NORTH SYDNEY GIRLS HIGH SCHOOL

HSC MATHEMATICS ASSESSMENT TASK

TERM 1 – 2006

Time Allowed: 1 hour + 2 minutes reading time

Instructions:

- Start each question on a new page
- Write on one side of the paper only, work down the page and do not work in columns
- Leave a margin on the left hand side of the page
- Show all necessary working
- Marks may not be awarded for untidy or poorly arranged work
- Diagrams are not drawn to scale
- There are five questions
- Marks are as indicated

This task is worth 20% of the HSC Assessment Mark

Name:_____

Question 1 (10 marks) **Marks**

(a) Find the third term of the sequence whose n^{th} term is given by $T_n = 3 \times 2^{n-2}$ **1**

(b) Consider the following series $101 + 96 + 91 + \ldots..$

(i) Explain why this series is arithmetic. **1**
(ii) State the next term. **1**
(iii) Which term is equal to 26? **2**
(iv) Find the sum of the first 10 terms. **2**

(c) Evaluate $\sum_{r=1}^{4} \frac{1}{r}$ **1**

(d) A geometric series has T_5 equal to $\frac{81}{8}$ and T_2 equal to 3.

Find an expression for T_n. **2**

Question 2 (11 marks) **Start a new page.** **Marks**

(a) A given geometric series has a limiting sum of 36 and its first term is 27.
Find the common ratio. **3**

(b) A retired woman decides to live off her savings. She has \$70 000 and invests it at an interest rate of 6% per annum, compounded monthly. At the end of each month after interest has been received, she withdraws \$$D$. Let the amount of money left at the end of the n^{th} month just after she has made her withdrawal be \$$A_n$

(i) Find an expression for A_1 and use it to show that
$A_2 = 70\,000(1.005)^2 - D(1.005 + 1)$ **2**

(ii) Write down an expression for A_n, the amount of money left after n months. **2**

(iii) Show that $D = \dfrac{70\,000(1{\cdot}005)^n - A_n}{\left[\dfrac{(1{\cdot}005)^n - 1}{0{\cdot}005}\right]}$ **2**

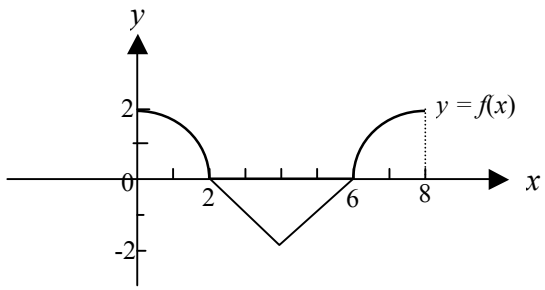
(iv) Find the monthly withdrawal, D, if the woman has no money left after 10 years. **2**

Question 3 (9 marks) **Start a new page.** **Marks**

a) Find the indefinite integral of $(5x+1)^2$ **3**

b) Evaluate $\int_{25}^{36} \frac{1}{\sqrt{x}}\,dx$ **3**

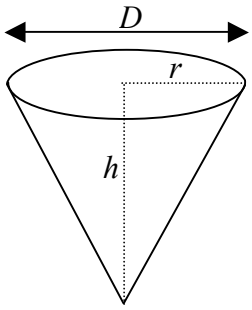
c) The graph of $y = f(x)$ is shown below. It consists of two circular arcs and intervals.



Evaluate $\int_0^8 f(x)\,dx$ **3**

Question 4 (8 marks) **Start a new page.**

The diagram below represents a conical water container.



In this cone, the sum of the base diameter, D, and the height, h, is 60 metres.

a) Write an expression for the height, h, in terms of the radius, r. **2**

b) Show that the volume of the container is given by

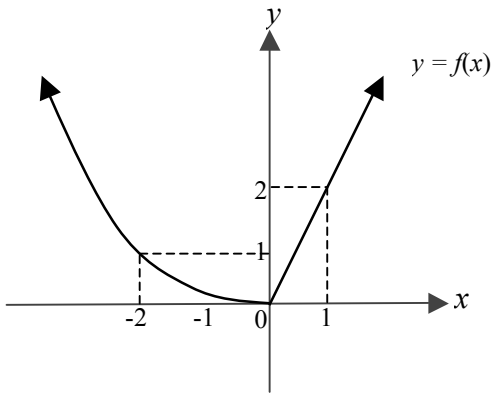
$$V = 20!\,r^2 - \frac{2}{3}\pi r^3$$

[You may use the formula $V = \frac{1}{3}\pi r^2 h$, the volume of a cone] **2**

c) Find the radius that makes the volume of the container a maximum. **4**

Question 5 (11 marks) **Start a new page.** **Marks**

a) The graph of $y = f(x)$ is drawn below



On your own paper, sketch the graph of $y = f'(x)$. **3**

b) Consider the curve $f(x) = 7 + 4x^3 - 3x^4$

i) Given that $y = f(x)$ has a stationary point of inflexion at (0, 7), find any other stationary point(s) and determine their(its) nature. **3**

ii) The graph of $y = f(x)$ passes through the point $\left(\frac{2}{3}, 7\frac{16}{27}\right)$. Show that this point is a point of inflexion. **2**

iii) Sketch the graph of $y = f(x)$ showing stationary points, points of inflexion and the intercept on the y-axis. **3**

End of paper

Solutions

QUESTION 1

1a) $T_3 = 6$

b) (i) $T_3 - T_2 = -5$
$T_2 - T_1 = -5$
So the common difference 'd' is equal

(ii) 86

(iii) $T_n = a + (n-1)d$
$T_n = 101 + (n-1) \times -5$
$= 106 - 5n$
Let $106 - 5n = 26$
$5n = 80$
$n = 16$
T_{16}

(iv) $S_{10} = \frac{10}{2}\{202 + 9 \times -15\}$
$= 5\{202 - 45\}$
$= 785$

c) $2\frac{1}{12}$

d) $ar^4 = \frac{81}{8}$ ①
$ar = 3$ ②
① ÷ ②
$r^3 = \frac{27}{8}$
$r = \frac{3}{2}$
FROM ②,
$a \times \frac{3}{2} = 3$
$a = 2$
$T_n = 2\left(\frac{3}{2}\right)^{n-1}$

OR $T_n = 2^{2-n} \cdot 3^{n-1}$

QUESTION 2

a) $36 = \frac{27}{1-r}$
$36(1-r) = 27$
$1 - r = \frac{3}{4}$
$r = \frac{1}{4}$

b)

(i) $A_1 = 70000(1.005) - D$
$A_2 = \{70000(1.005) - D\}1.005 - D$
$= 70000(1.005)^2 - 1.005D - D$
$= 70000(1.005)^2 - D(1.005 + 1)$

(ii)
$A_n = 70000(1.005)^n - D\left(1 + 1.005 + \ldots (1.005)^{n-1}\right)$

(iii)
$A_n = 70000(1.005)^n - D\left[\frac{(1.005)^n - 1}{0.005}\right]$

$D\left[\frac{(1.005)^n - 1}{0.005}\right] = 70000(1.005)^n - A_n$

$D = \frac{70000(1.005)^n - A_n}{\left[\frac{(1.005)^n - 1}{0.005}\right]}$

(iv) Let $A_n = 0$ and $n = 120$
$D = 70000(1.005)^{120} \times \frac{0.005}{(1.005)^{120} - 1}$
$= \$777.14\ (3)$

QUESTION 3

a) $\int (5x+1)^2\,dx = \frac{(5x+1)^3}{15} + C$ or $\int (25x^2+10x+1)\,dx = \frac{25x^3}{3} + 5x^2 + x + C$

b) $\int_{25}^{36} \frac{1}{\sqrt{x}}\,dx = \int_{25}^{36} x^{-\frac{1}{2}}\,dx$

$= 2\left[x^{\frac{1}{2}}\right]_{25}^{36}$

$= 2\left[\sqrt{x}\right]_{25}^{36}$

$= 2\left[\sqrt{36} - \sqrt{25}\right]$

$= 2[6-5]$

$= 2$

c) $\int_0^8 f(x)\,dx =$ AREA OF 2 QUADRANTS + 'NEGATIVE' AREA OF TRIANGLE

$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times 4 \times 2$

$= 2\pi - 4$

QUESTION 4

$2r + h = 60$

a) $h = 60 - 2r$

b) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (60 - 2r)$

$= 20\pi r^2 - \frac{2}{3}\pi r^3$

c) $\frac{dV}{dr} = 40\pi r - 2\pi r^2$

Put $\frac{dV}{dr} = 0$

$40\pi r - 2\pi r^2 = 0$

$2\pi r(20 - r) = 0$

$r = 20$

Now

$\frac{d^2V}{dr^2} = 40\pi - 4\pi r$

at $r = 20$

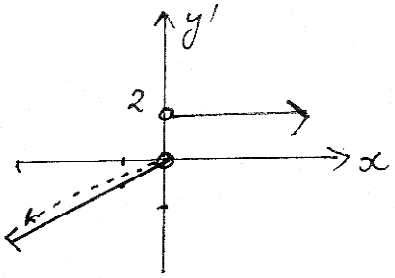
$\frac{d^2V}{dr^2} = 40\pi - 80\pi$

$= -40\pi$

$< 0 \Rightarrow$ MAX VALUE

QUESTION 5

a)



b (i) $f(x) = 7 + 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1-x)$

Put $f'(x) = 0$

$12x^2(1-x) = 0$

$x = 0, 1$

Consider $x = 1$

$(1, 8)$

$f''(x) = 24x - 36x^2$

$f''(1) = -12 < 0 \Rightarrow$ CONCAVE DOWN i.e MAXIMUM TURNING POINT

$(1, 8)$ is MAXIMUM TURNING POINT

(ii) POINT OF INFLECTION

$f''(x) = 24x - 36x^2$

Put $f''(x) = 0$

$24x - 36x^2 = 0$

$12x(2-3x) = 0$

$x = 0, \frac{2}{3}$

Check $x = \frac{2}{3}$

$f''\left(\frac{1}{2}\right) = 3 > 0 \Rightarrow$ concave upwards

$f''\left(\frac{3}{4}\right) = -2\frac{1}{2} < 0$ OR $f''(1) = -12 < 0 \Rightarrow$ CONCAVE DOWNWARD

$\left(\frac{2}{3}, 7\frac{16}{27}\right)$ IS A POINT OF INFLECTION

(iii)