# HSC Mathematics Assessment Task 2
## March 2009

Time Allowed: 50 minutes + 2 minutes reading time

Instructions: Start each question on a new page.
Show all necessary working, writing on one side of the paper only.
Work down the page and do not work in columns.
Marks may not be awarded for untidy or poorly arranged work.

**Name:** _____

**Teacher:** _____ **Class:** _____

| | H5 | H6 | H7 | |
|---|---|---|---|---|
| **Question 1** | /10 | | | |
| **Question 2** | /9 | | | |
| **Question 3** | /11 | | | |
| **Question 4a)** | | /5 | | |
| **Question 4b)** | | | /4 | |
| **Question 5** | /8 | | | |
| **Totals** | /38 | /5 | /4 | /47 |
| | | | | % |

**Question 1 (10 marks)** **Marks**

(a) Write $\sum_{k=1}^{4} \frac{1}{k}$ as a series of terms and find its sum as a fraction. **2**

(b) Find $T_7$ if $T_n = 2(-1)^n (n+1)$ **1**

(c) The following terms are consecutive terms of an arithmetic series

$$2b-1,\ 2b+1,\ 3b-2$$

(i) Find the value of $b$. **2**

(ii) Hence, find the 3 terms of this series in numerical form. **1**

(d) Consider the following series

$$8-2-12-.....-412$$

(i) Show that this series has 43 terms. **2**

(ii) Find the sum of this series. **2**

**Question 2 (9 marks) Start a new page**

(a) Find the primitive function of $x^3 - 3x^2 + 3x - 1$. **3**

(b) (i) Calculate the probability of obtaining at least one head when a coin is tossed

($\alpha$) 5 times; **1**

($\beta$) $n$ times. **1**

(ii) How many times does the coin have to be tossed so that there is greater than a 99% probability that there is at least one head? **2**

(c) A drawer contains 12 chocolates in wrapping. Six chocolates have blue wrapping, four have green wrapping and two have red wrapping. Two chocolates are withdrawn without replacement. Find the probability that the chocolates have the same coloured wrapping. **2**

**Question 3 (11 marks) Start a new page** **Marks**

(a) Find the $n$th term of an arithmetic series in which the sixth term is 8 and the tenth term is 20. **3**

(b) Consider the general geometric series with first term $a$ and ratio $r$.

(i) Write down the series formed from the alternate terms of this series, starting with $a$. **1**

(ii) Show that the alternate terms of this geometric series form another geometric series. **1**

(c) Insert three numbers between $\frac{1}{2}$ and 648 so that all five numbers form consecutive terms of a geometric series. **3**

(d) A rubber ball is dropped from a height of 50 cm onto a concrete surface and continues to rebound vertically in a straight line to $\frac{4}{5}$ of it previous height.

(i) How high does it reach after the sixth bounce? [Answer to nearest centimetre] **2**

(ii) What distance has the ball travelled when it hits the concrete for the *third* time? **1**

**Question 4 (9 marks) Start a new page**

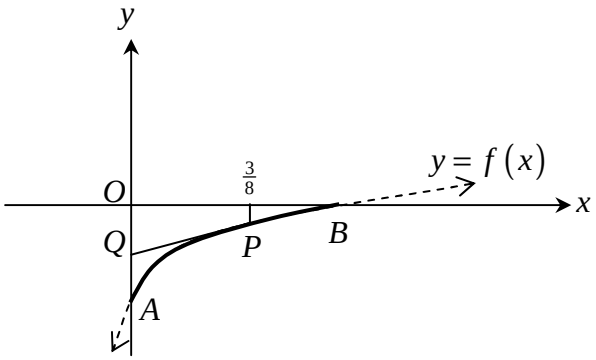
(a) The gradient of a particular curve varies inversely as the square of $x$, that is, $\frac{dy}{dx} = \frac{k}{x^2}$

(i) Find $y$ in terms of $x$, $k$ and the constant $c$. **2**

(ii) It is noted that the curve passes through the points $(1,3)$ and $(4,0)$. By forming simultaneous equations, express $y$ in terms of $x$. **3**

**Question 4 continued** **Marks**

(b) The curvature (concavity) of a section of the arch *AB* of a bridge with equation $y = f(x)$ is given by $\frac{d^2y}{dx^2} = -\sqrt{1-2x}$ where $x$ metres is the horizontal distance from *O* as shown below.




*AB* is the section of the arch.
*PQ* is the power cable.

(i) Show that the gradient function of the arch can be given by **3**

$$\frac{dy}{dx} = \frac{(1-2x)\sqrt{1-2x}}{3}$$

(ii) A power line *PQ*, as shown, connects point *Q* on the vertical axis tangentially to point *P* (at $x = \frac{3}{8}$). Find the slope of this power line. **1**

**Question 5 (8 marks) Start a new page**

(a) We note that $0\cdot\dot{2} = 0\cdot2 + 0\cdot02 + 0\cdot002 + \ldots$ and $0\cdot\dot{2}$ is approximated by $s$ where $s$ is given by $s = 0\cdot2 + 0\cdot02 + 0\cdot002 + \ldots$ to $n$ terms. **3**

Show that $s = \frac{2}{9}\left(1 - \frac{1}{10^n}\right)$

(b) A standard pack of 52 playing cards is used in a game where a card is selected and then replaced. Two people take alternate turns to select a card.
If a court card (Jack, King or Queen) is selected on a turn, the game is won.
If an Ace is selected on a turn, the game is lost. Otherwise, the game continues.

(i) Represent this situation for the first 2 turns using a probability tree. **3**

(ii) Find the probability that the game continues after the second turn. **1**

(iii) Find the probability that the game is lost on the third turn. **1**

**End of Paper**

## Question 1

T1 2009 [MATHEMATICS]

a) $\sum_{k=1}^{4} \frac{1}{k} = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
$= 2\frac{1}{12}$

b) $T_7 = 2(-1)^7(7+1)$
$= -16$

c) $2a-1, 2a+1, 3a-2$ ($T_1, T_2, T_3$)
$T_3 - T_2 = T_2 - T_1$
$(3a-2)-(2a+1) = (2a+1)-(2a-1)$
$3a-2-2a-1 = 2a+1-2a+1$
$a = 5$

$9, 11, 13$

d) (i) $a = 8$
$d = -10$
$T_n = -412$
$a+(n-1)d = -412$
$8+(n-1)-10 = -412$
$8-10n+10 = -412$
$10n = 412+18$
$10n = 430$
$n = 43$

(ii) $S_n = \frac{n}{2}(a+L)$
$S_{43} = \frac{43}{2}(8-412)$
$= -8686$

## Question 2

a) Let $f(x) = x^3 - 3x^2 + 3x - 1$
$F(x) = \frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - x + C$
or $f(x) = (x-1)^3$
$F(x) = \frac{(x-1)^4}{4} + C$

b) (i) $P(\text{at least one head}) = 1 - P(\text{5 tails})$
$= 1 - (\frac{1}{2})^5$
$= \frac{31}{32}$

(ii) $P(\text{at least one head}) = 1 - P(\text{no tails})$
$= 1 - (\frac{1}{2})^n$

$P(\text{at least one head}) \geq 0.99$
$1 - (\frac{1}{2})^n > 0.99$
$(\frac{1}{2})^n < 0.01$
$n \log_{10}(\frac{1}{2}) < \log_{10} 0.01$
$n > \frac{\log_{10} 0.01}{\log_{10} 0.5}$
$n > 6.6$
ie 7 times

(iii) $P(\text{2 chocolates same wrapping}) = \frac{1}{6}$


$P = \frac{1}{2}\cdot\frac{5}{11} + \frac{1}{3}\cdot\frac{3}{11} + \frac{1}{6}\cdot\frac{2}{11}$
$= \frac{1}{3}$

## Question 3

a) $T_6 = 8$
$T_{10} = 20$
$a+5d = 8$ (1)
$a+9d = 20$ (2)
(2)-(1)
$4d = 12$
$d = 3$
From (1)
$a+15 = 8$
$a = -7$
$T_n = a+(n-1)d$
$= -7+(n-1)3$
$= 3n-10$

b) Let the series be
$a + ar + ar^2 + ar^3 + ar^4 \ldots$
($T_1$ $T_2$ $T_3$ $T_4$ $T_5$)
Let the new series be
(i) $a + ar^2 + ar^4 + ar^6 + \ldots$
(ii) This is geometric as
$R = r^2$ (R for ratio)
$A = a$ (A for $T_1$)

c) $\frac{1}{2}, T_2, T_3, T_4, 648$
$a = \frac{1}{2}$
$ar^4 = 648$
$\frac{1}{2}r^4 = 648$
$r^4 = 1296$
$r = \pm 6$
$T_2 = \pm 3$, $T_3 = 18$, $T_4 = \pm 108$

d) (i) $a = 50$
$r = \frac{4}{5}$
$T_7 = ar^6$
$= 50 \times (\frac{4}{5})^6$
$= 13.1072\ldots$
$\approx 13$ cm

(ii) $T_n = 50(\frac{4}{5})^n$

50 $2T_2$ $2T_3$
$S = 50 + 2\times 50(\frac{4}{5}) + 2\times 50(\frac{4}{5})^2$
$= 50 + 80 + 64$
$= 194$

## Question 4

a) (i) $\frac{dy}{dx} = \frac{k}{x^2}$
$\frac{dy}{dx} = kx^{-2}$
$y = \frac{kx^{-1}}{-1} + C$
$y = -\frac{k}{x} + C$

(ii) When $x = 1$, $y = 3$
$3 = -k + C$ (1)
When $x = 4$, $y = 0$
$0 = -\frac{k}{4} + C$
$0 = -k + 4C$ (2)
(2) − (1)
$3C = -3$
$C = -1$
$k = -4$
$y = \frac{4}{x} - 1$

4(b)

(i) $y'' = -\sqrt{1-2x}$

$= -(1-2x)^{\frac{1}{2}}$

$y' = \frac{-2(1-2x)^{\frac{3}{2}}}{3 \times -2}$

$= \frac{(\sqrt{1-2x})^3}{3}$

$= \frac{\sqrt{1-2x}\,(1-2x)}{3}$

(ii) $x = \frac{3}{8}$

$y' = \frac{\sqrt{1-\frac{3}{4}}\,(1-\frac{3}{4})}{3}$

$= \frac{\frac{1}{2} \cdot \frac{1}{4}}{3}$

$= \frac{1}{24}$

b) The choices are
W (win)
L (lose)
C (continue)


(i) $P(CC) = \frac{9}{13} \cdot \frac{9}{13}$

(ii) $P(CL) = \frac{9}{13} \frac{9}{13} \times \frac{1}{13} = \frac{81}{169}$

$= \frac{81}{2197}$

Question 5

a) $0.2 = 0.2 + 0.02 + \ldots$

$S_n = 0.2 + 0.02 + 0.002 + \ldots$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{\frac{2}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}$

$= \frac{\frac{2}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{\frac{9}{10}}$

$= \frac{2}{9}\left(1-\left(\frac{1}{10}\right)^n\right)$