^2} + \dots\right)}_{r=\frac{1}{100} \quad |r|<1 \ \therefore \text{ limiting sum.}} \quad \checkmark 1$$

$$= \frac{1}{2} + \frac{32}{1000}\left(\frac{1}{1-\frac{1}{100}}\right) \quad \checkmark 1$$

$$= \frac{1}{2} + \frac{32}{1000} \times \frac{100}{99}$$

$$= \frac{495}{990} + \frac{32}{990}$$

$$= \frac{527}{990} \quad \checkmark 1$$

Must use G.P. expansion and limiting sum formula.
[1 mark if use other technique & correct answer]

(b) $S_n = 5n^2 - 9n$

$T_n = S_n - S_{n-1}$ $\checkmark 1$

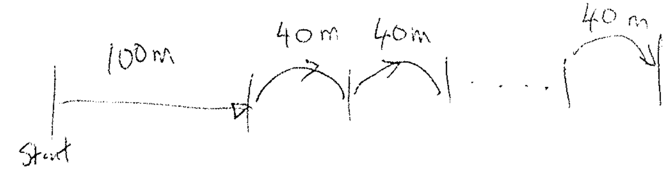
$= (5n^2 - 9n) - (5(n-1)^2 - 9(n-1))$ $\checkmark 1$

$= (5n^2 - 9n) - (5n^2 - 10n + 5 - 9n + 9)$

$= 5n^2 - 9n - 5n^2 + 10n - 5 + 9n - 9$

$\boxed{T_n = 10n - 14}$ $\checkmark 1$

Q8 (c)



(i) $T_n = 100 + 40n$ $\checkmark 1$

(ii) Total distance travelled $= 2 \times S_n$ (return trip)

$$= 2 \times \left(\frac{(140) + (100 + 40n)}{2}\right) n$$

$$= 240n + 40n^2 \text{ metres} \quad \checkmark 1$$

(iii) $240n + 40n^2 \leq 3000$ $\checkmark 1$

$40n^2 + 240n - 3000 \leq 0$

$2n^2 + 12n - 150 \leq 0$

Roots are $\dfrac{-12 \pm \sqrt{(12)^2 + 4(150)(2)}}{2(2)} \approx +6.16, -12.16$

+ve solution!

$\therefore n \leq 6.16$

So need $\boxed{6 \text{ pickaxes}}$ $\checkmark 1$

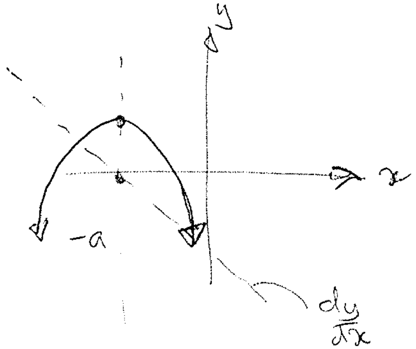
ECF from (ii): Distance $= 120n + 20n^2$

$\Rightarrow$ 9 pickaxes.

Guess & check OK so long as some working out shown.

Q9.

(a)



✓ for concave down parabola

✓ vertex at $x = -a$

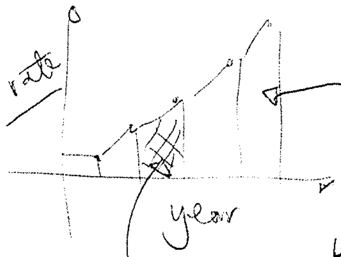
(b)(i) $A \approx \frac{15}{2}(0.10 + 2(0.24+0.60+1.46+3.58) + 8.75)$ ✓

$\approx \frac{15(20.61)}{2}$

≈ 154.58 million metric tonnes (2DP) ✓

(accept u^2)

(ii)



Area = rate × time
= total amount

* Area under curve integrates the rate over time, giving total accumulation.
* ~~Rat~~ Each trapezoid / rectangle ~~is~~ is approx linear rate × time = total amount.
* Plastic doesn't decay (fast) so it's accumulating over time.

Higher rate / longer time period → more plastic.

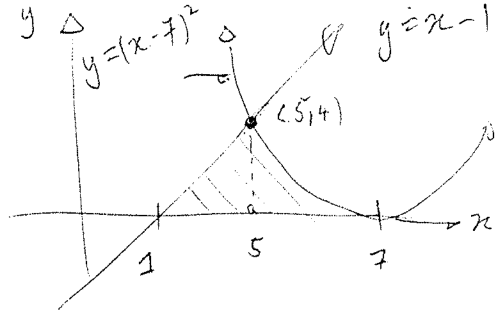
Marking.

✓2 Clear statement integrating accumulates rates

or

✓1 some evidence understanding [illegible] concept

(c)



(i) $y = (x-7)^2 \Rightarrow (5-7)^2 = (-2)^2 = 4$ ∴ (5,4) satisfies

$y = x - 1$ $\quad 5-4=1$ ∴ (5,4) satisfies.

(5,4) on both curves. ✓

(ii) Area is $\int_1^5 (x-1)\,dx + \int_5^7 (x-7)^2\,dx$ ✓

or: $\frac{1}{2}bh$
$= \frac{1}{2} \times 4 \times 4$
$= 8$

$= \left[\frac{1}{2}x^2 - x\right]_1^5 + \left[\frac{1}{3}(x-7)^3\right]_5^7$ ✓

$= \left(\frac{25}{2} - \frac{1}{2} - 5 + 1\right) + \left(\frac{1}{3}(0) + \frac{8}{3}\right)$

$= 8 + \frac{8}{3}$

$= \frac{32}{3}u^2$ ✓

Must have u